RESUMEN PROBLEMA DEL ARCO Y LA FLECHA

Flecha:	Aluminio Ligero	Madera	Aluminio Pesado
Peso [Kg]	0.023	0.020	0.031
TiempoEmpujeArco [s]	0.0126	0.0117	0.0146
VelocidadInicialFlecha [m/s]	61.9	66.5	53.3
VelocidadInicialFlecha [km/h]	222.97	239.11	192.05
TiempoVuelo [s]	8.975	9.614	7.742
DistanciaMáxima [m]	398.00	456.2	297.1
TiempoAlturaMaxima [s]	4.465	4.785	3.843
AltueaMaxima [m]	99.76	114.4	74.53

```
> restart
```

PROBLEMA DEL ARCO Y LA FLECHA

>

FLECHA DE ALUMNIO LIGERO

FASE DINÁMICA DEL PROBLEMA

```
> gravedad := 981/100; Hooke := (1825/100)/(5/10); Peso[aluminio] := 23/1000;
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$$gravedad := \frac{981}{100}$$

$$Hooke := \frac{73}{2}$$

$$Peso_{aluminio} := \frac{23}{1000} \quad \textbf{(1)}$$

```
> Ecuacion[dinamica] := diff(s(t), t, t)·(Peso[aluminio]/gravedad) = -Hooke·s(t);
```

$$Ecuacion_{dinamica} := \frac{23}{9810}\,\frac{d^2}{dt^2}s(t) = -\frac{73}{2}\,s(t) \quad \textbf{(2)}$$

```
> Condiciones := s(0) = -4964/10000, D(s)(0) = 0;
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$$Condiciones := s(0) = -\frac{1241}{2500}, D(s)(0) = 0 \quad \textbf{(3)}$$

```
> Solucion := dsolve({Ecuacion[dinamica], Condiciones}); evalf(%, 4);
```

$$Solucion := s(t) = -\frac{1241}{2500}\cos\left(\frac{3}{23}\sqrt{915055}\,t\right)$$

$$s(t) = -0.4964\cos(124.7\,t) \quad \textbf{(4)}$$

```
> tiempo[empuje] := solve(rhs(Solucion) = 0, t); evalf(%, 3);
```

$$tiempo_{empuje} := \frac{1}{238710}\pi\sqrt{915055}$$

$$0.0126 \quad \textbf{(5)}$$

```
> Velocidad[inicial] := subs(t = tiempo[empuje], rhs(diff(Solucion, t))); evalf(%, 3);
  evalf(%%·36/10, 5);
```

$$Velocidad_{inicial} := \frac{3723}{57500}\sin\left(\frac{1}{2}\pi\right)\sqrt{915055}$$

$$61.9$$

$$222.97 \quad \textbf{(6)}$$

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FASE CINEMÁTICA DEL PROBLEMA (TIRO PARABÓLICO)

```
> Velocidad[inicial];
```

$$\frac{3723}{57500}\sqrt{915055} \quad \textbf{(7)}$$

> $Ecuacion_{vertical} := diff(y(t), t, t) = -gravedad;\ Ecuacion_{horizontal} := diff(x(t), t) = Velocidad_{inicial} \cdot \cos\left(\frac{Pi}{4}\right);$

$$Ecuacion_{vertical} := \frac{d^2}{dt^2} y(t) = -\frac{981}{100}$$

$$Ecuacion_{horizontal} := \frac{d}{dt} x(t) = \frac{3723}{115000}\sqrt{915055}\sqrt{2} \quad \textbf{(8)}$$

> $Condicional_{verticales} := y(0) = 2, D(y)(0) = Velocidad_{inicial} \cdot \sin\left(\frac{Pi}{4}\right);$

$$Condicional_{verticales} := y(0) = 2, D(y)(0) = \frac{3723}{115000}\sqrt{915055}\sqrt{2} \quad \textbf{(9)}$$

> $Condicional_{horizontal} := x(0) = 5;$

$$Condicional_{horizontal} := x(0) = 5 \quad \textbf{(10)}$$

> $Solucion_{vertical} := dsolve(\{Ecuacion_{vertical}, Condicional_{verticales}\}); evalf(\%, 4);$

$$Solucion_{vertical} := y(t) = -\frac{981}{200} t^2 + \frac{3723}{115000}\sqrt{915055}\sqrt{2}\, t + 2$$

$$y(t) = -4.905\, t^2 + 43.80\, t + 2. \quad \textbf{(11)}$$

> $Solucion_{horizontal} := dsolve(\{Ecuacion_{horizontal}, Condicional_{horizontal}\}); evalf(\%, 4);$

$$Solucion_{horizontal} := x(t) = \frac{3723}{115000}\sqrt{1830110}\, t + 5$$

$$x(t) = 43.80\, t + 5. \quad \textbf{(12)}$$

> $tiempo_{vuelo} := solve(rhs(Solucion_{vertical}) = 0, t); evalf(\%, 4);$

$$tiempo_{vuelo} := \frac{1241}{376050}\sqrt{915055}\sqrt{2} - \frac{1}{376050}\sqrt{2876178638910}, \frac{1241}{376050}\sqrt{915055}\sqrt{2} + \frac{1}{376050}\sqrt{2876178638910}$$

$$-0.045, 8.975 \quad \textbf{(13)}$$

> $distancia_{maxima} := subs(t = tiempo_{vuelo}[2], rhs(Solucion_{horizontal})); evalf(\%, 4);$

$$distancia_{maxima} := \frac{3723}{115000}\sqrt{1830110}\left(\frac{1241}{376050}\sqrt{915055}\sqrt{2} + \frac{1}{376050}\sqrt{2876178638910}\right) + 5$$

$$398.0 \quad \textbf{(14)}$$

> $tiempo_{alturaMaxima} := solve(rhs(diff(Solucion_{vertical}, t)) = 0, t); evalf(\%, 4);$

$$tiempo_{alturaMaxima} := \frac{1241}{376050}\sqrt{915055}\sqrt{2}$$

$$4.465 \quad \textbf{(15)}$$

> $altura_{maxima} := subs(t = tiempo_{alturaMaxima}, rhs(Solucion_{vertical})); evalf(\%, 4);$

$$altura_{maxima} := \frac{114725913}{1150000}$$

$$99.76 \quad \textbf{(16)}$$

```
>
> restart
PROBLEMA DEL ARCO Y LA FLECHA
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FLECHA DE MADERA
FASE DINÁMICA DEL PROBLEMA
> gravedad := 981/100; Hooke := (1825/100)/(5/10); Peso_madera := 20/1000;
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$$gravedad := \frac{981}{100}$$

$$Hooke := \frac{73}{2}$$

$$Peso_{madera} := \frac{1}{50} \quad (17)$$

```
> Ecuacion_dinamica := diff(s(t), t, t)·(Peso_madera/gravedad) = -Hooke·s(t);
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$$Ecuacion_{dinamica} := \frac{2}{981}\,\frac{d^2}{dt^2}s(t) = -\frac{73}{2}\,s(t) \quad (18)$$

```
> Condiciones := s(0) = -4964/10000, D(s)(0) = 0;
```

$$Condiciones := s(0) = -\frac{1241}{2500},\ D(s)(0) = 0 \quad (19)$$

```
> Solucion := dsolve({Ecuacion_dinamica, Condiciones}); evalf(%, 4);
```

$$Solucion := s(t) = -\frac{1241}{2500}\cos\left(\frac{3}{2}\sqrt{7957}\,t\right)$$

$$s(t) = -0.4964\cos(133.8\,t) \quad (20)$$

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> tiempo_empuje := solve(rhs(Solucion) = 0, t); evalf(%, 3);
```

$$tiempo_{empuje} := \frac{1}{23871}\,\pi\sqrt{7957}$$

$$0.0117 \quad (21)$$

```
> Velocidad_inicial := subs(t = tiempo_empuje, rhs(diff(Solucion, t))); evalf(%, 3);
  evalf(%%·36/10, 5);
```

$$Velocidad_{inicial} := \frac{3723}{5000}\sin\left(\frac{1}{2}\,\pi\right)\sqrt{7957}$$

$$66.5$$

$$239.11 \quad (22)$$

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FASE CINEMÁTICA DEL PROBLEMA (TIRO PARABÓLICO)
> Velocidad_inicial;
```

$$(23)$$

$$\frac{3723}{5000}\sqrt{7957} \tag{23}$$

> $Ecuacion_{vertical} := diff(y(t), t, t) = -gravedad;\ Ecuacion_{horizontal} := diff(x(t), t) = Velocidad_{inicial}\cdot\cos\left(\frac{Pi}{4}\right);$

$$Ecuacion_{vertical} := \frac{d^2}{dt^2}y(t) = -\frac{981}{100}$$

$$Ecuacion_{horizontal} := \frac{d}{dt}x(t) = \frac{3723}{10000}\sqrt{7957}\sqrt{2} \tag{24}$$

> $Condicional_{verticales} := y(0) = 2, D(y)(0) = Velocidad_{inicial}\cdot\sin\left(\frac{Pi}{4}\right);$

$$Condicional_{verticales} := y(0) = 2, D(y)(0) = \frac{3723}{10000}\sqrt{7957}\sqrt{2} \tag{25}$$

> $Condicional_{horizontal} := x(0) = 5;$

$$Condicional_{horizontal} := x(0) = 5 \tag{26}$$

> $Solucion_{vertical} := dsolve(\{Ecuacion_{vertical}, Condicional_{verticales}\}); evalf(\%, 4);$

$$Solucion_{vertical} := y(t) = -\frac{981}{200}t^2 + \frac{3723}{10000}\sqrt{7957}\sqrt{2}\,t + 2$$

$$y(t) = -4.905\,t^2 + 46.95\,t + 2. \tag{27}$$

> $Solucion_{horizontal} := dsolve(\{Ecuacion_{horizontal}, Condicional_{horizontal}\}); evalf(\%, 4);$

$$Solucion_{horizontal} := x(t) = \frac{3723}{10000}\sqrt{15914}\,t + 5$$

$$x(t) = 46.95\,t + 5. \tag{28}$$

> $tiempo_{vuelo} := solve(rhs(Solucion_{vertical}) = 0, t); evalf(\%, 4);$

$$tiempo_{vuelo} := \frac{1241}{32700}\sqrt{7957}\sqrt{2} - \frac{1}{32700}\sqrt{24944849034}, \frac{1241}{32700}\sqrt{7957}\sqrt{2} + \frac{1}{32700}\sqrt{24944849034}$$

$$-0.044, 9.614 \tag{29}$$

> $distancia_{maxima} := subs(t = tiempo_{vuelo}[2], rhs(Solucion_{horizontal})); evalf(\%, 4);$

$$distancia_{maxima} := \frac{3723}{10000}\sqrt{15914}\left(\frac{1241}{32700}\sqrt{7957}\sqrt{2} + \frac{1}{32700}\sqrt{24944849034}\right) + 5$$

$$456.2 \tag{30}$$

> $tiempo_{alturaMaxima} := solve(rhs(diff(Solucion_{vertical}, t)) = 0, t); evalf(\%, 4);$

$$tiempo_{alturaMaxima} := \frac{1241}{32700}\sqrt{7957}\sqrt{2}$$

$$4.785 \tag{31}$$

> $altura_{maxima} := subs(t = tiempo_{alturaMaxima}, rhs(Solucion_{vertical})); evalf(\%, 4);$

$$altura_{maxima} := \frac{114425913}{1000000}$$

$$114.4 \tag{32}$$

>

> restart

PROBLEMA DEL ARCO Y LA FLECHA

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FLECHA DE ALUMNIO PESADO

FASE DINÁMICA DEL PROBLEMA

> $gravedad := \frac{981}{100}$; $Hooke := \frac{\left(\frac{1825}{100}\right)}{\left(\frac{5}{10}\right)}$; $Peso_{aluminioPesado} := \frac{31}{1000}$;

$$gravedad := \frac{981}{100}$$

$$Hooke := \frac{73}{2}$$

$$Peso_{aluminioPesado} := \frac{31}{1000} \quad \textbf{(33)}$$

> $Ecuacion_{dinamica} := diff(s(t), t, t) \cdot \left(\frac{Peso_{aluminioPesado}}{gravedad}\right) = -Hooke \cdot s(t)$;

$$Ecuacion_{dinamica} := \frac{31}{9810} \frac{d^2}{dt^2} s(t) = -\frac{73}{2} s(t) \quad \textbf{(34)}$$

> $Condiciones := s(0) = -\frac{4964}{10000}, D(s)(0) = 0$;

$$Condiciones := s(0) = -\frac{1241}{2500}, D(s)(0) = 0 \quad \textbf{(35)}$$

> $Solucion := dsolve(\{Ecuacion_{dinamica}, Condiciones\})$; $evalf(\%, 4)$;

$$Solucion := s(t) = -\frac{1241}{2500} \cos\left(\frac{3}{31}\sqrt{1233335}\, t\right)$$

$$s(t) = -0.4964 \cos(107.4\, t) \quad \textbf{(36)}$$

> $tiempo_{empuje} := solve(rhs(Solucion) = 0, t)$; $evalf(\%, 3)$;

$$tiempo_{empuje} := \frac{1}{238710} \pi \sqrt{1233335}$$

$$0.0146 \quad \textbf{(37)}$$

> $Velocidad_{inicial} := subs(t = tiempo_{empuje}, rhs(diff(Solucion, t)))$; $evalf(\%, 3)$;
$evalf\left(\frac{\%\% \cdot 36}{10}, 5\right)$;

$$Velocidad_{inicial} := \frac{3723}{77500} \sin\left(\frac{1}{2}\pi\right) \sqrt{1233335}$$

$$53.3$$

$$192.05 \quad \textbf{(38)}$$

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FASE CINEMÁTICA DEL PROBLEMA (TIRO PARABÓLICO)

> $Velocidad_{inicial}$;

(39)

$$\frac{3723}{77500}\sqrt{1233335} \tag{39}$$

> $Ecuacion_{vertical} := diff(y(t), t, t) = -gravedad;\ Ecuacion_{horizontal} := diff(x(t), t) = Velocidad_{inicial}\cdot\cos\left(\frac{Pi}{4}\right);$

$$Ecuacion_{vertical} := \frac{d^2}{dt^2}y(t) = -\frac{981}{100}$$

$$Ecuacion_{horizontal} := \frac{d}{dt}x(t) = \frac{3723}{155000}\sqrt{1233335}\sqrt{2} \tag{40}$$

> $Condicional_{verticales} := y(0) = 2, D(y)(0) = Velocidad_{inicial}\cdot\sin\left(\frac{Pi}{4}\right);$

$$Condicional_{verticales} := y(0) = 2, D(y)(0) = \frac{3723}{155000}\sqrt{1233335}\sqrt{2} \tag{41}$$

> $Condicional_{horizontal} := x(0) = 5;$

$$Condicional_{horizontal} := x(0) = 5 \tag{42}$$

> $Solucion_{vertical} := dsolve(\{Ecuacion_{vertical}, Condicional_{verticales}\}); evalf(\%, 4);$

$$Solucion_{vertical} := y(t) = -\frac{981}{200}t^2 + \frac{3723}{155000}\sqrt{1233335}\sqrt{2}\,t + 2$$

$$y(t) = -4.905\,t^2 + 37.71\,t + 2. \tag{43}$$

> $Solucion_{horizontal} := dsolve(\{Ecuacion_{horizontal}, Condicional_{horizontal}\}); evalf(\%, 4);$

$$Solucion_{horizontal} := x(t) = \frac{3723}{155000}\sqrt{2466670}\,t + 5$$

$$x(t) = 37.74\,t + 5. \tag{44}$$

> $tiempo_{vuelo} := solve(rhs(Solucion_{vertical}) = 0, t); evalf(\%, 4);$

$$tiempo_{vuelo} := \frac{1241}{506850}\sqrt{1233335}\sqrt{2} - \frac{1}{506850}\sqrt{3903620600270},\ \frac{1241}{506850}\sqrt{1233335}\sqrt{2} + \frac{1}{506850}\sqrt{3903620600270}$$

$$-0.056, 7.742 \tag{45}$$

> $distancia_{maxima} := subs(t = tiempo_{vuelo}[2], rhs(Solucion_{horizontal})); evalf(\%, 4);$

$$distancia_{maxima} := \frac{3723}{155000}\sqrt{2466670}\left(\frac{1241}{506850}\sqrt{1233335}\sqrt{2} + \frac{1}{506850}\sqrt{3903620600270}\right) + 5$$

$$297.1 \tag{46}$$

> $tiempo_{alturaMaxima} := solve(rhs(diff(Solucion_{vertical}, t)) = 0, t); evalf(\%, 4);$

$$tiempo_{alturaMaxima} := \frac{1241}{506850}\sqrt{1233335}\sqrt{2}$$

$$3.843 \tag{47}$$

> $altura_{maxima} := subs(t = tiempo_{alturaMaxima}, rhs(Solucion_{vertical})); evalf(\%, 4);$

$$altura_{maxima} := \frac{115525913}{1550000}$$

$$74.53 \qquad \textbf{(48)}$$

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