

> restart

SOLUCIONES GENERALES, PARTICULARES Y FUNDAMENTALES

> SolucionGeneral := y(x) = C1·exp(x) + C2·exp(2·x) + C3·exp(3·x) + C4·exp(-2·x);

$$\text{SolucionGeneral} := y(x) = C1 e^x + C2 e^{2x} + C3 e^{3x} + C4 e^{-2x}$$

(1)

> SolFund[1] := exp(x); SolFund[2] := exp(2·x); SolFund[3] := exp(3·x); SolFund[4] := exp(-2·x);

$$\text{SolFund}_1 := e^x$$

$$\text{SolFund}_2 := e^{2x}$$

$$\text{SolFund}_3 := e^{3x}$$

$$\text{SolFund}_4 := e^{-2x}$$

(2)

> WW := array([[SolFund₁, SolFund₂, SolFund₃, SolFund₄], [diff(SolFund₁, x), diff(SolFund₂, x), diff(SolFund₃, x), diff(SolFund₄, x)], [diff(SolFund₁, x\$2), diff(SolFund₂, x\$2), diff(SolFund₃, x\$2), diff(SolFund₄, x\$2)], [diff(SolFund₁, x\$3), diff(SolFund₂, x\$3), diff(SolFund₃, x\$3), diff(SolFund₄, x\$3)]]);

$$WW := \begin{bmatrix} e^x & e^{2x} & e^{3x} & e^{-2x} \\ e^x & 2e^{2x} & 3e^{3x} & -2e^{-2x} \\ e^x & 4e^{2x} & 9e^{3x} & 4e^{-2x} \\ e^x & 8e^{2x} & 27e^{3x} & -8e^{-2x} \end{bmatrix}$$

(3)

> with(linalg) :

> WWW := wronskian([SolFund₁, SolFund₂, SolFund₃, SolFund₄], x);

$$WWW := \begin{bmatrix} e^x & e^{2x} & e^{3x} & e^{-2x} \\ e^x & 2e^{2x} & 3e^{3x} & -2e^{-2x} \\ e^x & 4e^{2x} & 9e^{3x} & 4e^{-2x} \\ e^x & 8e^{2x} & 27e^{3x} & -8e^{-2x} \end{bmatrix}$$

(4)

> comprobacion := det(WWW) ≠ 0;

$$\text{comprobacion} := -120 e^x e^{2x} e^{3x} e^{-2x} \neq 0$$

(5)

> SolucionGeneral;

$$y(x) = C1 e^x + C2 e^{2x} + C3 e^{3x} + C4 e^{-2x}$$

(6)

> Sistema := diff(SolucionGeneral, x), diff(SolucionGeneral, x\$2), diff(SolucionGeneral, x\$3), diff(SolucionGeneral, x\$4) : Sistema₁; Sistema₂; Sistema₃; Sistema₄;

$$\frac{d}{dx} y(x) = C1 e^x + 2 C2 e^{2x} + 3 C3 e^{3x} - 2 C4 e^{-2x}$$

$$\frac{d^2}{dx^2} y(x) = C1 e^x + 4 C2 e^{2x} + 9 C3 e^{3x} + 4 C4 e^{-2x}$$

$$\frac{d^3}{dx^3} y(x) = C1 e^x + 8 C2 e^{2x} + 27 C3 e^{3x} - 8 C4 e^{-2x}$$

$$\frac{d^4}{dx^4} y(x) = C1 e^x + 16 C2 e^{2x} + 81 C3 e^{3x} + 16 C4 e^{-2x} \quad (7)$$

> SOL := solve({Sistema}, {C1, C2, C3, C4}) : SOL₁; SOL₂; SOL₃; SOL₄;

$$C1 = \frac{1}{6} \frac{-4 \left(\frac{d^2}{dx^2} y(x) \right) + 12 \left(\frac{d}{dx} y(x) \right) - 3 \left(\frac{d^3}{dx^3} y(x) \right) + \frac{d^4}{dx^4} y(x)}{e^x}$$

$$C2 = -\frac{1}{8} \frac{-5 \left(\frac{d^2}{dx^2} y(x) \right) + 6 \left(\frac{d}{dx} y(x) \right) - 2 \left(\frac{d^3}{dx^3} y(x) \right) + \frac{d^4}{dx^4} y(x)}{e^{2x}}$$

$$C3 = \frac{1}{30} \frac{-4 \left(\frac{d^2}{dx^2} y(x) \right) + 4 \left(\frac{d}{dx} y(x) \right) - \left(\frac{d^3}{dx^3} y(x) \right) + \frac{d^4}{dx^4} y(x)}{e^{3x}}$$

$$C4 = -\frac{1}{120} \frac{-11 \left(\frac{d^2}{dx^2} y(x) \right) + 6 \left(\frac{d}{dx} y(x) \right) + 6 \left(\frac{d^3}{dx^3} y(x) \right) - \left(\frac{d^4}{dx^4} y(x) \right)}{e^{-2x}} \quad (8)$$

> SolucionGeneral;

$$y(x) = C1 e^x + C2 e^{2x} + C3 e^{3x} + C4 e^{-2x} \quad (9)$$

> EcuacionIntermedia := subs(C1 = rhs(SOL₁), C2 = rhs(SOL₂), C3 = rhs(SOL₃), C4 = rhs(SOL₄), lhs(SolucionGeneral) - rhs(SolucionGeneral) = 0);

$$EcuacionIntermedia := y(x) + \frac{1}{12} \frac{d^2}{dx^2} y(x) - \frac{4}{3} \frac{d}{dx} y(x) + \frac{1}{3} \frac{d^3}{dx^3} y(x) - \frac{1}{12} \frac{d^4}{dx^4} y(x) = 0 \quad (10)$$

> EcuacionDiferencialFinal := lhs(EcuacionIntermedia) · (-12) = 0;

$$EcuacionDiferencialFinal := -12 y(x) - \left(\frac{d^2}{dx^2} y(x) \right) + 16 \left(\frac{d}{dx} y(x) \right) - 4 \left(\frac{d^3}{dx^3} y(x) \right) + \frac{d^4}{dx^4} y(x) = 0 \quad (11)$$

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