

> restart :

METODO DE PARAMETROS VARIABLES

> Ecuacion := diff(y(x), x\$2) - 3\*diff(y(x), x) + 2\*y(x) = 10\*exp(3\*x) - 18\*exp(4\*x);

$$Ecuacion := \frac{d^2}{dx^2} y(x) - 3 \left( \frac{d}{dx} y(x) \right) + 2 y(x) = 10 e^{3x} - 18 e^{4x} \quad (1)$$

> EcuacionHomogenea := lhs(Ecuacion) = 0;

$$EcuacionHomogenea := \frac{d^2}{dx^2} y(x) - 3 \left( \frac{d}{dx} y(x) \right) + 2 y(x) = 0 \quad (2)$$

> Q(x) := rhs(Ecuacion);

$$Q(x) := 10 e^{3x} - 18 e^{4x} \quad (3)$$

> EcuacionCaracteristica := m\*2 - 3\*m + 2 = 0;

$$EcuacionCaracteristica := m^2 - 3m + 2 = 0 \quad (4)$$

> Raiz := solve(EcuacionCaracteristica);

$$Raiz := 2, 1 \quad (5)$$

> Sol1 := exp(Raiz1\*x); Sol2 := exp(Raiz2\*x);

$$Sol_1 := e^{2x}$$

$$Sol_2 := e^x$$

(6)

> SolucionHomogenea := y(x) = C1\*Sol1 + C2\*Sol2;

$$SolucionHomogenea := y(x) = C1 e^{2x} + C2 e^x \quad (7)$$

> SolucionNoHomogenea := y(x) = A(x)\*Sol1 + B(x)\*Sol2;

$$SolucionNoHomogenea := y(x) = A(x) e^{2x} + B(x) e^x \quad (8)$$

> with(linalg) :

> AA := wronskian([Sol1, Sol2], x);

$$AA := \begin{bmatrix} e^{2x} & e^x \\ 2e^{2x} & e^x \end{bmatrix} \quad (9)$$

> BB := array([0, Q(x)]);

$$BB := \begin{bmatrix} 0 & 10 e^{3x} - 18 e^{4x} \end{bmatrix} \quad (10)$$

> SOL := linsolve(AA, BB);

$$SOL := \begin{bmatrix} \frac{2(5e^{3x} - 9e^{4x})}{e^{2x}} & -\frac{2(5e^{3x} - 9e^{4x})}{e^x} \end{bmatrix} \quad (11)$$

> Aprima := SOL1; Bprima := SOL2;

$$Aprima := \frac{2(5e^{3x} - 9e^{4x})}{e^{2x}}$$

$$Bprima := -\frac{2(5e^{3x} - 9e^{4x})}{e^x} \quad (12)$$

> A(x) := int(Aprima, x) + C1; B(x) := int(Bprima, x) + C2;

$$A(x) := 10 e^x - 9 (e^x)^2 + C1$$

$$B(x) := -5 (e^x)^2 + 6 (e^x)^3 + C2 \quad (13)$$

$$\begin{aligned} &> \text{SolucionFinal} := \text{simplify}(\text{expand}(\text{SolucionNoHomogenea})); \\ &\text{SolucionFinal} := y(x) = 5 e^{3x} - 3 e^{4x} + C1 e^{2x} + C2 e^x \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{restart}; \\ &> \text{Ecuacion} := \text{diff}(y(x), x\$3) - 27 \cdot y(x) = 8 \cdot \exp(2 \cdot x) + 4 \cdot x \cdot 2; \\ &\text{Ecuacion} := \frac{d^3}{dx^3} y(x) - 27 y(x) = 8 e^{2x} + 4 x^2 \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{EcuacionHomogenea} := \text{lhs}(\text{Ecuacion}) = 0; \\ &\text{EcuacionHomogenea} := \frac{d^3}{dx^3} y(x) - 27 y(x) = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} &> Q(x) := \text{rhs}(\text{Ecuacion}); \\ &Q(x) := 8 e^{2x} + 4 x^2 \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{EcuacionCaracteristica} := m \cdot 3 - 27 = 0; \\ &\text{EcuacionCaracteristica} := m^3 - 27 = 0 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{Raiz} := \text{solve}(\text{EcuacionCaracteristica}); \\ &\text{Raiz} := 3, -\frac{3}{2} + \frac{3}{2} i \sqrt{3}, -\frac{3}{2} - \frac{3}{2} i \sqrt{3} \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{Sol}_1 := \exp(\text{Raiz}_1 \cdot x); \\ &\text{Sol}_1 := e^{3x} \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{Sol}_2 := \exp(\text{Re}(\text{Raiz}_2) \cdot x) \cdot \cos(\text{Im}(\text{Raiz}_2) \cdot x); \\ &\text{Sol}_2 := e^{-\frac{3}{2}x} \cos\left(\frac{3}{2} \sqrt{3} x\right) \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{Sol}_3 := \exp(\text{Re}(\text{Raiz}_2) \cdot x) \cdot \sin(\text{Im}(\text{Raiz}_2) \cdot x); \\ &\text{Sol}_3 := e^{-\frac{3}{2}x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{SolucionHomogenea} := y(x) = C1 \cdot \text{Sol}_1 + C2 \cdot \text{Sol}_2 + C3 \cdot \text{Sol}_3; \\ &\text{SolucionHomogenea} := y(x) = C1 e^{3x} + C2 e^{-\frac{3}{2}x} \cos\left(\frac{3}{2} \sqrt{3} x\right) + C3 e^{-\frac{3}{2}x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{SolucionNoHomogenea} := y(x) = A(x) \cdot \text{Sol}_1 + B(x) \cdot \text{Sol}_2 + D(x) \cdot \text{Sol}_3; \\ &\text{SolucionNoHomogenea} := y(x) = A(x) e^{3x} + B(x) e^{-\frac{3}{2}x} \cos\left(\frac{3}{2} \sqrt{3} x\right) \\ &\quad + D(x) e^{-\frac{3}{2}x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{with}(\text{linalg}) : \\ &> \text{AA} := \text{wronskian}([ \text{Sol}_1, \text{Sol}_2, \text{Sol}_3 ], x); \end{aligned}$$

$$AA := \left[ \left[ e^{3x}, e^{-\frac{3}{2}x} \cos\left(\frac{3}{2}\sqrt{3}x\right), e^{-\frac{3}{2}x} \sin\left(\frac{3}{2}\sqrt{3}x\right) \right], \right. \\ \left[ 3e^{3x}, -\frac{3}{2}e^{-\frac{3}{2}x} \cos\left(\frac{3}{2}\sqrt{3}x\right) - \frac{3}{2}e^{-\frac{3}{2}x} \sin\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}, \right. \\ \left. -\frac{3}{2}e^{-\frac{3}{2}x} \sin\left(\frac{3}{2}\sqrt{3}x\right) + \frac{3}{2}e^{-\frac{3}{2}x} \cos\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3} \right], \\ \left[ 9e^{3x}, -\frac{9}{2}e^{-\frac{3}{2}x} \cos\left(\frac{3}{2}\sqrt{3}x\right) + \frac{9}{2}e^{-\frac{3}{2}x} \sin\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}, \right. \\ \left. -\frac{9}{2}e^{-\frac{3}{2}x} \sin\left(\frac{3}{2}\sqrt{3}x\right) - \frac{9}{2}e^{-\frac{3}{2}x} \cos\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3} \right] \Big] \quad (25)$$

>  $BB := \text{array}([ [0], [0], [Q(x)] ]);$

$$BB := \begin{bmatrix} 0 \\ 0 \\ 8e^{2x} + 4x^2 \end{bmatrix} \quad (26)$$

>  $SOL := \text{linsolve}(AA, BB);$

$$SOL := \begin{bmatrix} \frac{4}{27} \frac{2e^{2x} + x^2}{e^{3x}} \\ -\frac{4}{81} \frac{\left(-3\sin\left(\frac{3}{2}\sqrt{3}x\right) + \cos\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}\right)\sqrt{3}(2e^{2x} + x^2)}{e^{-\frac{3}{2}x}\left(\cos\left(\frac{3}{2}\sqrt{3}x\right)^2 + \sin\left(\frac{3}{2}\sqrt{3}x\right)^2\right)} \\ -\frac{4}{81} \frac{\sqrt{3}\left(3\cos\left(\frac{3}{2}\sqrt{3}x\right) + \sin\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}\right)(2e^{2x} + x^2)}{e^{-\frac{3}{2}x}\left(\cos\left(\frac{3}{2}\sqrt{3}x\right)^2 + \sin\left(\frac{3}{2}\sqrt{3}x\right)^2\right)} \end{bmatrix} \quad (27)$$

>  $Aprima := SOL_{1,1};$

$$Aprima := \frac{4}{27} \frac{2e^{2x} + x^2}{e^{3x}} \quad (28)$$

>  $Bprima := \text{simplify}(\text{expand}(SOL_{2,1}));$

$$Bprima := \frac{4}{27} \left( 2\sin\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}e^{2x} + \sin\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}x^2 - 2\cos\left(\frac{3}{2}\sqrt{3}x\right)e^{2x} \right. \\ \left. - \cos\left(\frac{3}{2}\sqrt{3}x\right)x^2 \right) e^{\frac{3}{2}x} \quad (29)$$

>  $Dprima := \text{simplify}(\text{expand}(SOL_{3,1}));$

$$Dprima := -\frac{4}{27} \left( 2\cos\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}e^{2x} + \cos\left(\frac{3}{2}\sqrt{3}x\right)\sqrt{3}x^2 + 2\sin\left(\frac{3}{2}\sqrt{3}x\right)e^{2x} \right. \\ \left. - \sin\left(\frac{3}{2}\sqrt{3}x\right)x^2 \right) e^{\frac{3}{2}x} \quad (30)$$

$$+ \sin\left(\frac{3}{2} \sqrt{3} x\right) x^2 \right) e^{\frac{3}{2} x}$$

>  $A(x) := \text{int}(A_{\text{prima}}, x) + C1;$

$$A(x) := -\frac{8}{27 e^x} - \frac{4}{81} \frac{x^2}{(e^x)^3} - \frac{8}{243} \frac{x}{(e^x)^3} - \frac{8}{729 (e^x)^3} + C1 \quad (31)$$

>  $B(x) := \text{int}(B_{\text{prima}}, x) + C2;$

$$B(x) := \frac{8}{27} \sqrt{3} \left( -\frac{3}{38} \sqrt{3} e^{\frac{7}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) + \frac{7}{38} \sin\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} \right) + \frac{4}{27} \sqrt{3} \left( \left( -\frac{1}{6} \sqrt{3} x^2 + \frac{1}{9} \sqrt{3} x \right) e^{\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) + \left( \frac{1}{6} x^2 + \frac{1}{9} x - \frac{2}{27} \right) e^{\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \right) - \frac{28}{513} \cos\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} - \frac{4}{171} \sqrt{3} \sin\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} - \frac{4}{27} \left( \frac{1}{6} x^2 + \frac{1}{9} x - \frac{2}{27} \right) e^{\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) + \frac{4}{27} \left( -\frac{1}{6} \sqrt{3} x^2 + \frac{1}{9} \sqrt{3} x \right) e^{\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) + C2 \quad (32)$$

>  $D(x) := \text{int}(D_{\text{prima}}, x) + C3;$

$$D(x) := -\frac{8}{27} \sqrt{3} \left( \frac{7}{38} \cos\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} + \frac{3}{38} \sqrt{3} \sin\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} \right) - \frac{4}{27} \sqrt{3} \left( \left( \frac{1}{6} x^2 + \frac{1}{9} x - \frac{2}{27} \right) e^{\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) - \left( -\frac{1}{6} \sqrt{3} x^2 + \frac{1}{9} \sqrt{3} x \right) e^{\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \right) + \frac{4}{171} \sqrt{3} e^{\frac{7}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) - \frac{28}{513} \sin\left(\frac{3}{2} \sqrt{3} x\right) e^{\frac{7}{2} x} - \frac{4}{27} \left( -\frac{1}{6} \sqrt{3} x^2 + \frac{1}{9} \sqrt{3} x \right) e^{\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) - \frac{4}{27} \left( \frac{1}{6} x^2 + \frac{1}{9} x - \frac{2}{27} \right) e^{\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) + C3 \quad (33)$$

>  $\text{SolucionFinal} := \text{simplify}(\text{expand}(\text{SolucionNoHomogenea}));$

$$\text{SolucionFinal} := y(x) = -\frac{8}{19} e^{2x} - \frac{4}{27} x^2 + C1 e^{3x} + C2 e^{-\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) + C3 e^{-\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \quad (34)$$

>  $\text{EcuacionUltima} := \text{dsolve}(\text{Ecuacion});$

$$\text{EcuacionUltima} := y(x) = -\frac{8}{19} e^{2x} - \frac{4}{27} x^2 + \_C1 e^{3x} + \_C2 e^{-\frac{3}{2} x} \cos\left(\frac{3}{2} \sqrt{3} x\right) \quad (35)$$

$$\left[ \begin{array}{l} +_C 3 e^{-\frac{3}{2} x} \sin\left(\frac{3}{2} \sqrt{3} x\right) \end{array} \right]$$