

> restart :

PROBLEMA DEL ARCO Y FLECHA CON LOS MÉTODOS DEL PROGRAMA DE ESTUDIOS

$$\begin{aligned} > \text{gravedad} := \frac{981}{100}; \text{Peso}_{\text{flecha}} := \frac{23}{1000}; \text{Hooke} := \frac{\left(\frac{1825}{100}\right)}{\left(\frac{5}{10}\right)}; \\ &\quad \text{gravedad} := \frac{981}{100} \\ &\quad \text{Peso}_{\text{flecha}} := \frac{23}{1000} \\ &\quad \text{Hooke} := \frac{73}{2} \end{aligned} \quad (1)$$

$$\begin{aligned} > \text{Ecuacion} := \text{diff}(s(t), t\$2) + \left(\frac{\text{Hooke} \cdot \text{gravedad}}{\text{Peso}_{\text{flecha}}}\right) \cdot s(t) = 0; \\ &\quad \text{Ecuacion} := \frac{d^2}{dt^2} s(t) + \frac{358065}{23} s(t) = 0 \end{aligned} \quad (2)$$

$$\begin{aligned} > \text{EcuacionCaracteristica} := m \cdot 2 + \left(\frac{\text{Hooke} \cdot \text{gravedad}}{\text{Peso}_{\text{flecha}}}\right) = 0; \\ &\quad \text{EcuacionCaracteristica} := m^2 + \frac{358065}{23} = 0 \end{aligned} \quad (3)$$

$$\begin{aligned} > \text{Raiz} := \text{solve}(\text{EcuacionCaracteristica}); \\ &\quad \text{Raiz} := \frac{3}{23} \text{I} \sqrt{915055}, -\frac{3}{23} \text{I} \sqrt{915055} \end{aligned} \quad (4)$$

$$\begin{aligned} > \text{Sol}_1 := \cos(\text{Im}(\text{Raiz}_1) \cdot t); \text{Sol}_2 := \sin(\text{Im}(\text{Raiz}_1) \cdot t); \\ &\quad \text{Sol}_1 := \cos\left(\frac{3}{23} \sqrt{915055} t\right) \\ &\quad \text{Sol}_2 := \sin\left(\frac{3}{23} \sqrt{915055} t\right) \end{aligned} \quad (5)$$

$$\begin{aligned} > \text{SolucionGeneral} := s(t) = C1 \cdot \text{Sol}_1 + C2 \cdot \text{Sol}_2; \\ &\quad \text{SolucionGeneral} := s(t) = C1 \cos\left(\frac{3}{23} \sqrt{915055} t\right) + C2 \sin\left(\frac{3}{23} \sqrt{915055} t\right) \end{aligned} \quad (6)$$

$$\begin{aligned} > \text{Condiciones} := s(0) = -\frac{4964}{10000}, D(s)(0) = 0; \\ &\quad \text{Condiciones} := s(0) = -\frac{1241}{2500}, D(s)(0) = 0 \end{aligned} \quad (7)$$

$$\begin{aligned} > \text{sistema} := \text{subs}\left(t=0, \text{rhs}(\text{SolucionGeneral}) = -\frac{4964}{10000}\right), \text{subs}(t=0, \\ &\quad \text{rhs}(\text{diff}(\text{SolucionGeneral}, t)) = 0) : \text{sistema}_1; \text{sistema}_2; \\ &\quad C1 = -\frac{1241}{2500} \\ &\quad \frac{3}{23} C2 \sqrt{915055} = 0 \end{aligned} \quad (8)$$

$$> \text{SOL} := \text{solve}(\{\text{sistema}\}, \{C1, C2\});$$

$$SOL := \left\{ C1 = -\frac{1241}{2500}, C2 = 0 \right\} \quad (9)$$

> *SolucionParticular* := subs(*C1* = rhs(*SOL*<sub>1</sub>), *C2* = rhs(*SOL*<sub>2</sub>), *SolucionGeneral*);

$$SolucionParticular := s(t) = -\frac{1241}{2500} \cos\left(\frac{3}{23} \sqrt{915055} t\right) \quad (10)$$

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COMPROBACION

> *SolucionDinamica* := dsolve({*Ecuacion*, *Condiciones*});

$$SolucionDinamica := s(t) = -\frac{1241}{2500} \cos\left(\frac{3}{23} \sqrt{915055} t\right) \quad (11)$$

CALCULAR TIEMPO Y VELOCIDAD IMPULSADA

> *tiempo\_impulso* := solve(rhs(*SolucionParticular*) = 0, *t*); evalf(%, 4);

$$tiempo_{impulso} := \frac{1}{238710} \pi \sqrt{915055} \\ 0.01259 \quad (12)$$

> *VelocidadSalida* := subs(*t* = *tiempo\_impulso*, rhs(diff(*SolucionParticular*, *t*))); evalf(%, 4);  
evalf(%%, 4) · 3.6;

$$VelocidadSalida := \frac{3723}{57500} \sin\left(\frac{1}{2} \pi\right) \sqrt{915055} \\ 61.94 \\ 222.984 \quad (13)$$

RESOLVER EL TIRO PARABÓLICO

> *EcuacionVertical* := diff(*y*(*t*), t\$2) = -*gravedad*;

$$EcuacionVertical := \frac{d^2}{dt^2} y(t) = -\frac{981}{100} \quad (14)$$

> *EcuacionVerticalHomogenea* := lhs(*EcuacionVertical*) = 0; *Q*(*t*) := rhs(*EcuacionVertical*);

$$EcuacionVerticalHomogenea := \frac{d^2}{dt^2} y(t) = 0$$

$$Q(t) := -\frac{981}{100} \quad (15)$$

> *EcuacionVerticalCaracteristica* := *m* · 2 = 0;

$$EcuacionVerticalCaracteristica := m^2 = 0 \quad (16)$$

> *RaizVertical* := solve(*EcuacionVerticalCaracteristica*);

$$RaizVertical := 0, 0 \quad (17)$$

> *SolVert*<sub>1</sub> := t · exp(*RaizVertical*<sub>1</sub> · t); *SolVert*<sub>2</sub> := exp(*RaizVertical*<sub>2</sub> · t);

$$SolVert_1 := t$$

$$SolVert_2 := 1 \quad (18)$$

> *SolucionVerticalHomogenea* := *y*(*t*) = *C3* · *SolVert*<sub>1</sub> + *C4* · *SolVert*<sub>2</sub>;

$$SolucionVerticalHomogenea := y(t) = C3 t + C4 \quad (19)$$

> *SolucionVerticalNoHomogenea* := *y*(*t*) = *A*(*t*) · *SolVert*<sub>1</sub> + *B*(*t*) · *SolVert*<sub>2</sub>;

$$SolucionVerticalNoHomogenea := y(t) = A(t) t + B(t) \quad (20)$$

$$\begin{aligned}
&> \text{with(linalg)} : \\
&> AA := \text{wronskian}([SolVert_1, SolVert_2], t); BB := \text{array}([0, Q(t)]); \\
&\quad AA := \begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix} \\
&\quad BB := \begin{bmatrix} 0 & -\frac{981}{100} \end{bmatrix} \tag{21}
\end{aligned}$$

$$\begin{aligned}
&> SOLvertical := \text{linsolve}(AA, BB); \\
&\quad SOLvertical := \begin{bmatrix} -\frac{981}{100} & \frac{981}{100} t \end{bmatrix} \tag{22}
\end{aligned}$$

$$\begin{aligned}
&> Aprima := SOLvertical_1; Bprima := SOLvertical_2; \\
&\quad Aprima := -\frac{981}{100} \\
&\quad Bprima := \frac{981}{100} t \tag{23}
\end{aligned}$$

$$\begin{aligned}
&> A(t) := \text{int}(Aprima, t) + C3; B(t) := \text{int}(Bprima, t) + C4; \\
&\quad A(t) := -\frac{981}{100} t + C3 \\
&\quad B(t) := \frac{981}{200} t^2 + C4 \tag{24}
\end{aligned}$$

$$\begin{aligned}
&> SolucionVerticalNoHomogenea; \\
&\quad y(t) = \left( -\frac{981}{100} t + C3 \right) t + \frac{981}{200} t^2 + C4 \tag{25}
\end{aligned}$$

$$\begin{aligned}
&> SolucionVerticalGeneral := \text{simplify}(SolucionVerticalNoHomogenea); \\
&\quad SolucionVerticalGeneral := y(t) = -\frac{981}{200} t^2 + C3 t + C4 \tag{26}
\end{aligned}$$

$$\begin{aligned}
&> CondicionesVerticales := y(0) = 2, D(y)(0) = \text{VelocidadSalida} \cdot \sin\left(\frac{\text{Pi}}{4}\right); \\
&\quad CondicionesVerticales := y(0) = 2, D(y)(0) = \frac{3723}{115000} \sqrt{915055} \sqrt{2} \tag{27}
\end{aligned}$$

$$\begin{aligned}
&> sistemaVertical := \text{subs}(t=0, \text{rhs}(SolucionVerticalGeneral) = 2), \text{subs}\left(t=0, \right. \\
&\quad \left. \text{rhs}(\text{diff}(SolucionVerticalGeneral, t)) = \text{VelocidadSalida} \cdot \sin\left(\frac{\text{Pi}}{4}\right)\right) : sistemaVertical_1; \\
&\quad sistemaVertical_2; \\
&\quad C4 = 2 \\
&\quad C3 = \frac{3723}{115000} \sqrt{915055} \sqrt{2} \tag{28}
\end{aligned}$$

$$\begin{aligned}
&> SolucionVerticalParticular := \text{subs}(C3 = \text{rhs}(sistemaVertical_2), C4 = \text{rhs}(sistemaVertical_1), \\
&\quad SolucionVerticalGeneral); \text{evalf}(\%, 4); \\
&\quad SolucionVerticalParticular := y(t) = -\frac{981}{200} t^2 + \frac{3723}{115000} \sqrt{915055} \sqrt{2} t + 2 \tag{29}
\end{aligned}$$

$$y(t) = -4.905 t^2 + 43.80 t + 2. \quad (29)$$

## COMPROBACIÓN 2

> *SolucionFinalVertical* := dsolve( {*EcuacionVertical*, *CondicionesVerticales*} );

$$\text{SolucionFinalVertical} := y(t) = -\frac{981}{200} t^2 + \frac{3723}{115000} \sqrt{915055} \sqrt{2} t + 2 \quad (30)$$

## CONTINUAR TIRO PARABÓLICO

> *EcuacionHorizontal* := diff( *x(t)*, *t* ) = *VelocidadSalida* · cos(  $\frac{\text{Pi}}{4}$  );

$$\text{EcuacionHorizontal} := \frac{d}{dt} x(t) = \frac{3723}{115000} \sqrt{915055} \sqrt{2} \quad (31)$$

> *p(t)* := 0; *q(t)* := rhs(*EcuacionHorizontal*);  
*p(t)* := 0

$$q(t) := \frac{3723}{115000} \sqrt{915055} \sqrt{2} \quad (32)$$

> *SolucionGeneralHorizontal* := *x(t)* = C5 · exp( -int( *p(t)*, *t* ) ) + exp( -int( *p(t)*, *t* ) ) · int( exp( int( *p(t)*, *t* ) ) · *q(t)*, *t* )

$$\text{SolucionGeneralHorizontal} := x(t) = C5 + \frac{3723}{115000} \sqrt{915055} \sqrt{2} t \quad (33)$$

> *CondicionesHorizontales* := *x(0)* = 5;

$$\text{CondicionesHorizontales} := x(0) = 5 \quad (34)$$

> *parametro* := subs( *t* = 0, rhs( *SolucionGeneralHorizontal* ) ) = 5 )

$$\text{parametro} := C5 = 5 \quad (35)$$

> *SolucionParticularHorizontal* := subs( C5 = rhs( *parametro* ), *SolucionGeneralHorizontal* );

$$\text{SolucionParticularHorizontal} := x(t) = 5 + \frac{3723}{115000} \sqrt{915055} \sqrt{2} t \quad (36)$$

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## COMPROBACION 3

> *SolucionFinalHorizontal* := dsolve( {*EcuacionHorizontal*, *CondicionesHorizontales*} );

$$\text{SolucionFinalHorizontal} := x(t) = \frac{3723}{115000} \sqrt{1830110} t + 5 \quad (37)$$

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## CÁLCULO FINAL RESULTADOS

> *TiempoVuelo* := solve( rhs( *SolucionVerticalParticular* ) = 0, *t* ); evalf( %, 4 );

$$\begin{aligned} \text{TiempoVuelo} := & \frac{1241}{376050} \sqrt{915055} \sqrt{2} - \frac{1}{376050} \sqrt{2876178638910}, \\ & \frac{1241}{376050} \sqrt{915055} \sqrt{2} + \frac{1}{376050} \sqrt{2876178638910} \\ & -0.045, 8.975 \end{aligned} \quad (38)$$

> *DistanciaMaxima* := subs( *t* = *TiempoVuelo*<sub>2</sub>, rhs( *SolucionParticularHorizontal* ) ); evalf( %, 4 );

$$\begin{aligned} \text{DistanciaMaxima} := & 5 + \frac{3723}{115000} \sqrt{915055} \sqrt{2} \left( \frac{1241}{376050} \sqrt{915055} \sqrt{2} \right. \\ & \left. + \frac{1}{376050} \sqrt{2876178638910} \right) \end{aligned}$$

398.0

(39)

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> plot([rhs(SolucionParticularHorizontal), rhs(SolucionVerticalParticular), t = 0  
      ..TiempoVuelo2], scaling = CONSTRAINED);
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