

> restart :

>  $xy' = \sqrt{y^2 - x^2}.$

> Ecuacion := x·diff(y(x), x) = sqrt(y(x)·2 - x·2);

$$Ecuacion := x \left( \frac{d}{dx} y(x) \right) = \sqrt{y(x)^2 - x^2} \quad (1)$$

> with(DEtools) :

> odeadvisor(Ecuacion);

[[\_homogeneous, class A], \_rational, \_dAlembert] (2)

> EcuacionBis := eval(subs(y(x) = u(x)·x, Ecuacion));

$$EcuacionBis := x \left( \left( \frac{d}{dx} u(x) \right) x + u(x) \right) = \sqrt{u(x)^2 x^2 - x^2} \quad (3)$$

> EcuacionDespejada := simplify(factor(isolate(EcuacionBis, diff(u(x), x))));

$$EcuacionDespejada := \frac{d}{dx} u(x) = - \frac{-\sqrt{x^2 (u(x)^2 - 1)} + u(x) x}{x^2} \quad (4)$$

> SolucionSimbolica := Int(1/(-sqrt(u·2 - 1) + u), u) + Int(1/x, x) = CI;

$$SolucionSimbolica := \int \frac{1}{-\sqrt{u^2 - 1} + u} du + \int \frac{1}{x} dx = CI \quad (5)$$

> SolucionIntermedia := int(1/(-sqrt(u·2 - 1) + u), u) + int(1/x, x) = CI;

$$SolucionIntermedia := \frac{1}{2} u^2 + \frac{1}{2} u \sqrt{u^2 - 1} - \frac{1}{2} \ln(u + \sqrt{u^2 - 1}) + \ln(x) = CI \quad (6)$$

> SolucionFinal := simplify(subs(u = y/x, SolucionIntermedia));

$$SolucionFinal := \frac{1}{2} \frac{y^2 + y \sqrt{-\frac{y^2 + x^2}{x^2}} x - \ln\left(\frac{y + \sqrt{-\frac{y^2 + x^2}{x^2}} x}{x}\right) x^2 + 2 \ln(x) x^2}{x^2} \quad (7)$$

= CI

> SolucionGeneral

$$:= \frac{1}{2} \frac{1}{x^2} \left( y(x)^2 + y(x) \sqrt{-\frac{-y(x)^2 + x^2}{x^2}} x - \ln\left(\frac{y(x) + \sqrt{-\frac{-y(x)^2 + x^2}{x^2}} x}{x}\right) x^2 + 2 \ln(x) x^2 \right) = CI$$

SolucionGeneral :=

(8)

$$\frac{1}{2} \frac{1}{x^2} \left( y(x)^2 + y(x) \sqrt{-\frac{y(x)^2 + x^2}{x^2}} x - \ln \left( \frac{y(x) + \sqrt{-\frac{y(x)^2 + x^2}{x^2}} x}{x} \right) x^2 + 2 \ln(x) x^2 \right) = CI$$

> *DerivadaSolucion* := *simplify(isolate(simplify(diff(SolucionGeneral, x)), diff(y(x), x)))*;  
*DerivadaSolucion* :=  $\frac{d}{dx} y(x)$  (9)

$$= \frac{y(x)^3 + y(x)^2 \sqrt{\frac{y(x)^2 - x^2}{x^2}} x - y(x) x^2 - \sqrt{\frac{y(x)^2 - x^2}{x^2}} x^3}{x \left( y(x)^2 + y(x) \sqrt{\frac{y(x)^2 - x^2}{x^2}} x - x^2 \right)}$$

> *DerivadaEcuacion* := *isolate(Ecuacion, diff(y(x), x))*;

$$DerivadaEcuacion := \frac{d}{dx} y(x) = \frac{\sqrt{y(x)^2 - x^2}}{x} \quad (10)$$

> *comprobacion* := *simplify(rhs(DerivadaSolucion) - rhs(DerivadaEcuacion))* = 0;

$$comprobacion := \frac{1}{x \left( y(x)^2 + y(x) \sqrt{\frac{y(x)^2 - x^2}{x^2}} x - x^2 \right)} \left( y(x)^3 + y(x)^2 \sqrt{\frac{y(x)^2 - x^2}{x^2}} x - y(x) x^2 - \sqrt{\frac{y(x)^2 - x^2}{x^2}} x^3 - \sqrt{y(x)^2 - x^2} y(x)^2 - \sqrt{y(x)^2 - x^2} y(x) \sqrt{\frac{y(x)^2 - x^2}{x^2}} x + \sqrt{y(x)^2 - x^2} x^2 \right) = 0 \quad (11)$$

> *SolDespejada* := *isolate(diff(dsolve(Ecuacion), x), diff(y(x), x))*;

$$SolDespejada := \frac{d}{dx} y(x) = \left( - \left( - \frac{2 y(x)^2}{x^3} - \frac{2 y(x) \sqrt{y(x)^2 - x^2}}{x^3} + \frac{3}{x} \right) x^2 \sqrt{y(x)^2 - x^2} \left( \sqrt{y(x)^2 - x^2} + y(x) \right) + \sqrt{y(x)^2 - x^2} y(x) x + y(x)^2 x - x^3 \right) / \left( 4 y(x)^3 - 4 y(x) x^2 + 3 \sqrt{y(x)^2 - x^2} y(x)^2 + (y(x)^2 - x^2)^{3/2} - \sqrt{y(x)^2 - x^2} x^2 \right) \quad (12)$$

> *comprobacion2* := *simplify(rhs(DerivadaSolucion) - rhs(SolDespejada))* = 0;

$$comprobacion2 := \left( x \left( x^4 - y(x)^2 \sqrt{\frac{y(x)^2 - x^2}{x^2}} x \sqrt{y(x)^2 - x^2} - \sqrt{y(x)^2 - x^2} y(x) x^2 \right) \right) \quad (13)$$

$$\begin{aligned}
& + \sqrt{y(x)^2 - x^2} \, y(x)^3 - y(x)^3 \sqrt{\frac{y(x)^2 - x^2}{x^2}} \, x + x^3 y(x) \sqrt{\frac{y(x)^2 - x^2}{x^2}} \\
& + \sqrt{\frac{y(x)^2 - x^2}{x^2}} \, x^3 \sqrt{y(x)^2 - x^2} - 2 x^2 y(x)^2 + y(x)^4 \Big) \Big) \Big/ \left( \left( y(x)^2 \right. \right. \\
& + y(x) \sqrt{\frac{y(x)^2 - x^2}{x^2}} \, x - x^2 \Big) \left( 2 y(x)^3 - 2 y(x) \, x^2 + 2 \sqrt{y(x)^2 - x^2} \, y(x)^2 \right. \\
& \left. \left. - \sqrt{y(x)^2 - x^2} \, x^2 \right) \right) = 0
\end{aligned}$$

