# Capítulo 2.- La ecuación diferencial lineal

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)\,y = Q(x)$$

EDO(n)L $\begin{cases} \text{coeficientes variables} \\ \text{coeficientes constantes: } \forall a_i(x) = k_i \quad k_i \in \mathbb{R} \end{cases}$

$i = 0 \dots n$

EDO(n)L $\begin{cases} \text{Homogénea cuando } Q(x) = 0 \\ \text{No-homogénea cuando } Q(x) \neq 0 \end{cases}$

EDO(n) L CC H

EDO(n) L CC NH

EDO(n) L CV H

EDO(n) L CV NH

$$a_0(x)\frac{d^n y}{dx^n} + \ldots + a_n(x)\,y = Q(x)$$

EDOL $\begin{cases} 1^{\underline{er}} \text{ orden} & n=1 \\ \text{orden superior a } 1 & n>1 \end{cases}$

EDO(1) L CV NH

$$a_0(x)\frac{dy}{dx}+a_1(x)\,y=Q(x)$$

Regla de Oro: Todos los métodos de solución del programa de estudios suponen que la ecuación diferencial sea normalizada (estandarizada). Una ecuación estará normalizada cuando el coeficiente de la derivada de mayor orden sea 1.

$$a_0(x)\frac{dy}{dx} + a_1(x)\,y = Q(x)$$

normalizar

$$\frac{dy}{dx} + \frac{a_1(x)}{a_0(x)}\,y = \frac{Q(x)}{a_0(x)}$$

$$\boxed{\frac{dy}{dx} + p(x)\,y = q(x)}$$

EDO(1)L cv NH.

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$Q(x) = 0$

EDO(1)L cv $\underline{H}$

$$\frac{dy}{dx} + p(x)\, y = 0$$

# Resolución de la EDO(1)L CV H.

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{dx} = -p(x)y$$

$$dy = -p(x)\,y\,dx$$

Separación de Variables

$$\frac{dy}{y} = -p(x)\,dx$$

$$\int \frac{dy}{y} = -\int p(x)\,dx$$

$$\int \frac{dy}{y} = -\int p(x)\,dx$$

$$\mathcal{L}y + k_1 = -\left[\int p(x)\,dx\right] + k_2$$

$$\mathcal{L}y = -\left[\quad\right] + (k_2 - k_1)$$

$$y = e^{\left((k_2-k_1) - \int p(x)\,dx\right)}$$

$$y = e^{k_2-k_1}\, e^{-\int p(x)\,dx}$$

SOL

GRAL

$$\boxed{y = c_1 e^{-\int p(x)\,dx}}$$

$$\frac{dy}{dx} + p(x)\,y = 0$$

$$\boxed{\frac{dy}{dx} - 2y = 0} \rightarrow p(x) = -2$$

$$e^{-\int p(x)dx} \Rightarrow e^{-\int(-2)dx} \Rightarrow e^{+2\int dx} \Rightarrow e^{2x}$$

$$\boxed{y_g = c_1 e^{2x}}$$

$$\frac{d}{dx}$$

$$\frac{dy}{dx} = 2c_1 e^{2x}$$

$$\left[2c_1 e^{2x}\right] - 2\left[c_1 e^{2x}\right] = 0$$

$$(2c_1 - 2c_1)e^{2x} = 0$$

$$(0)e^{2x} = 0 \Rightarrow 0 \equiv 0$$

$$\boxed{\frac{dy}{dx} + \frac{y}{x} = 0} \qquad \frac{dy}{dx} + \left(\frac{1}{x}\right) y = 0$$

$$p(x) = \frac{1}{x} \qquad \int p(x)\,dx = \int \frac{dx}{x} \Rightarrow \ln x$$

$$e^{-\int p(x)\,dx} = e^{-\ln(x)}$$

$$y = c_1 e^{-\ln(x)} \Rightarrow y = c_1 e^{\ln\left(\frac{1}{x}\right)} \rightarrow \boxed{y_g = \frac{c_1}{x}}$$

$$\frac{y}{c_1} = e^{\ln\left(\frac{1}{x}\right)} \rightarrow \ln\left(\frac{y}{c_1}\right) = \ln e^{\ln\left(\frac{1}{x}\right)}$$

$$\ln\left(\frac{y}{c_1}\right) = \ln\left(\frac{1}{x}\right) \cancelto{1}{\ln e} \rightarrow \ln\left(\frac{y}{c_1}\right) = \ln\left(\frac{1}{x}\right) \rightarrow \frac{y}{c_1} = \frac{1}{x}$$

$$\frac{dy}{dx} + \frac{y}{x} = 0 \quad — \quad y_g = \frac{c_1}{x}$$

$$y = c_1 x^{-1} \longrightarrow y = \frac{c_1}{x}$$

$$\frac{dy}{dx} = -c_1 x^{-2}$$

$$\frac{dy}{dx} = -\frac{c_1}{x^2}$$

$$\left[-\frac{c_1}{x^2}\right] + \frac{\left[\frac{c_1}{x}\right]}{x} = 0$$

$$0 = 0$$

$$\boxed{\frac{\frac{a}{b}}{\frac{c}{d}} \Rightarrow \frac{ad}{cb}}$$

$$\frac{dy}{dx} - \frac{y}{x^2} = 0 \longrightarrow p(x) = \frac{-1}{x^2}$$

$$\int p(x)\,dx = \int \left(-\frac{1}{x^2}\right) dx \Rightarrow -\int \frac{dx}{x^2} \Rightarrow -\int x^{-2}\,dx$$

$$\int p(x)\,dx = -\left[\frac{x^{-1}}{-1}\right] \Rightarrow x^{-1} \Rightarrow \frac{1}{x}$$

$$y_g = c_1 e^{-\left(\frac{1}{x}\right)} \Rightarrow \boxed{y_g = c_1 e^{-\frac{1}{x}}}$$

$$\frac{dy}{dx} - \frac{y}{x^2} = 0 \longrightarrow y_g = C_1 e^{-\frac{1}{x}}$$

$$\frac{dy}{dx} = C_1 e^{-\frac{1}{x}} \frac{d}{dx}\left(-\frac{1}{x}\right)$$

$$\frac{dy}{dx} = C_1 e^{-\frac{1}{x}} \left(\frac{1}{x^2}\right)$$

$$\left[\frac{C_1 e^{-\frac{1}{x}}}{x^2}\right] - \frac{\left[C_1 e^{-\frac{1}{x}}\right]}{x^2} = 0$$

$$(C_1 - C_1)\frac{e^{-\frac{1}{x}}}{x^2} = 0$$

$$(0)\frac{e^{-\frac{1}{x}}}{x^2} = 0 \longrightarrow 0 = 0$$

$$y_g = C_1 e^{-\int p(x)dx} \qquad y = \frac{C_1}{e^{\int p(x)dx}}$$

$$y e^{\int p(x)dx} = C_1 \qquad F(x,y) = C_1$$

Solución general

$$\frac{\partial F}{\partial x} = y\left[e^{\int p\,dx}\, p(x)\right] \qquad \frac{d}{dx}F(x,y) = 0$$

$$\frac{\partial F}{\partial y} = e^{\int p(x)dx} \qquad \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$y e^{\int p(x)dx} p(x) + \left(e^{\int p(x)dx}\right)\frac{dy}{dx} = 0$$

$$e^{\int p(x)dx}\frac{dy}{dx} + e^{\int p(x)dx}p(x)\,y = 0$$

FACTOR INTEGRANTE

$$e^{\int p(x)dx}\left[\frac{dy}{dx} + p(x)\,y\right] = 0$$

$$y = e^{\int p(x)dx}$$

$$\Rightarrow y = 0 \qquad \frac{dy}{dx} = 0 \qquad \underline{\text{trivial}}$$

$$\frac{dy}{dx} + p(x)\,y = 0 \qquad y = c_1 e^{-\int p(x)dx}$$

$$\frac{dy}{dx} + p(x)y = q(x)$$

$$e^{\int p(x)dx}\left[\frac{dy}{dx} + p(x)y\right] = q(x)e^{\int p(x)dx}$$

$$\boxed{u = ye^{\int p(x)dx}}$$

$$\frac{d}{dx}\left[ye^{\int p(x)dx}\right] = q(x)e^{\int p(x)dx}$$

$$d\left[ye^{\int p(x)dx}\right] = q(x)e^{\int p(x)dx}dx$$

$$du = e^{\int p(x)dx} q(x)\,dx$$

$$\int du = \int e^{\int p(x)dx} q(x)\,dx$$

$$u + k_1 = \left[\int \quad\right] + k_2$$

$$u = (k_2 - k_1) + \left[\int \quad\right]$$