

Resolver la EDO(2) LCC H.

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$\boxed{\frac{dy}{dx} + a_1 y = 0} \quad p(x) = a_1$$

$$y_g = C_1 e^{-\int a_1 dx}$$

$$\Rightarrow y_g = C_1 e^{-a_1 x}$$

$$\Rightarrow \boxed{y_g = C_1 e^{-a_1 x}}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{orden} = 2$$

$$y_g = C_1 y_1 + C_2 y_2 \quad \left\{ \begin{array}{l} y_1, \text{ "Soluciones} \\ \text{particulares"} \\ y_2 \text{ "fundamentales"} \end{array} \right.$$

Hipótesis $y_i = e^{mx} \quad m \in \mathbb{R}$

$$\frac{dy_i}{dx} = m e^{mx}$$

$$\frac{d^2 y_i}{dx^2} = m (m e^{mx}) \Rightarrow m^2 e^{mx}$$

$$\left[m^2 e^{mx} \right] + a_1 \left[m e^{mx} \right] + a_2 \left[e^{mx} \right] = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0$$

$$m^2 + a_1 m + a_2 = 0$$

Ecuación Característica
de la EDO(2) LCC H.

raíces $\left\{ \begin{array}{l} m_1 \\ m_2 \end{array} \right.$ (supongo)
 $m_1 \neq m_2$

$$y_1 = e^{m_1 x}$$

$$y_2 = e^{m_2 x}$$

$$e^{mx} = 0$$

$$y_i = 0.$$

trivial.

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$\rightarrow m^2 + a_1 m + a_2 = 0$$

$$(m - m_1)(m - m_2) = 0$$

$$y_1 = e^{m_1 x} \quad y_2 = e^{m_2 x}$$

$$W \Rightarrow \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$$

$$\begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$m_2 e^{m_1 x} e^{m_2 x} - m_1 e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$m_2 - m_1 \neq 0$$

$$m_2 \neq m_1$$

$$\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

$$y_i = e^{mx}$$

$$m^2 - 5m + 6 = 0$$

$$(m-2)(m-3) = 0$$

$$m_1 = 2 \quad m_2 = 3$$

$$m_2 \neq m_1$$

$$y_1 = e^{2x} \quad y_2 = e^{3x}$$

$$y_g = c_1 e^{2x} + c_2 e^{3x}$$

$$\begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix} \neq 0$$

$$3e^{5x} - 2e^{5x} \neq 0$$

$$e^{5x} \neq 0$$



Equación característica

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} m_1 \\ m_2 \end{array} \right.$$

$$m_1, m_2 \in \mathbb{R} \quad \left\{ \begin{array}{l} m_2 \neq m_1 \quad \leftarrow \text{Tipo I} \\ m_2 = m_1 \quad \leftarrow \text{Tipo II} \end{array} \right.$$

$$m_1, m_2 \in \mathbb{C} \quad \leftarrow \text{Tipo III}$$

Tipo I.-

$$y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x} \quad m_1 \neq m_2 \in \mathbb{R}$$

TIP0 III

$$\frac{dy^2}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{array}{l} m_2, m_1 \in \mathbb{C} \end{array} \right.$$

$$\left. \begin{array}{l} m_1 = a + bi \\ m_2 = a - bi \end{array} \right\} m_1 \neq m_2 \quad a \in \mathbb{R} \quad b \in \mathbb{R}^+$$

$$\begin{array}{l} y_1 = e^{(a+bi)x} \\ y_2 = e^{(a-bi)x} \end{array} \quad \left| \quad \begin{array}{l} y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x} \\ y \in \mathbb{R} \quad x \in \mathbb{R} \end{array} \right.$$

Teorema Euler

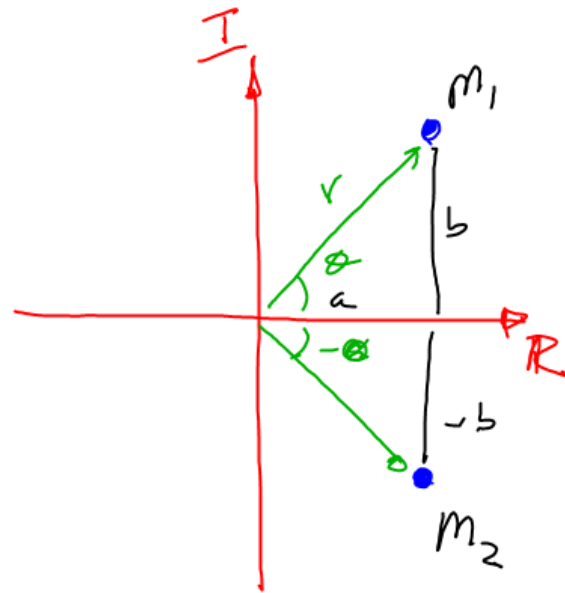
$$e = 2.71$$

$$\pi = 3.1416$$

$$i = \sqrt{-1}$$

$$1 \Rightarrow 1$$

$$e^{\pi i} + 1 = 0$$



$$m_1 = a + bi \Rightarrow r e^{\theta i}$$

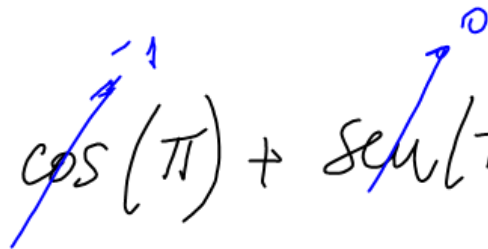
$$m_2 = a - bi \Rightarrow r e^{-\theta i}$$

$$r e^{\theta i} = r \cos(\theta) + r \sin(\theta) i$$

$$r e^{-\theta i} = r \cos(\theta) - r \sin(\theta) i$$

$$e^{\theta i} = \cos(\theta) + \operatorname{sen}(\theta) i$$

\overline{W} radians

$$e^{\pi i} = \cos(\pi) + \operatorname{sen}(\pi) i$$


$$e^{\pi i} = -1 \Rightarrow e^{\pi i} + 1 = 0$$

$$e^{(a+bi)x} \Rightarrow e^{ax} e^{bxi} \Rightarrow e^{ax} (\cos(bx) + \operatorname{sen}(bx) i)$$

$$e^{(a-bi)x} \Rightarrow e^{ax} e^{-bxi} \Rightarrow e^{ax} (\cos(bx) - \operatorname{sen}(bx) i)$$

$$y = C_1 \left(e^{ax} \cos(bx) + e^{ax} \operatorname{sen}(bx) i \right) + \\ + C_2 \left(e^{ax} \cos(bx) - e^{ax} \operatorname{sen}(bx) i \right).$$

$$y = (C_1 + C_2) e^{ax} \cos(bx) + (C_1 i - C_2 i) e^{ax} \operatorname{sen}(bx)$$

$$y = C_{10} e^{ax} \cos(bx) + C_{20} e^{ax} \operatorname{sen}(bx) \dots$$

$$m_1, m_2 = \{ a \pm bi \}$$