```
> restart
> Ecuacion := y'' - 5 y' + 6 y = 5·exp(x)
```
$$Ecuacion := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 5\, e^x \tag{1}$$

```
> EcuacionHom := lhs(Ecuacion) = 0
```
$$EcuacionHom := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 0 \tag{2}$$

```
> Q(x) := rhs(Ecuacion)
```
$$Q(x) := 5\, e^x \tag{3}$$

```
> EcuacionCarac := m··2 - 5 m + 6 = 0
```
$$EcuacionCarac := m^2 - 5\, m + 6 = 0 \tag{4}$$

```
> Raiz := solve(EcuacionCarac)
```
$$Raiz := 3, 2 \tag{5}$$

```
> Sol_1 := y(x) = exp(Raiz_1·x); Sol_2 := y(x) = exp(Raiz_2·x)
```
$$Sol_1 := y(x) = e^{3x}$$
$$Sol_2 := y(x) = e^{2x} \tag{6}$$

```
> SolucionHom := y(x) = C1·rhs(Sol_1) + C2·rhs(Sol_2)
```
$$SolucionHom := y(x) = C1\, e^{3x} + C2\, e^{2x} \tag{7}$$

```
> SolucionNoHom := y(x) = A(x)·rhs(Sol_1) + B(x)·rhs(Sol_2)
```
$$SolucionNoHom := y(x) = A(x)\, e^{3x} + B(x)\, e^{2x} \tag{8}$$

```
> with(linalg) :
> AA := wronskian([rhs(Sol_1), rhs(Sol_2)], x)
```
$$AA := \begin{bmatrix} e^{3x} & e^{2x} \\ 3\, e^{3x} & 2\, e^{2x} \end{bmatrix} \tag{9}$$

```
> BB := array([0, Q(x)])
```
$$BB := \begin{bmatrix} 0 & 5\, e^x \end{bmatrix} \tag{10}$$

```
> Sol := simplify(linsolve(AA, BB))
```
$$Sol := \begin{bmatrix} 5\, e^{-2x} & -5\, e^{-x} \end{bmatrix} \tag{11}$$

```
> Aprima := Sol_1; Bprima := Sol_2
```
$$Aprima := 5\, e^{-2x}$$
$$Bprima := -5\, e^{-x} \tag{12}$$

```
> A(x) := int(Aprima, x) + C1; B(x) := int(Bprima, x) + C2
```
$$A(x) := -\frac{5}{2}\, e^{-2x} + C1$$
$$B(x) := 5\, e^{-x} + C2 \tag{13}$$

```
> SolucionGeneral := simplify(SolucionNoHom)
```
$$\tag{14}$$

$$SolucionGeneral := y(x) = \frac{5}{2}\,e^x + C1\,e^{3x} + C2\,e^{2x} \tag{14}$$

`> SolGral := dsolve(Ecuacion)`

$$SolGral := y(x) = e^{2x}\_C2 + e^{3x}\_C1 + \frac{5}{2}\,e^x \tag{15}$$

`> restart`

`>`

`> Ecuacion := y''''-3 y'''-6 y''+9 y'+27 y = 3·exp(3 x)cos(4 x) + x··3`

$$Ecuacion := \frac{d^4}{dx^4}\,y(x) - 3\left(\frac{d^3}{dx^3}\,y(x)\right) - 6\left(\frac{d^2}{dx^2}\,y(x)\right) + 9\left(\frac{d}{dx}\,y(x)\right) + 27\,y(x) \tag{16}$$
$$= 3\,e^{3x}\cos(4x) + x^3$$

`> EcuacionHom := lhs(Ecuacion) = 0`

$$EcuacionHom := \frac{d^4}{dx^4}\,y(x) - 3\left(\frac{d^3}{dx^3}\,y(x)\right) - 6\left(\frac{d^2}{dx^2}\,y(x)\right) + 9\left(\frac{d}{dx}\,y(x)\right) + 27\,y(x) = 0 \tag{17}$$

`> Q(x) := rhs(Ecuacion)`

$$Q(x) := 3\,e^{3x}\cos(4x) + x^3 \tag{18}$$

`> EcuaCarac := expand((m-3)··2·(m··2 + 3 m + 3)) = 0`

$$EcuaCarac := m^4 - 3\,m^3 - 6\,m^2 + 9\,m + 27 = 0 \tag{19}$$

`> Raiz := solve(EcuaCarac)`

$$Raiz := 3, 3, -\frac{3}{2} + \frac{1}{2}\,I\sqrt{3}, -\frac{3}{2} - \frac{1}{2}\,I\sqrt{3} \tag{20}$$

`> Sol₁ := y(x) = exp(Raiz₁·x); Sol₂ := y(x) = x·exp(Raiz₂·x)`

$$Sol_1 := y(x) = e^{3x}$$
$$Sol_2 := y(x) = x\,e^{3x} \tag{21}$$

`> Sol₃ := y(x) = exp(Re(Raiz₃)·x)·cos(Im(Raiz₃)·x); Sol₄ := y(x) = exp(Re(Raiz₃)·x)·sin(Im(Raiz₃)·x)`

$$Sol_3 := y(x) = e^{-\frac{3}{2}x}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)$$
$$Sol_4 := y(x) = e^{-\frac{3}{2}x}\sin\left(\frac{1}{2}\sqrt{3}\,x\right) \tag{22}$$

`> EcuacionNoHom := y(x) = A(x)·rhs(Sol₁) + B(x)·rhs(Sol₂) + C(x)·rhs(Sol₃) + D(x)·rhs(Sol₄)`

$$EcuacionNoHom := y(x) = A(x)\,e^{3x} + B(x)\,x\,e^{3x} + C(x)\,e^{-\frac{3}{2}x}\cos\left(\frac{1}{2}\sqrt{3}\,x\right) \tag{23}$$
$$+ D(x)\,e^{-\frac{3}{2}x}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)$$

`> with(linalg):`

`> AA := wronskian([rhs(Sol₁), rhs(Sol₂), rhs(Sol₃), rhs(Sol₄)], x):`

> $BB := array([0, 0, 0, Q(x)])$

$$BB := \begin{bmatrix} 0 & 0 & 0 & 3\,e^{3x}\cos(4x) + x^3 \end{bmatrix} \tag{24}$$

> $Sol := simplify(linsolve(AA, BB)) : Aprima := Sol_1$

$$Aprima := -\frac{1}{147}\,(7x+3)\,\left(3\,e^{3x}\cos(4x) + x^3\right)e^{-3x} \tag{25}$$

> $Bprima := Sol_2$

$$Bprima := \frac{1}{21}\,\left(3\,e^{3x}\cos(4x) + x^3\right)e^{-3x} \tag{26}$$

> $Cprima := Sol_3$

$$Cprima := \frac{1}{441}\,\sqrt{3}\,e^{\frac{3}{2}x}\left(-39\sin\left(\frac{1}{2}\sqrt{3}\,x\right)e^{3x}\cos(4x) - 13\sin\left(\frac{1}{2}\sqrt{3}\,x\right)x^3\right. \tag{27}$$
$$\left.+ 9\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\,e^{3x}\cos(4x) + 3\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\,x^3\right)$$

> $Dprima := Sol_4$

$$Dprima := \frac{1}{441}\,\sqrt{3}\,e^{\frac{3}{2}x}\left(39\cos\left(\frac{1}{2}\sqrt{3}\,x\right)e^{3x}\cos(4x) + 13\cos\left(\frac{1}{2}\sqrt{3}\,x\right)x^3\right. \tag{28}$$
$$\left.+ 9\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\,e^{3x}\cos(4x) + 3\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\,x^3\right)$$

> $A(x) := int(Aprima, x) + C1 : B(x) := int(Bprima, x) + C2 : C(x) := int(Cprima, x) + C3 :$
  $D(x) := int(Dprima, x) + C4 :$

> $SolucionGeneralNoHomogenea := simplify(EcuacionNoHom) : evalf(\%, 2)$

$$y(x) = 3.2\,10^{-8}\,e^{-1.5x}\left(1.3\,10^5\,e^{4.5x}\sin(0.85x)\cos(4.8x) - 1.1\,10^5\,e^{4.5x}\cos(0.85x)\cos(\right. \tag{29}$$
$$-3.2x) + 3.7\,10^5\,e^{4.5x}\cos(0.85x)\cos(4.8x) + 3.1\,10^7\,x\,e^{4.5x}\,C2$$
$$+ 4.5\,10^5\,e^{4.5x}\sin(0.85x)\cos(-3.2x) - 1.1\,10^5\,e^{4.5x}\sin(0.85x)\sin(-3.2x)$$
$$- 3.9\,10^6\,e^{4.5x}\cos(x)^3\sin(x) - 4.5\,10^5\,e^{4.5x}\cos(0.85x)\sin(-3.2x)$$
$$+ 1.9\,10^6\,e^{4.5x}\cos(x)\sin(x) + 3.7\,10^5\,e^{4.5x}\sin(0.85x)\sin(4.8x)$$
$$- 1.3\,10^5\,e^{4.5x}\cos(0.85x)\sin(4.8x) + 3.1\,10^7\cos(0.85x)\,C3 + 3.1\,10^7\sin(0.85x)\,C4$$
$$+ 2.3\,10^6\,x\,e^{1.5x} + 1.2\,10^6\,e^{1.5x}\,x^3 - 1.2\,10^6\,e^{1.5x}\,x^2 + 2.2\,10^6\,e^{4.5x}\cos(x)^2$$
$$+ 3.1\,10^7\,e^{4.5x}\,C1 - 2.2\,10^6\,e^{4.5x}\cos(x)^4 - 5.2\,10^5\,e^{1.5x})$$

> $SolGral := dsolve(Ecuacion) : evalf(\%, 2)$

$$y(x) = -0.016 + 0.074\,x - 0.00071\,e^{3.x}\cos(4.x) + 0.037\,x^3 + 0.0089\,e^{3.x} \tag{30}$$
$$- 0.0051\,e^{3.x}\sin(4.x) - 0.037\,x^2 + \_C1\,e^{3.x} + \_C2\,x\,e^{3.x} + \_C3\,e^{-1.5x}\cos(0.85x)$$
$$+ \_C4\,e^{-1.5x}\sin(0.85x)$$

> $comprobacion := simplify(rhs(SolGral) - rhs(SolucionGeneralNoHomogenea)) = 0$

$$comprobacion := \frac{1}{14}\,e^{3x}\cos(x)^4 - \frac{1}{14}\,e^{3x}\cos(x)^2 + \_C1\,e^{3x} - \frac{15}{21136}\,e^{3x}\cos(4x) \tag{31}$$
$$+ \frac{1}{112}\,e^{3x} - e^{3x}\,C1 - \frac{27}{5284}\,e^{3x}\sin(4x) - \frac{330}{64729}\,\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sin\left(\frac{1}{2}\,(\sqrt{3}\right.$$

$$+8\right)x\right)\mathrm{e}^{3x}-\frac{38}{9247}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\cos\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{38}{9247}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\cos\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$+\frac{330}{64729}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sin\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$+\frac{330}{64729}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\cos\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{38}{9247}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sin\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{330}{64729}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\cos\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{38}{9247}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sin\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}-\mathrm{e}^{-\frac{3}{2}x}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)C4$$

$$-\mathrm{e}^{-\frac{3}{2}x}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)C3+\frac{6}{49}\mathrm{e}^{3x}\cos(x)^{3}\sin(x)-\frac{3}{49}\mathrm{e}^{3x}\cos(x)\sin(x)+\_C2\,x\,\mathrm{e}^{3x}$$

$$+\_C3\,\mathrm{e}^{-\frac{3}{2}x}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)+\_C4\,\mathrm{e}^{-\frac{3}{2}x}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)$$

$$+\frac{842}{194187}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\cos\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$+\frac{51}{9247}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\sin\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{51}{9247}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\cos\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{842}{194187}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\sin\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{51}{9247}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\cos\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$+\frac{842}{194187}\sin\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\sin\left(\frac{1}{2}\left(\sqrt{3}-8\right)x\right)\mathrm{e}^{3x}$$

$$+\frac{51}{9247}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\sin\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}$$

$$-\frac{842}{194187}\cos\left(\frac{1}{2}\sqrt{3}\,x\right)\sqrt{3}\cos\left(\frac{1}{2}\left(\sqrt{3}+8\right)x\right)\mathrm{e}^{3x}-\mathrm{e}^{3x}x\,C2=0$$

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