```
> restart
```

```
> SolucionGeneral := y(x) = C1·x··2 + C2·x + C3/x + 4 exp(x) + 5 sin(2 x)
```

$$SolucionGeneral := y(x) = C1\,x^2 + C2\,x + \frac{C3}{x} + 4\,e^x + 5\sin(2\,x) \qquad (1)$$

```
> SolucionHomogenea := y(x) = C1·x··2 + C2·x + C3/x
```

$$SolucionHomogenea := y(x) = C1\,x^2 + C2\,x + \frac{C3}{x} \qquad (2)$$

```
> SolucionParticular := y(x) = 4 exp(x) + 5 sin(2 x)
```

$$SolucionParticular := y(x) = 4\,e^x + 5\sin(2\,x) \qquad (3)$$

```
> Sistema := diff(SolucionHomogenea, x), diff(SolucionHomogenea, x$2),
    diff(SolucionHomogenea, x$3) : Sistema_1; Sistema_2; Sistema_3;
```

$$\frac{d}{dx}\,y(x) = 2\,C1\,x + C2 - \frac{C3}{x^2}$$

$$\frac{d^2}{dx^2}\,y(x) = 2\,C1 + \frac{2\,C3}{x^3}$$

$$\frac{d^3}{dx^3}\,y(x) = -\frac{6\,C3}{x^4} \qquad (4)$$

```
> Parametro := solve({Sistema}, {C1, C2, C3})
```

$$Parametro := \left\{ C1 = \frac{1}{2}\,\frac{d^2}{dx^2}\,y(x) + \frac{1}{6}\left(\frac{d^3}{dx^3}\,y(x)\right)x,\ C2 = \frac{d}{dx}\,y(x) - \left(\frac{d^2}{dx^2}\,y(x)\right)x \qquad (5)$$

$$- \frac{1}{2}\left(\frac{d^3}{dx^3}\,y(x)\right)x^2,\ C3 = -\frac{1}{6}\left(\frac{d^3}{dx^3}\,y(x)\right)x^4 \right\}$$

```
> Parametro_1;
```

$$C1 = \frac{1}{2}\,\frac{d^2}{dx^2}\,y(x) + \frac{1}{6}\left(\frac{d^3}{dx^3}\,y(x)\right)x \qquad (6)$$

```
> Parametro_2;
```

$$C2 = \frac{d}{dx}\,y(x) - \left(\frac{d^2}{dx^2}\,y(x)\right)x - \frac{1}{2}\left(\frac{d^3}{dx^3}\,y(x)\right)x^2 \qquad (7)$$

```
> Parametro_3;
```

$$C3 = -\frac{1}{6}\left(\frac{d^3}{dx^3}\,y(x)\right)x^4 \qquad (8)$$

```
> EcuacionIntermedia := expand(simplify(subs(C1 = rhs(Parametro_1), C2
    = rhs(Parametro_2), C3 = rhs(Parametro_3), SolucionHomogenea)))
```

$$EcuacionIntermedia := y(x) = -\frac{1}{2}\left(\frac{d^2}{dx^2}\,y(x)\right)x^2 - \frac{1}{2}\left(\frac{d^3}{dx^3}\,y(x)\right)x^3 + x\left(\frac{d}{dx}\,y(x)\right) \qquad (9)$$

> $EcuacionHomogenea := expand\left(\dfrac{lhs(EcuacionIntermedia)}{\dfrac{(x\cdot\cdot 3)}{2}} - \dfrac{rhs(EcuacionIntermedia)}{\dfrac{(x\cdot\cdot 3)}{2}}\right)$
  $= 0$

$$EcuacionHomogenea := \frac{2\,y(x)}{x^3} + \frac{\dfrac{d^2}{dx^2}\,y(x)}{x} + \frac{d^3}{dx^3}\,y(x) - \frac{2\left(\dfrac{d}{dx}\,y(x)\right)}{x^2} = 0 \tag{10}$$

> $Q(x) := expand(simplify(eval(subs(y(x) = rhs(SolucionParticular),$
  $lhs(EcuacionHomogenea)))))$

$$Q(x) := \frac{8\,e^x}{x^3} + \frac{20\sin(x)\cos(x)}{x^3} + \frac{4\,e^x}{x} - \frac{40\sin(x)\cos(x)}{x} + 4\,e^x - 80\cos(x)^2 + 40 \tag{11}$$

$$-\frac{8\,e^x}{x^2} - \frac{40\cos(x)^2}{x^2} + \frac{20}{x^2}$$

> $EcuacionNoHomogenea := lhs(EcuacionHomogenea) = Q(x)$

$$EcuacionNoHomogenea := \frac{2\,y(x)}{x^3} + \frac{\dfrac{d^2}{dx^2}\,y(x)}{x} + \frac{d^3}{dx^3}\,y(x) - \frac{2\left(\dfrac{d}{dx}\,y(x)\right)}{x^2} = \frac{8\,e^x}{x^3} \tag{12}$$

$$+ \frac{20\sin(x)\cos(x)}{x^3} + \frac{4\,e^x}{x} - \frac{40\sin(x)\cos(x)}{x} + 4\,e^x - 80\cos(x)^2 + 40 - \frac{8\,e^x}{x^2}$$

$$- \frac{40\cos(x)^2}{x^2} + \frac{20}{x^2}$$

> $EcuacionHomogeneaAsociada := lhs(EcuacionNoHomogenea) = 0$

$$EcuacionHomogeneaAsociada := \frac{2\,y(x)}{x^3} + \frac{\dfrac{d^2}{dx^2}\,y(x)}{x} + \frac{d^3}{dx^3}\,y(x) - \frac{2\left(\dfrac{d}{dx}\,y(x)\right)}{x^2} = 0 \tag{13}$$

> $SolucionHomogenea;$

$$y(x) = C1\,x^2 + C2\,x + \frac{C3}{x} \tag{14}$$

> $Sol_1 := y(x) = x\cdot\cdot 2;\ Sol_2 := y(x) = x;\ Sol_3 := y(x) = \dfrac{1}{x};$

$$Sol_1 := y(x) = x^2$$

$$Sol_2 := y(x) = x$$

$$Sol_3 := y(x) = \frac{1}{x} \tag{15}$$

> $SolucionNoHom := y(x) = A(x)\cdot rhs(Sol_1) + B(x)\cdot rhs(Sol_2) + D(x)\cdot rhs(Sol_3)$

$$SolucionNoHom := y(x) = A(x)\,x^2 + B(x)\,x + \frac{D(x)}{x} \tag{16}$$

> $with(linalg):$
> $AA := wronskian([rhs(Sol_1), rhs(Sol_2), rhs(Sol_3)], x)$

$$AA := \begin{bmatrix} x^2 & x & \dfrac{1}{x} \\[2mm] 2x & 1 & -\dfrac{1}{x^2} \\[2mm] 2 & 0 & \dfrac{2}{x^3} \end{bmatrix} \tag{17}$$

`> BB := array([0, 0, Q(x)])`

$$BB := \left[0, 0, \frac{8\,e^x}{x^3} + \frac{20\sin(x)\cos(x)}{x^3} + \frac{4\,e^x}{x} - \frac{40\sin(x)\cos(x)}{x} + 4\,e^x - 80\cos(x)^2 + 40 \right. \tag{18}$$

$$\left. - \frac{8\,e^x}{x^2} - \frac{40\cos(x)^2}{x^2} + \frac{20}{x^2} \right]$$

`> SOL := linsolve(AA, BB)`

$$SOL := \left[\frac{4}{3}\,\frac{1}{x^3}\left(x^2\,e^x - 10\,x^2\sin(x)\cos(x) + 2\,e^x + 5\sin(x)\cos(x) + 10\,x^3 - 2\,x\,e^x + e^x x^3 \right.\right. \tag{19}$$

$$\left. - 20\,x^3\cos(x)^2 - 10\,x\cos(x)^2 + 5\,x \right), -\frac{1}{x^2}\left(2\left(x^2\,e^x - 10\,x^2\sin(x)\cos(x) + 2\,e^x \right.\right.$$

$$\left.\left. + 5\sin(x)\cos(x) + 10\,x^3 - 2\,x\,e^x + e^x x^3 - 20\,x^3\cos(x)^2 - 10\,x\cos(x)^2 + 5\,x \right)\right), \frac{2}{3}\,x^2\,e^x$$

$$- \frac{20}{3}\,x^2\sin(x)\cos(x) + \frac{4}{3}\,e^x + \frac{10}{3}\sin(x)\cos(x) + \frac{20}{3}\,x^3 - \frac{4}{3}\,x\,e^x + \frac{2}{3}\,e^x x^3$$

$$\left. - \frac{40}{3}\,x^3\cos(x)^2 - \frac{20}{3}\,x\cos(x)^2 + \frac{10}{3}\,x \right]$$

`> Aprima := SOL₁; Bprima := SOL₂; Dprima := SOL₃;`

$$Aprima := \frac{4}{3}\,\frac{1}{x^3}\left(x^2\,e^x - 10\,x^2\sin(x)\cos(x) + 2\,e^x + 5\sin(x)\cos(x) + 10\,x^3 - 2\,x\,e^x + e^x x^3 \right.$$

$$\left. - 20\,x^3\cos(x)^2 - 10\,x\cos(x)^2 + 5\,x \right)$$

$$Bprima := -\frac{1}{x^2}\left(2\left(x^2\,e^x - 10\,x^2\sin(x)\cos(x) + 2\,e^x + 5\sin(x)\cos(x) + 10\,x^3 - 2\,x\,e^x + e^x x^3 \right.\right.$$

$$\left.\left. - 20\,x^3\cos(x)^2 - 10\,x\cos(x)^2 + 5\,x \right)\right)$$

$$Dprima := \frac{2}{3}\,x^2\,e^x - \frac{20}{3}\,x^2\sin(x)\cos(x) + \frac{4}{3}\,e^x + \frac{10}{3}\sin(x)\cos(x) + \frac{20}{3}\,x^3 - \frac{4}{3}\,x\,e^x \tag{20}$$

$$+ \frac{2}{3}\,e^x x^3 - \frac{40}{3}\,x^3\cos(x)^2 - \frac{20}{3}\,x\cos(x)^2 + \frac{10}{3}\,x$$

`> A(x) := int(Aprima, x) + C1; B(x) := int(Bprima, x) + C2; D(x) := int(Dprima, x) + C3;`

$$A(x) := -\frac{4}{3}\,\frac{e^x}{x^2} + \frac{4}{3}\,\frac{e^x}{x} - \frac{5}{3}\,\frac{\sin(2x)}{x^2} + \frac{10}{3}\,\frac{\cos(2x)}{x} + \frac{4}{3}\,e^x - \frac{40}{3}\sin(x)\cos(x)$$

$$+ C1$$

$$B(x) := \frac{4\,e^x}{x} + \frac{5\sin(2x)}{x} - 20\,x^2 - 2\,x\,e^x + 40\,x\left(\frac{1}{2}\sin(x)\cos(x) + \frac{1}{2}\,x \right) + C2$$

$$D(x) := -\frac{4}{3} x^2 e^x + \frac{4}{3} x e^x + \frac{2}{3} e^x x^3 + \frac{20}{3} x \left(\frac{1}{2} \sin(x) \cos(x) + \frac{1}{2} x \right) - \frac{20}{3} x^2 \cos(x)^2 \qquad \textbf{(21)}$$

$$+ \frac{20}{3} x^4 - \frac{40}{3} x^3 \left(\frac{1}{2} \sin(x) \cos(x) + \frac{1}{2} x \right) + C3$$

> $SolucionFinal := expand(simplify(SolucionNoHom))$

$$SolucionFinal := y(x) = 4\, e^x + 10\, \sin(x)\, \cos(x) + C1\, x^2 + C2\, x + \frac{C3}{x} \qquad \textbf{(22)}$$

> $SolucionGeneral;$

$$y(x) = C1\, x^2 + C2\, x + \frac{C3}{x} + 4\, e^x + 5\, \sin(2\, x) \qquad \textbf{(23)}$$

>