

> restart

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$$\frac{d}{dt} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} e^{3t} + 4t^2 \\ 6t + 8 \end{bmatrix} \quad \bar{x}(0) = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

> AA := array([ [3, 4], [2, 5] ])

$$AA := \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad (1)$$

> BB := array([ exp(3·t) + 4·t·2, 6·t + 8 ])

$$BB := \begin{bmatrix} e^{3t} + 4t^2 & 6t + 8 \end{bmatrix} \quad (2)$$

> Xcero := array([ 5, -6 ])

$$Xcero := \begin{bmatrix} 5 & -6 \end{bmatrix} \quad (3)$$

> with(linalg) :

> MatExp := exponential(AA, t)

$$MatExp := \begin{bmatrix} \frac{2}{3} e^t + \frac{1}{3} e^{7t} & \frac{2}{3} e^{7t} - \frac{2}{3} e^t \\ \frac{1}{3} e^{7t} - \frac{1}{3} e^t & \frac{1}{3} e^t + \frac{2}{3} e^{7t} \end{bmatrix} \quad (4)$$

> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)

$$MatExpTau := \begin{bmatrix} \frac{2}{3} e^{t-\tau} + \frac{1}{3} e^{7t-7\tau} & \frac{2}{3} e^{7t-7\tau} - \frac{2}{3} e^{t-\tau} \\ \frac{1}{3} e^{7t-7\tau} - \frac{1}{3} e^{t-\tau} & \frac{1}{3} e^{t-\tau} + \frac{2}{3} e^{7t-7\tau} \end{bmatrix} \quad (5)$$

> BBtau := map(rcurry(eval, t='tau'), BB)

$$BBtau := \begin{bmatrix} e^{3\tau} + 4\tau^2 & 6\tau + 8 \end{bmatrix} \quad (6)$$

> Prod := evalm( MatExpTau &\* BBtau ) : Prod<sub>1</sub>; Prod<sub>2</sub>

$$\begin{aligned} & \left( \frac{2}{3} e^{t-\tau} + \frac{1}{3} e^{7t-7\tau} \right) (e^{3\tau} + 4\tau^2) + \left( \frac{2}{3} e^{7t-7\tau} - \frac{2}{3} e^{t-\tau} \right) (6\tau + 8) \\ & \left( \frac{1}{3} e^{7t-7\tau} - \frac{1}{3} e^{t-\tau} \right) (e^{3\tau} + 4\tau^2) + \left( \frac{1}{3} e^{t-\tau} + \frac{2}{3} e^{7t-7\tau} \right) (6\tau + 8) \end{aligned} \quad (7)$$

> IntProd := map(int, Prod, tau=0..t) : IntProd<sub>1</sub>; IntProd<sub>2</sub>

$$\begin{aligned} & -\frac{13}{3} e^t + \frac{3847}{4116} e^{7t} + \frac{1}{4} e^{3t} + \frac{1080}{343} - \frac{20}{7} t^2 - \frac{96}{49} t \\ & \frac{3847}{4116} e^{7t} + \frac{13}{6} e^t - \frac{1}{4} e^{3t} - \frac{978}{343} + \frac{8}{7} t^2 + \frac{2}{49} t \end{aligned} \quad (8)$$

> *comprobacion*<sub>1</sub> := *map*(*rcurry*(*eval*, *t*:=0'), *IntProd*)

$$\text{comprobacion}_1 := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (9)$$

> *Sol* := *evalm*(*evalm*(*MatExp* &\* *Xcero*) + *IntProd*) : *Solucion*<sub>1</sub> := *x*<sub>1</sub>(*t*) = *Sol*<sub>1</sub>; *Solucion*<sub>2</sub> := *x*<sub>2</sub>(*t*) = *Sol*<sub>2</sub>;

$$\begin{aligned} \text{Solucion}_1 &:= x_1(t) = 3 e^t - \frac{1919}{1372} e^{7t} + \frac{1}{4} e^{3t} + \frac{1080}{343} - \frac{20}{7} t^2 - \frac{96}{49} t \\ \text{Solucion}_2 &:= x_2(t) = -\frac{1919}{1372} e^{7t} - \frac{3}{2} e^t - \frac{1}{4} e^{3t} - \frac{978}{343} + \frac{8}{7} t^2 + \frac{2}{49} t \end{aligned} \quad (10)$$

> *Sistema* := *diff*(*X*<sub>1</sub>(*t*), *t*) = 3·*X*<sub>1</sub>(*t*) + 4·*X*<sub>2</sub>(*t*) + *exp*(3·*t*) + 4·*t*·2, *diff*(*X*<sub>2</sub>(*t*), *t*) = 2·*X*<sub>1</sub>(*t*) + 5·*X*<sub>2</sub>(*t*) + 6·*t* + 8 : *Sistema*<sub>1</sub>; *Sistema*<sub>2</sub>

$$\begin{aligned} \frac{d}{dt} X_1(t) &= 3 X_1(t) + 4 X_2(t) + e^{3t} + 4 t^2 \\ \frac{d}{dt} X_2(t) &= 2 X_1(t) + 5 X_2(t) + 6 t + 8 \end{aligned} \quad (11)$$

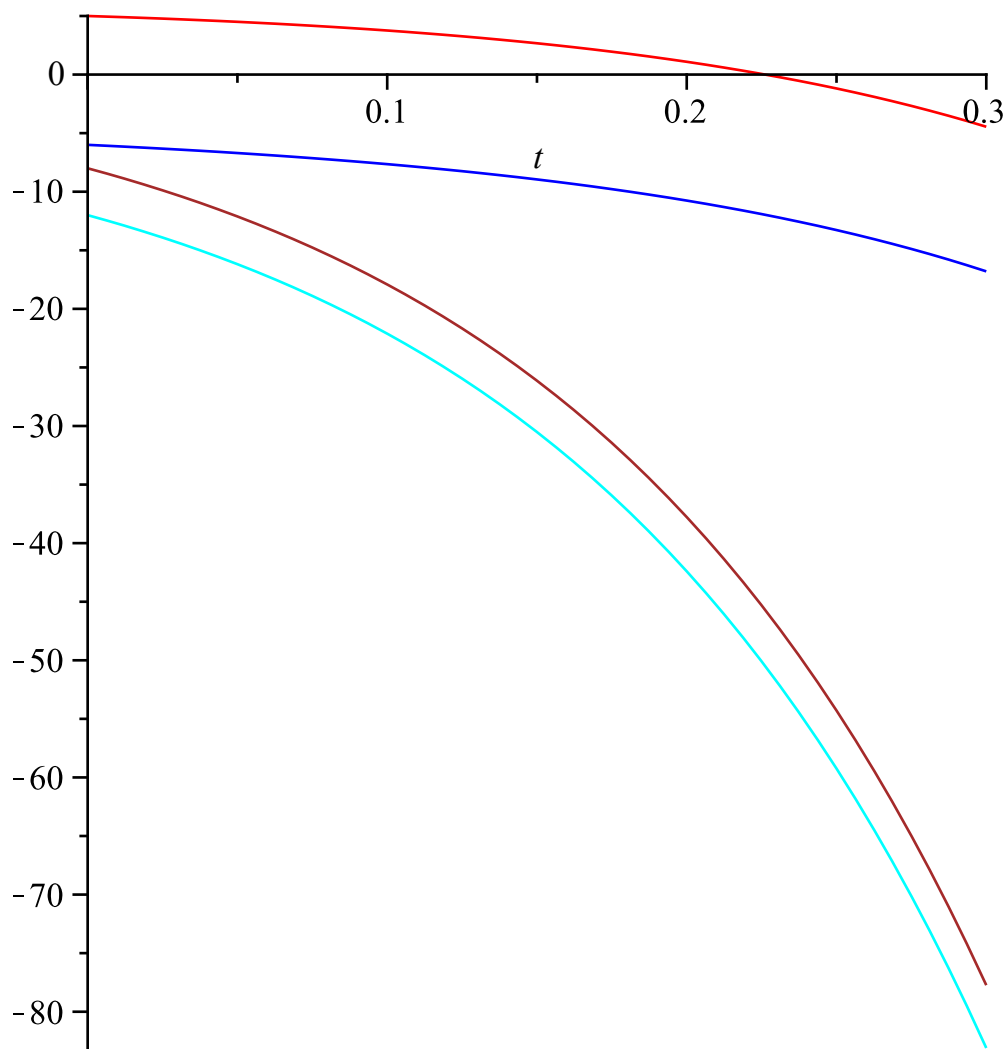
> *Condiciones* := *X*<sub>1</sub>(0) = 5, *X*<sub>2</sub>(0) = -6;

$$\text{Condiciones} := X_1(0) = 5, X_2(0) = -6 \quad (12)$$

> *SOLUCION* := *dsolve*( {*Sistema*, *Condiciones*} )

$$\begin{aligned} \text{SOLUCION} &:= \left\{ X_1(t) = 3 e^t - \frac{1919}{1372} e^{7t} + \frac{1}{4} e^{3t} + \frac{1080}{343} - \frac{20}{7} t^2 - \frac{96}{49} t, X_2(t) = \right. \\ &\quad \left. - \frac{1919}{1372} e^{7t} - \frac{3}{2} e^t - \frac{1}{4} e^{3t} - \frac{978}{343} + \frac{8}{7} t^2 + \frac{2}{49} t \right\} \end{aligned} \quad (13)$$

> *plot*( [*rhs*(*Solucion*<sub>1</sub>), *rhs*(*Solucion*<sub>2</sub>), *rhs*(*diff*(*Solucion*<sub>1</sub>, *t*)), *rhs*(*diff*(*Solucion*<sub>2</sub>, *t*)) ], *t* = 0 .. 0.3, *color* = [*red*, *blue*, *brown*, *cyan*])



> Sistema<sub>1</sub>; Sistema<sub>2</sub>

$$\frac{d}{dt} X_1(t) = 3 X_1(t) + 4 X_2(t) + e^{3t} + 4 t^2$$

$$\frac{d}{dt} X_2(t) = 2 X_1(t) + 5 X_2(t) + 6 t + 8$$

(14)

> Incognita := isolate(Sistema<sub>1</sub>, X<sub>2</sub>(t))

$$Incognita := X_2(t) = \frac{1}{4} \frac{d}{dt} X_1(t) - \frac{3}{4} X_1(t) - \frac{1}{4} e^{3t} - t^2$$

(15)

> Ecuacion := simplify(eval(subs(X<sub>2</sub>(t) = rhs(Incognita), Sistema<sub>2</sub>)))

$$Ecuacion := \frac{1}{4} \frac{d^2}{dt^2} X_1(t) - \frac{3}{4} \frac{d}{dt} X_1(t) - \frac{3}{4} e^{3t} - 2t = -\frac{7}{4} X_1(t) + \frac{5}{4} \frac{d}{dt} X_1(t) - \frac{5}{4} e^{3t} - 5t^2 + 6t + 8 \quad (16)$$

$$\begin{aligned} > EcuacionFinal := lhs(Ecuacion) \cdot 4 - (-3 e^{3t} - 8t) - \left( -7 X_1(t) + 5 \left( \frac{d}{dt} X_1(t) \right) \right) \\ &= rhs(Ecuacion) \cdot 4 - (-3 e^{3t} - 8t) - \left( -7 X_1(t) + 5 \left( \frac{d}{dt} X_1(t) \right) \right); \end{aligned}$$

(17)

$$EcuacionFinal := \frac{d^2}{dt^2} X_1(t) - 8 \left( \frac{d}{dt} X_1(t) \right) + 7 X_1(t) = -2 e^{3t} - 20 t^2 + 32 t + 32 \quad (17)$$

$$> EcuacionHomogenea := lhs(EcuacionFinal) = 0$$

$$EcuacionHomogenea := \frac{d^2}{dt^2} X_1(t) - 8 \left( \frac{d}{dt} X_1(t) \right) + 7 X_1(t) = 0 \quad (18)$$

$$> Q(t) := rhs(EcuacionFinal);$$

$$Q(t) := -2 e^{3t} - 20 t^2 + 32 t + 32 \quad (19)$$

$$> EcuaCaract := m \cdot 2 - 8 m + 7 = 0$$

$$EcuaCaract := m^2 - 8 m + 7 = 0 \quad (20)$$

$$> Raiz := solve(EcuaCaract)$$

$$Raiz := 7, 1 \quad (21)$$

$$> Sol_1 := X(t) = \exp(Raiz_1 \cdot t); Sol_2 := X(t) = \exp(Raiz_2 \cdot t)$$

$$Sol_1 := X(t) = e^{7t}$$

$$Sol_2 := X(t) = e^t \quad (22)$$

$$> with(linalg) :$$

$$> WW := wronskian([rhs(Sol_1), rhs(Sol_2)], t)$$

$$WW := \begin{bmatrix} e^{7t} & e^t \\ 7 e^{7t} & e^t \end{bmatrix} \quad (23)$$

$$> BBB := array([0, Q(t)])$$

$$BBB := \begin{bmatrix} 0 & -2 e^{3t} - 20 t^2 + 32 t + 32 \end{bmatrix} \quad (24)$$

$$> SOLU := linsolve(WW, BBB)$$

$$SOLU := \begin{bmatrix} -\frac{1}{3} \frac{-16 t - 16 + e^{3t} + 10 t^2}{e^{7t}} & \frac{1}{3} \frac{-16 t - 16 + e^{3t} + 10 t^2}{e^t} \end{bmatrix} \quad (25)$$

$$> Aprima := SOLU_1; Bprima := SOLU_2;$$

$$Aprima := -\frac{1}{3} \frac{-16 t - 16 + e^{3t} + 10 t^2}{e^{7t}}$$

$$Bprima := \frac{1}{3} \frac{-16 t - 16 + e^{3t} + 10 t^2}{e^t} \quad (26)$$

$$> A(t) := int(Aprima, t) + C1; B(t) := int(Bprima, t) + C2$$

$$A(t) := -\frac{92}{147} \frac{t}{(e^t)^7} - \frac{292}{343 (e^t)^7} + \frac{1}{12 (e^t)^4} + \frac{10}{21} \frac{t^2}{(e^t)^7} + C1$$

$$B(t) := -\frac{4}{3} \frac{t}{e^t} + \frac{4}{e^t} + \frac{1}{6} (e^t)^2 - \frac{10}{3} \frac{t^2}{e^t} + C2 \quad (27)$$

$$> SolucionGeneral := X_1(t) = expand(A(t) \cdot rhs(Sol_1) + B(t) \cdot rhs(Sol_2));$$

$$SolucionGeneral := X_1(t) = -\frac{96}{49} t + \frac{1080}{343} + \frac{1}{4} (e^t)^3 - \frac{20}{7} t^2 + (e^t)^7 C1 + e^t C2 \quad (28)$$

$$\begin{aligned} &> \text{SolGral} := \text{simplify}(\text{eval}(\text{subs}(X_1(t) = \text{rhs}(\text{SolucionGeneral}), \text{Incognita}))) \\ &\quad \text{SolGral} := X_2(t) = -\frac{978}{343} + \frac{2}{49}t + e^{7t}C1 - \frac{1}{2}e^tC2 + \frac{8}{7}t^2 - \frac{1}{4}e^{3t} \end{aligned} \quad (29)$$

$$\begin{aligned} &> \text{sistemita} := \text{subs}(t=0, \text{rhs}(\text{SolucionGeneral}) = 5), \text{subs}(t=0, \text{rhs}(\text{SolGral}) = -6) : \\ &\quad \text{sistemita}_1; \text{sistemita}_2 \\ &\quad \frac{4663}{1372} + C1 + C2 = 5 \\ &\quad -\frac{4255}{1372} + C1 - \frac{1}{2}C2 = -6 \end{aligned} \quad (30)$$

$$\begin{aligned} &> \text{SolSol} := \text{solve}(\{\text{sistemita}\}, \{C1, C2\}) \\ &\quad \text{SolSol} := \left\{ C1 = -\frac{1919}{1372}, C2 = 3 \right\} \end{aligned} \quad (31)$$

$$\begin{aligned} &> \text{SOLUC} := \text{simplify}(\text{subs}(C1 = \text{rhs}(\text{SolSol}_1), C2 = \text{rhs}(\text{SolSol}_2), \text{SolucionGeneral})), \\ &\quad \text{simplify}(\text{subs}(C1 = \text{rhs}(\text{SolSol}_1), C2 = \text{rhs}(\text{SolSol}_2), \text{SolGral})) : \text{SOLUC}_1; \text{SOLUC}_2 \\ &\quad X_1(t) = 3e^t - \frac{1919}{1372}e^{7t} + \frac{1}{4}e^{3t} + \frac{1080}{343} - \frac{20}{7}t^2 - \frac{96}{49}t \\ &\quad X_2(t) = -\frac{1919}{1372}e^{7t} - \frac{3}{2}e^t - \frac{1}{4}e^{3t} - \frac{978}{343} + \frac{8}{7}t^2 + \frac{2}{49}t \end{aligned} \quad (32)$$

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