```maple
> restart
> AA := array([[0, 1], [-1, 0]])
```

$$AA := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{1}$$

```maple
> with(linalg):
> MatExp := exponential(AA, t)
```

$$MatExp := \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix} \tag{2}$$

```maple
> DerMatExp := map(diff, MatExp, t)
```

$$DerMatExp := \begin{bmatrix} -\sin(t) & \cos(t) \\ -\cos(t) & -\sin(t) \end{bmatrix} \tag{3}$$

```maple
> AAA := map(rcurry(eval, t='0'), DerMatExp)
```

$$AAA := \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{4}$$

```maple
> Sistema := diff(x(t), t) = y(t), diff(y(t), t) = -x(t) : Sistema_1; Sistema_2;
```

$$\frac{d}{dt} x(t) = y(t)$$

$$\frac{d}{dt} y(t) = -x(t) \tag{5}$$

```maple
> restart
> SolGral := y(x) = C1·exp(x) + C2·cos(2·x) + 5·sin(2·x)
```

$$SolGral := y(x) = C1\, e^x + C2 \cos(2x) + 5 \sin(2x) \tag{6}$$

```maple
> Sistema := diff(SolGral, x), diff(SolGral, x$2) : Sistema_1; Sistema_2
```

$$\frac{d}{dx} y(x) = C1\, e^x - 2\, C2 \sin(2x) + 10 \cos(2x)$$

$$\frac{d^2}{dx^2} y(x) = C1\, e^x - 4\, C2 \cos(2x) - 20 \sin(2x) \tag{7}$$

```maple
> Parametro := solve({Sistema}, {C1, C2})
```

$$Parametro := \left\{ C1 = \right.$$

$$-\frac{\sin(2x)\left(\frac{d^2}{dx^2} y(x)\right) + 20 \sin(2x)^2 - 2\left(\frac{d}{dx} y(x)\right)\cos(2x) + 20 \cos(2x)^2}{e^x\left(-\sin(2x) + 2\cos(2x)\right)}, C2 =$$

$$\left. -\frac{1}{2}\, \frac{\frac{d^2}{dx^2} y(x) - \left(\frac{d}{dx} y(x)\right) + 10 \cos(2x) + 20 \sin(2x)}{-\sin(2x) + 2\cos(2x)} \right\} \tag{8}$$

```maple
> Ecuacion := simplify(eval(subs(C1 = rhs(Parametro_1), C2 = rhs(Parametro_2), SolGral)))
```

Ecuacion := $y(x)$ **(9)**

$$= \frac{1}{2}\; \frac{-2\sin(2\,x)\left(\dfrac{d^2}{dx^2}\,y(x)\right) - 50 + 5\left(\dfrac{d}{dx}\,y(x)\right)\cos(2\,x) - \cos(2\,x)\left(\dfrac{d^2}{dx^2}\,y(x)\right)}{-\sin(2\,x) + 2\cos(2\,x)}$$

> *EcuacionFinal* := $lhs(Ecuacion)\cdot 2\cdot(-\sin(2\,x)+2\cos(2\,x)) - rhs(Ecuacion)\cdot 2\cdot(-\sin(2\,x)+2\cos(2\,x)) = 0$

EcuacionFinal := $2\,y(x)\,(-\sin(2\,x)+2\cos(2\,x)) + 2\sin(2\,x)\left(\dfrac{d^2}{dx^2}\,y(x)\right) + 50$ **(10)**

$$-5\left(\frac{d}{dx}\,y(x)\right)\cos(2\,x) + \cos(2\,x)\left(\frac{d^2}{dx^2}\,y(x)\right) = 0$$

> *restart*

> *Solucion* := $y(x) = 4\cdot\exp(x) + C2\cdot\cos(2\cdot x) + C3\cdot\sin(2\cdot x)$

$$Solucion := y(x) = 4\,e^x + C2\cos(2\,x) + C3\sin(2\,x) \quad\text{(11)}$$

> *Sistema* := $diff(Solucion, x), diff(Solucion, x\$2):$

> *Parametro* := $solve(\{Sistema\}, \{C2, C3\})$

Parametro := $\Bigg\{ C2$ **(12)**

$$= \frac{1}{4}\; \frac{-\cos(2\,x)\left(\dfrac{d^2}{dx^2}\,y(x)\right) + 4\cos(2\,x)\,e^x - 2\left(\dfrac{d}{dx}\,y(x)\right)\sin(2\,x) + 8\,e^x\sin(2\,x)}{\cos(2\,x)^2 + \sin(2\,x)^2},$$

$$C3$$

$$= \frac{1}{4}\; \frac{-\left(\dfrac{d^2}{dx^2}\,y(x)\right)\sin(2\,x) + 4\,e^x\sin(2\,x) + 2\cos(2\,x)\left(\dfrac{d}{dx}\,y(x)\right) - 8\cos(2\,x)\,e^x}{\cos(2\,x)^2 + \sin(2\,x)^2} \Bigg\}$$

> *Ecuacion* := $simplify(eval(subs(C2 = rhs(Parametro_1), C3 = rhs(Parametro_2), Solucion)))$

$$Ecuacion := y(x) = 5\,e^x - \frac{1}{4}\,\frac{d^2}{dx^2}\,y(x) \quad\text{(13)}$$

> *restart*

> *EcuacionNoHom* := $diff(x(t), t\$2) + 4\cdot diff(x(t), t) + 4\,x(t) = 8\cdot t\cdot\cdot 2\cdot\exp(-2\,t)$

$$EcuacionNoHom := \frac{d^2}{dt^2}\,x(t) + 4\left(\frac{d}{dt}\,x(t)\right) + 4\,x(t) = 8\,t^2\,e^{-2\,t} \quad\text{(14)}$$

> *EcuaHomAsoc* := $lhs(EcuacionNoHom) = 0$

$$EcuaHomAsoc := \frac{d^2}{dt^2}\,x(t) + 4\left(\frac{d}{dt}\,x(t)\right) + 4\,x(t) = 0 \quad\text{(15)}$$

> $Q(t) := rhs(EcuacionNoHom);$

$$Q(t) := 8\,t^2\,e^{-2\,t} \quad\text{(16)}$$

> $EcuaCaract := m \cdot \cdot 2 + 4\,m + 4 = 0$

$$EcuaCaract := m^2 + 4\,m + 4 = 0 \tag{17}$$

> $Raiz := solve(EcuaCaract)$

$$Raiz := -2, -2 \tag{18}$$

> $Sol_1 := x(t) = \exp(Raiz_1 \cdot t); Sol_2 := x(t) = t \cdot \exp(Raiz_1 \cdot t)$

$$Sol_1 := x(t) = e^{-2t}$$

$$Sol_2 := x(t) = t\,e^{-2t} \tag{19}$$

> $SolHom := x(t) = C1 \cdot rhs(Sol_1) + C2 \cdot rhs(Sol_2)$

$$SolHom := x(t) = C1\,e^{-2t} + C2\,t\,e^{-2t} \tag{20}$$

> $SolNoHom := x(t) = A(t) \cdot rhs(Sol_1) + B(t) \cdot rhs(Sol_2)$

$$SolNoHom := x(t) = A(t)\,e^{-2t} + B(t)\,t\,e^{-2t} \tag{21}$$

> $with(linalg):$
> $WW := wronskian([rhs(Sol_1), rhs(Sol_2)], t)$

$$WW := \begin{bmatrix} e^{-2t} & t\,e^{-2t} \\ -2\,e^{-2t} & e^{-2t} - 2\,t\,e^{-2t} \end{bmatrix} \tag{22}$$

> $BB := array([0, Q(t)])$

$$BB := \begin{bmatrix} 0 & 8\,t^2\,e^{-2t} \end{bmatrix} \tag{23}$$

> $SOL := linsolve(WW, BB)$

$$SOL := \begin{bmatrix} -8\,t^3 & 8\,t^2 \end{bmatrix} \tag{24}$$

> $Aprima := SOL_1; Bprima := SOL_2;$

$$Aprima := -8\,t^3$$

$$Bprima := 8\,t^2 \tag{25}$$

> $A(t) := int(Aprima, t) + C1; B(t) := int(Bprima, t) + C2;$

$$A(t) := -2\,t^4 + C1$$

$$B(t) := \frac{8}{3}\,t^3 + C2 \tag{26}$$

> $simplify(expand(SolNoHom));$

$$x(t) = \frac{1}{3}\,e^{-2t}\left(2\,t^4 + 3\,C1 + 3\,t\,C2\right) \tag{27}$$

> $Comprobacion := dsolve(EcuacionNoHom)$

$$Comprobacion := x(t) = e^{-2t}\_C2 + t\,e^{-2t}\_C1 + \frac{2}{3}\,e^{-2t}\,t^4 \tag{28}$$

>