SOLUCIÓN GENERAL $\quad x^3y^2 + x^2y^4 + xy^5 = C_1$

inc.

$F(x, y) = C_1$

$y(x)$

V.i.

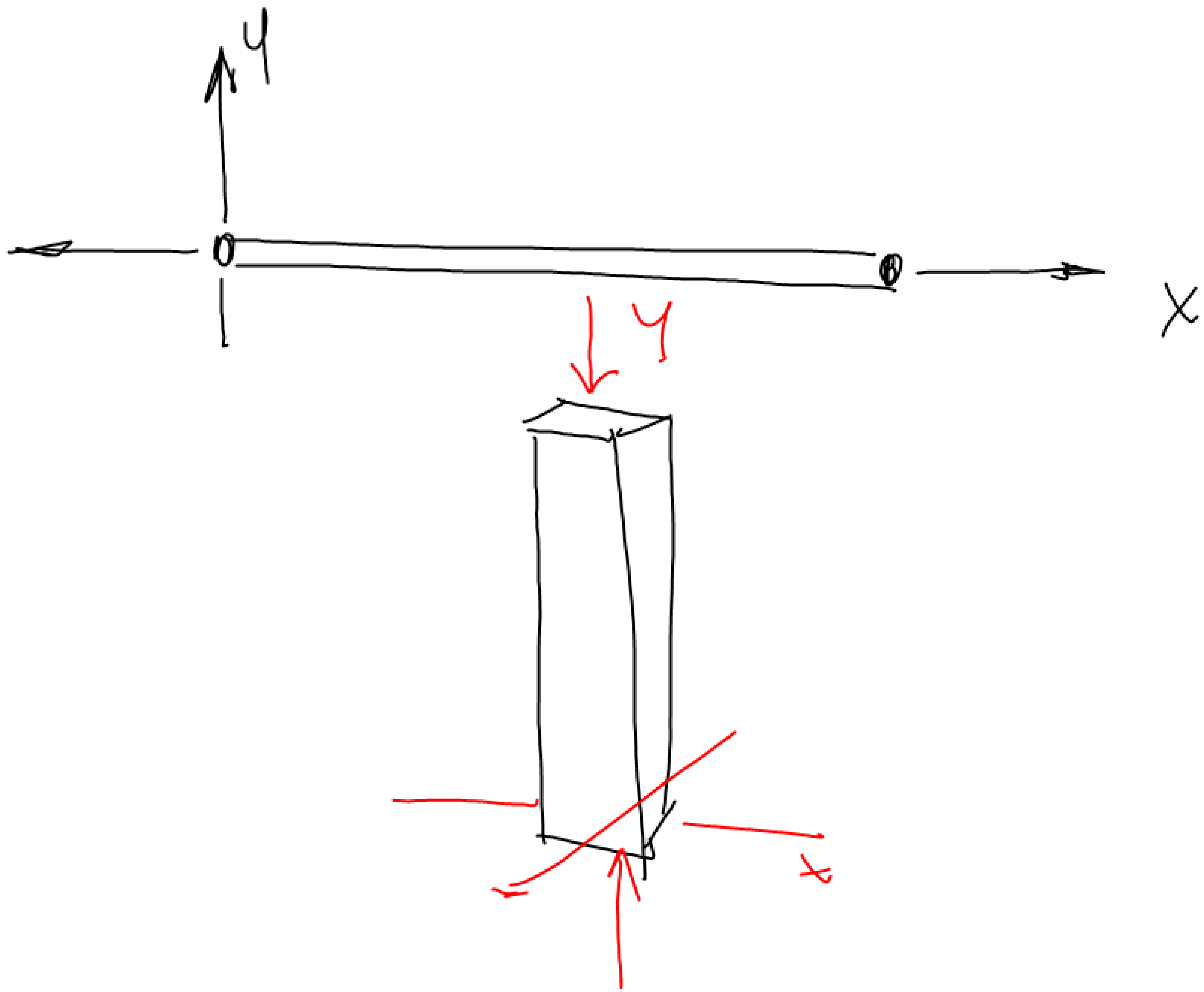
$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$

Ecuación Diferencial

$$\rightarrow \left(3x^2y^2 + 2xy^4 + y^5\right) + \left(2x^3y + 4x^2y^3 + 5xy^4\right) \cdot \frac{dy}{dx} = 0$$

EDO(1)NL

$$\frac{\partial F}{\partial y} + \frac{\partial F}{\partial x} \cdot \frac{dx}{dy} = 0$$


y
x
y
x

$$\left(3x^2y^2+2xy^4+y^5\right)+\left(2x^3y+4x^2y^3+5xy^4\right)\frac{dy}{dx}=0$$

$$\frac{\partial F}{\partial x} \qquad \frac{\partial F}{\partial y}$$

Teorema Schwartz.

$M(x,y)$

siempre

$$\frac{\partial^2 M}{\partial x \partial y}=\frac{\partial^2 M}{\partial y \partial x}$$

$$M(x,y)=\frac{\partial F}{\partial x} \qquad N(x,y)=\frac{\partial F}{\partial y}$$

$$\frac{\partial M}{\partial y} \Rightarrow \frac{\partial^2 F}{\partial x \partial y} \qquad \frac{\partial N}{\partial x} \Rightarrow \frac{\partial^2 F}{\partial y \partial x}$$

$$\underbrace{(3x^2y^2+2xy^4+y^5)}_{M(x,y)}+\underbrace{(2x^3y+4x^2y^3+5xy^4)}_{N(x,y)}\frac{dy}{dx}=0$$

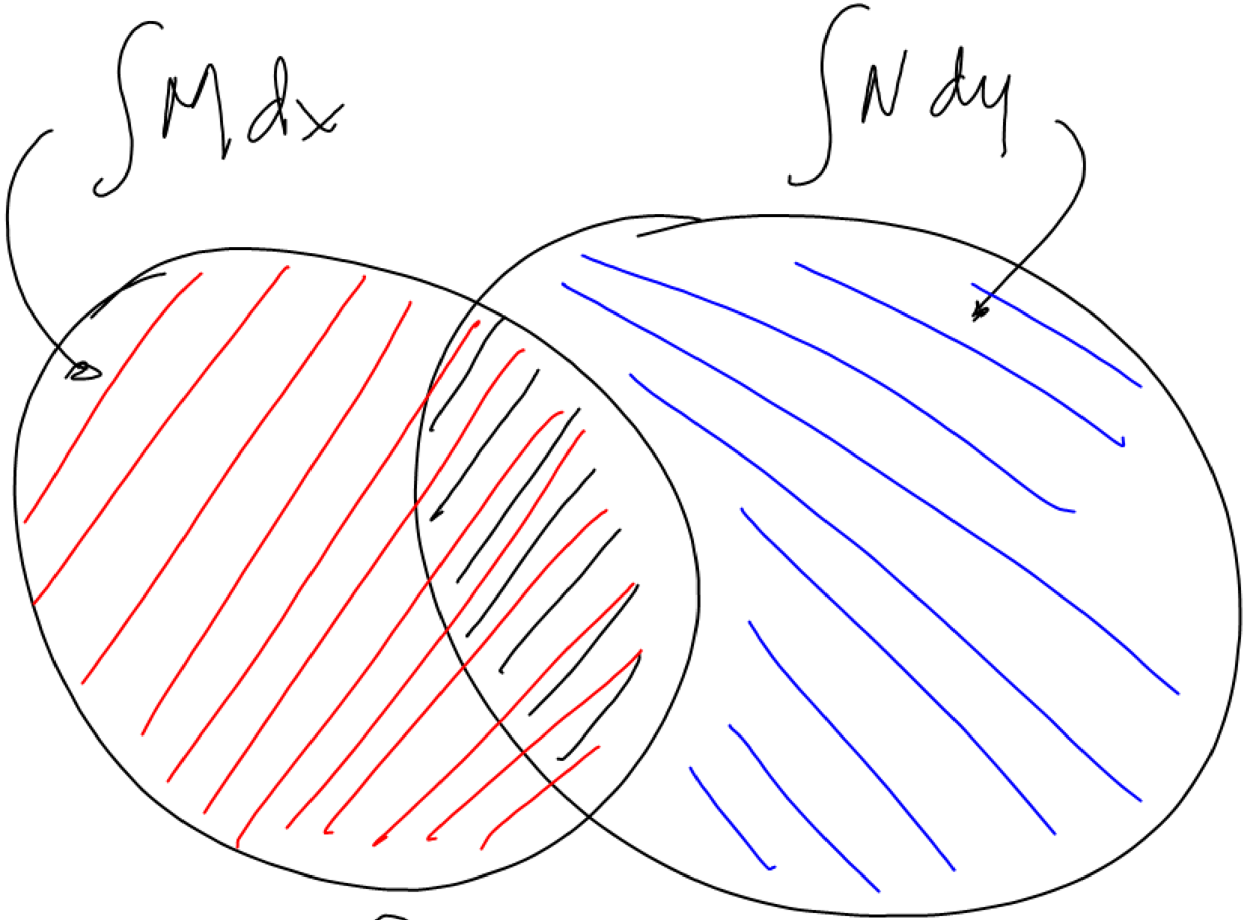
$$\frac{\partial M}{\partial y}=6x^2y+8xy^3+5y^4$$

$$\frac{\partial N}{\partial x}=6x^2y+8xy^3+5y^4$$

Como

$\frac{\partial M}{\partial y}\equiv\frac{\partial N}{\partial x}$ entonces EDO(1)NL

es EXACTA.



$$\text{Sol} \Rightarrow \int M\,dx \cup \int N\,dy = C$$

$$SG \Rightarrow \int M\,dx + \int \left[N - \frac{\partial}{\partial y}\int M\,dx \right] dy = C_1$$

$$SG \Rightarrow \int N\,dy + \int \left[M - \frac{\partial}{\partial x}\int N\,dy \right] dx = C_1$$

$$\left(3x^2y^2 + 2xy^4 + y^5\right) + \left(2x^3y + 4x^2y^3 + 5xy^4\right)\cdot\frac{dy}{dx} = 0$$

$$\int M\,dx = 3y^2\int x^2\,dx + 2y^4\int x\,dx + y^5\int dx$$

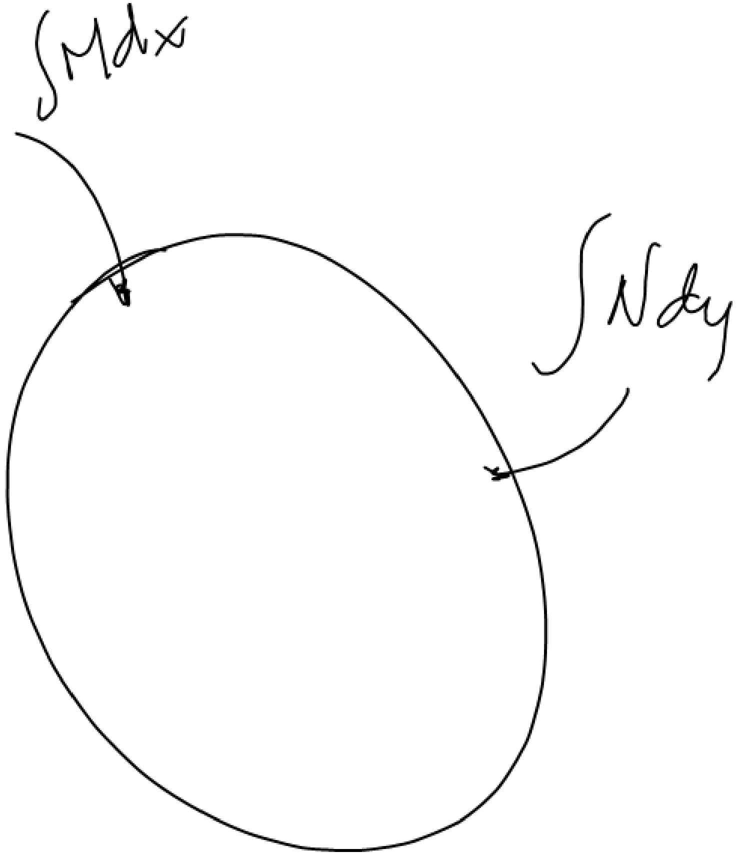
$$= 3y^2\left[\frac{x^3}{3}\right] + 2y^4\left[\frac{x^2}{2}\right] + y^5x$$

$$\int M\,dx = x^3y^2 + x^2y^4 + xy^5$$

$$\frac{\partial}{\partial y}\int M\,dx = 2x^3y + 4x^2y^3 + 5xy^4$$

$$\left[N(x,y) - \frac{\partial}{\partial y}\int M\,dx\right] = 0$$

$$\boxed{SG \Rightarrow x^3y^2 + x^2y^4 + xy^5 = C_1}$$


$\int M dx$
$\int N dy$

SG $e^x\cos(5y) + x^3 + 4y^2 = C_1$

E DO(1)NL

$$\left(e^x\cos(5y) + 3x^2\right) + \left(-5e^x \operatorname{sen}(5y) + 8y\right)\frac{dy}{dx} = 0$$

$$\frac{\partial M}{\partial y} = -5e^x \operatorname{sen}(5y) \qquad \frac{\partial N}{\partial x} = -5e^x \operatorname{sen}(5y)$$

EXACTA.

$$\int M\,dx = \cos(5y)\int e^x\,dx + 3\int x^2\,dx$$

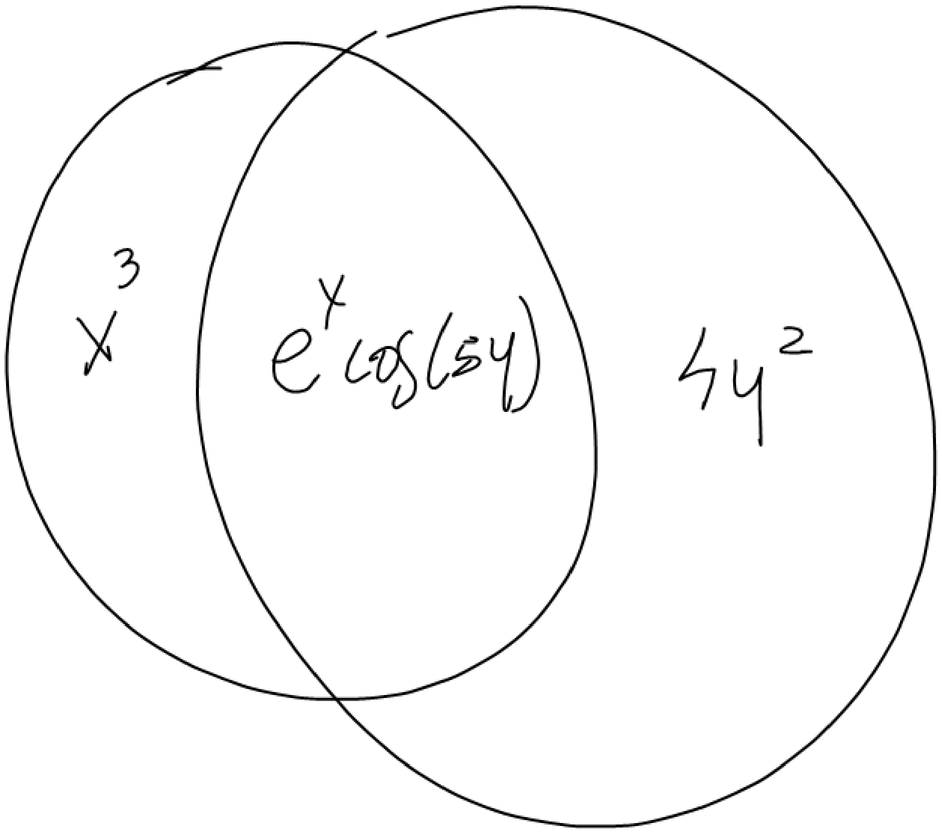
$$= e^x\cos(5y) + x^3$$

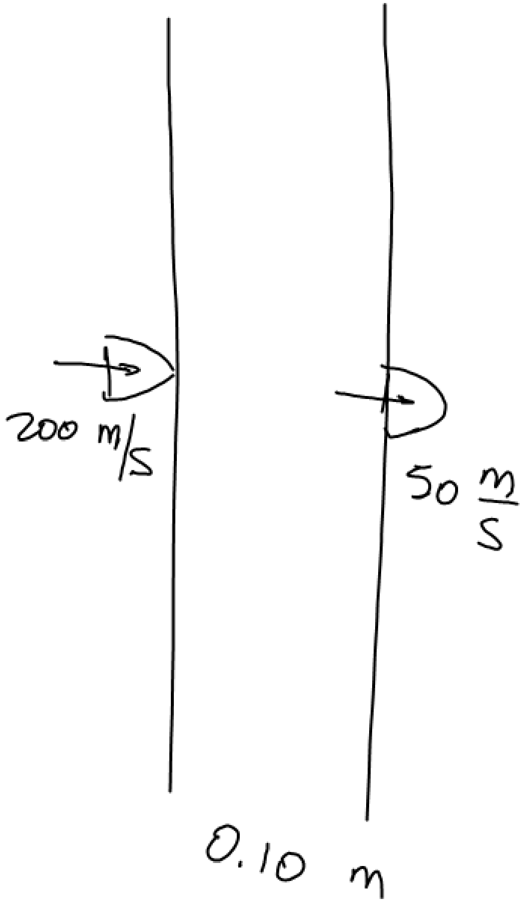
$$\frac{\partial}{\partial y}\int M\,dx = -5e^x \operatorname{sen}(5y) +$$

$$\left[N - \frac{\partial}{\partial y}\int M\,dx\right] = \left(-5e^x \operatorname{sen} 5y + 8y\right) - \left(-5e^x \operatorname{sen}(5y)\right) \Rightarrow 8y$$

$$\int\left[N - \frac{\partial}{\partial y}\int M\,dx\right]dy = 8\int y\,dy \Rightarrow 4y^2$$

$$SG \Rightarrow e^x\cos(5y) + x^3 + 4y^2 = C_1$$


x^3
$e^x \cos(5y)$
$4y^2$



$$\frac{dV}{dt} = -kV^2$$