

# MÉTODO DEL FACTOR INTEGRANTE.

ECUACIONES, NO-LINEALES, NO-EXACTAS

$$x^2y + x^3y + x^4y^5 = C_1 \quad \text{SG}$$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} \cdot \frac{dy}{dx} = 0$$

$$2xy^2 + 3x^2y + 4x^3y^5 + \left(2xy + x^3 + 5x^4y^4\right) \frac{dy}{dx} = 0$$

$$\underbrace{2xy^2 + 3x^2y + 4x^3y^5}_{M(x,y)} + \underbrace{\left(2xy + x^2 + 5x^3y^4\right) \frac{dy}{dx}}_{N(x,y)} = 0$$

EXACTA.

$$\frac{\partial M}{\partial y} = 4xy + 3x^2 + 20x^3y^4$$

$$\frac{\partial N}{\partial x} = 4xy + 3x^2 + 20x^3y^4$$


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$$x \left(2y^2 + 3xy + 4x^3y^5\right) + x \left(2xy + x^2 + 5x^3y^4\right) \frac{dy}{dx} = 0$$

$$\left(2y^2 + 3xy + 4x^3y^5\right) + \left(2xy + x^2 + 5x^3y^4\right) \frac{dy}{dx} = 0$$

$$(2y^2 + 3xy + 4x^2y^5) + (2xy + x^2 + 5x^3y^4) \frac{dy}{dx} = 0$$

$M(x, y)$        $N(x, y)$

$$\frac{\partial M}{\partial y} = 4y + 3x + 20x^2y^4$$

NO-EXACTA

$$\frac{\partial N}{\partial x} = 2y + 2x + 15x^2y^4$$

EDO(1) NL. NE.

$$M(x,y) + N(x,y) \frac{dy}{dx} = 0$$

$\mu(x,y)$  es una función conocida como  
Factor Integrante cuando

$$\mu(x,y)M(x,y) + \mu(x,y)N(x,y) \frac{dy}{dx} = 0$$

EXACTA.

$$\mu M + \mu N \frac{dy}{dx} = 0 \quad \text{EXACTA.}$$

$$\frac{\partial}{\partial y}(\mu M) = \frac{\partial}{\partial x}(\mu N)$$

$$\frac{\partial \mu}{\partial y} M + \mu \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \frac{\partial N}{\partial x}$$

$$\mu(x, y)$$

EDeDP(L)

Hipótesis<sub>1</sub>,  $\mu \Rightarrow \mu(x)$

Hipótesis<sub>2</sub>  $\Rightarrow \mu \Rightarrow \mu(y)$

$$M(x) \frac{\partial M}{\partial y} = \frac{d\mu}{dx} N + M(x) \frac{\partial N}{\partial x}$$

$$\frac{d\mu}{dx} N = M(x) \frac{\partial M}{\partial y} - M(x) \frac{\partial N}{\partial x}$$

$$\frac{d\mu}{dx} N = M(x) \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\frac{d\mu}{m} = \left( \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) dx$$

$$f(x)$$

$$(2y^2 + 3xy + 4x^2y^5) + (2xy + x^2 + 5x^3y^4) \frac{dy}{dx} = 0$$

*M*                            *N*

$$\frac{\partial M}{\partial y} = 4y + 3x + 20x^2y^4 \quad \frac{\partial N}{\partial x} = 2y + 2x + 15x^2y^4$$

$$f(x) \left( \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \right) = \frac{(4y + 3x + 20x^2y^4) - (2y + 2x + 15x^2y^4)}{2xy + x^2 + 5x^3y^4}$$

$$= \frac{2y + x + 5x^2y^4}{x(2y + x + 5x^2y^4)}$$

$$f(x) = \frac{1}{x}$$

$$\frac{d\mu}{m} = \left(\frac{1}{x}\right)dx$$

$$\int \frac{d\mu}{m} = \int \frac{dx}{x}$$

$$d\mu = dx$$

$$\boxed{a(x) = x}$$

$$\frac{\partial \mu}{\partial y} M + \mu \frac{\partial M}{\partial y} = \frac{\partial \mu}{\partial x} N + \mu \frac{\partial N}{\partial x}$$

$$\mu \Rightarrow \mu(y)$$

$$\frac{d\mu}{dy} M + \mu(y) \frac{\partial M}{\partial y} = \mu(y) \frac{\partial N}{\partial x}$$

$$\frac{d\mu}{dy} M = \mu(y) \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right)$$

$$\frac{d\mu}{M} = \left( \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} \right) dy$$

$$\frac{dy}{dx} + p(x)y = 0$$

$$\frac{dy}{y} + p(x)dx = 0$$

$$y = C_1 e^{-\int p(x)dx}$$

$$\frac{p(x)y + \frac{dy}{dx}}{M(x,y)} = 1$$

$$\frac{\partial M}{\partial y} = p(x) \quad \frac{\partial N}{\partial x} = 0$$

$$f(x) \rightarrow \mu(x) \quad \frac{d\mu}{\mu} = \left( \frac{p(x) - (0)}{1} \right) dx$$

$$\frac{d\mu}{\mu} = p(x) dx$$

$$\int \frac{d\mu}{\mu} = \int p(x) dx$$

$$\ln \mu = \int p(x) dx$$

$$\mu = e^{\int p(x) dx}$$

$$e^{\int p(x)dx} M y + e^{\int p(x)dx} \frac{dy}{dx} = 0$$

MM                  NN

$$\frac{\partial M y}{\partial y} = e^{\int p(x)dx}$$

$$\frac{\partial N y}{\partial x} = e^{\int p(x)dx}$$

$$d\left(e^{\int p(x)dx} y\right) = 0$$

$$\int d\left(e^{\int p(x)dx} y\right) = C$$

$$e^{\int p(x)dx} y = C_1$$

$$y = C_1 e^{-\int p(x)dx}$$