```
> restart
> Ecuacion := x·diff(y(x), x) = sqrt(x··2 − y(x)··2) + y(x)
```

$$\text{Ecuacion} := x\left(\frac{d}{dx}\, y(x)\right) = \sqrt{x^2 - y(x)^2} + y(x) \tag{1}$$

```
> with(DEtools):
> odeadvisor(Ecuacion)
```

$$[[\_homogeneous,\, class\ A],\, \_rational,\, \_dAlembert] \tag{2}$$

```
> intfactor(Ecuacion)
```

$$\frac{1}{\sqrt{-(y(x)-x)\,(y(x)+x)}\;x} \tag{3}$$

```
> FactInt := 1/(sqrt(−(y−x)(y+x)) x)
```

$$\text{FactInt} := \frac{1}{\sqrt{-(y-x)\,(x+y)}\;x} \tag{4}$$

```
> M(x, y) := sqrt(x··2 − y··2) + y
```

$$M(x, y) := \sqrt{x^2 - y^2} + y \tag{5}$$

```
> N(x, y) := −x
```

$$N(x, y) := -x \tag{6}$$

```
> comprobacion₁ := simplify(diff(M(x, y), y) − diff(N(x, y), x)) = 0
```

$$\text{comprobacion}_1 := \frac{2\sqrt{x^2-y^2}-y}{\sqrt{x^2-y^2}} = 0 \tag{7}$$

```
> MM(x, y) := simplify(FactInt·M(x, y));
```

$$MM(x, y) := \frac{\sqrt{x^2-y^2}+y}{\sqrt{x^2-y^2}\;x} \tag{8}$$

```
> NN(x, y) := simplify(FactInt·N(x, y))
```

$$NN(x, y) := -\frac{1}{\sqrt{x^2-y^2}} \tag{9}$$

```
> comprobacion₂ := simplify(diff(MM(x, y), y) − diff(NN(x, y), x)) = 0
```

$$\text{comprobacion}_2 := 0 = 0 \tag{10}$$

```
> IntMMx := expand(int(MM(x, y), x))
```

$$\text{IntMMx} := \ln(x) - \frac{y\,\ln\left(\dfrac{-2y^2 + 2\sqrt{-y^2}\,\sqrt{x^2-y^2}}{x}\right)}{\sqrt{-y^2}} \tag{11}$$

```
> RestaInt := simplify(NN(x, y) − diff(IntMMx, y))
```

$$\text{RestaInt} := \frac{1}{\sqrt{-y^2}} \tag{12}$$

```
> IntRestaInt := int(RestaInt, y)
```

$$\tag{13}$$

$$IntRestaInt := \frac{y \ln(y)}{\sqrt{-y^2}} \tag{13}$$

> $SolucionGeneral := simplify(\ IntMMx + IntRestaInt\ ) = C1$

$$SolucionGeneral := \frac{\ln(x)\sqrt{-y^2} - y \ln(2) - y \ln\left(\dfrac{-y^2 + \sqrt{-y^2}\,\sqrt{x^2 - y^2}}{x}\right) + y \ln(y)}{\sqrt{-y^2}} = C1 \tag{14}$$

> $Ecuacion$

$$x \left(\frac{d}{dx}\, y(x)\right) = \sqrt{x^2 - y(x)^2} + y(x) \tag{15}$$

> $EcuacionSeparable := simplify(subs(y(x) = u(x) \cdot x, Ecuacion))$

$$EcuacionSeparable := x \left(\frac{d}{dx}\,(u(x)\,x)\right) = \sqrt{-x^2\,(-1 + u(x)^2)} + u(x)\,x \tag{16}$$

> $EcuacionSeparada := expand(simplify(\ isolate(EcuacionSeparable, diff(u(x), x))))$

$$EcuacionSeparada := \frac{d}{dx}\, u(x) = \frac{\sqrt{x^2 - u(x)^2\,x^2}}{x^2} \tag{17}$$

> $S(u) := \sqrt{1 - u^2}$

$$S(u) := \sqrt{1 - u^2} \tag{18}$$

> $P(x) := \dfrac{x}{x \cdot \cdot 2}$

$$P(x) := \frac{1}{x} \tag{19}$$

> $SolucionGral := int(P(x), x) + int\left(\dfrac{1}{S(u)}, u\right) = C1$

$$SolucionGral := \ln(x) + \arcsin(u) = C1 \tag{20}$$

> $SolFinal := subs\left(u = \dfrac{y}{x}, SolucionGral\right)$

$$SolFinal := \ln(x) + \arcsin\left(\frac{y}{x}\right) = C1 \tag{21}$$

> $SolucionGeneral;$

$$\frac{\ln(x)\sqrt{-y^2} - y \ln(2) - y \ln\left(\dfrac{-y^2 + \sqrt{-y^2}\,\sqrt{x^2 - y^2}}{x}\right) + y \ln(y)}{\sqrt{-y^2}} = C1 \tag{22}$$

> $SolFinalDos := \ln(x) + \arcsin\left(\dfrac{y(x)}{x}\right) = C1$

$$SolFinalDos := \ln(x) + \arcsin\left(\frac{y(x)}{x}\right) = C1 \tag{23}$$

> $DerSolFinal := simplify(isolate(diff(SolFinalDos, x), diff(y(x), x)))$

$$DerSolFinal := \frac{d}{dx}\, y(x) = -\frac{\sqrt{-\dfrac{-x^2 + y(x)^2}{x^2}}\,x - y(x)}{x} \tag{24}$$

> $Raiz_1 := isolate\left(DerSolFinal, \text{sqrt}\left(-\dfrac{-x^2 + y(x)^2}{x^2}\right)\right)$

$$Raiz_1 := \sqrt{-\dfrac{-x^2 + y(x)^2}{x^2}} = \dfrac{-x\left(\dfrac{d}{dx}\,y(x)\right) + y(x)}{x} \tag{25}$$

> $CuadRaiz_1 := lhs\left(Raiz_1\right)\cdot\cdot 2 = expand\left(rhs\left(Raiz_1\right)\cdot\cdot 2\right)$

$$CuadRaiz_1 := -\dfrac{-x^2 + y(x)^2}{x^2} = \left(\dfrac{d}{dx}\,y(x)\right)^2 - \dfrac{2\left(\dfrac{d}{dx}\,y(x)\right)y(x)}{x} + \dfrac{y(x)^2}{x^2} \tag{26}$$

> $DerEcua := isolate(Ecuacion, diff(y(x), x))$

$$DerEcua := \dfrac{d}{dx}\,y(x) = \dfrac{\sqrt{x^2 - y(x)^2} + y(x)}{x} \tag{27}$$

> $Raiz_2 := isolate\left(DerEcua, \text{sqrt}\left(x^2 - y(x)^2\right)\right)$

$$Raiz_2 := \sqrt{x^2 - y(x)^2} = x\left(\dfrac{d}{dx}\,y(x)\right) - y(x) \tag{28}$$

> $CuadRaiz_2 := lhs\left(Raiz_2\right)\cdot\cdot 2 = expand\left(rhs\left(Raiz_2\right)\cdot\cdot 2\right)$

$$CuadRaiz_2 := x^2 - y(x)^2 = x^2\left(\dfrac{d}{dx}\,y(x)\right)^2 - 2x\left(\dfrac{d}{dx}\,y(x)\right)y(x) + y(x)^2 \tag{29}$$

> $Der_1 := simplify\left(lhs\left(CuadRaiz_1\right) - rhs\left(CuadRaiz_1\right)\right) = 0$

$$Der_1 := -\dfrac{-x^2 + 2y(x)^2 + x^2\left(\dfrac{d}{dx}\,y(x)\right)^2 - 2x\left(\dfrac{d}{dx}\,y(x)\right)y(x)}{x^2} = 0 \tag{30}$$

> $Der_2 := lhs\left(CuadRaiz_2\right) - rhs\left(CuadRaiz_2\right) = 0$

$$Der_2 := x^2 - 2y(x)^2 - x^2\left(\dfrac{d}{dx}\,y(x)\right)^2 + 2x\left(\dfrac{d}{dx}\,y(x)\right)y(x) = 0 \tag{31}$$

> $comprobacion_3 := simplify\left(expand\left(lhs\left(Der_1\right) - lhs\left(Der_2\right)\right)\right) = 0$

$$comprobacion_3 := \dfrac{1}{x^2}\left(x^2 - 2y(x)^2 - x^2\left(\dfrac{d}{dx}\,y(x)\right)^2 + 2x\left(\dfrac{d}{dx}\,y(x)\right)y(x) - x^4\right. \tag{32}$$
$$\left. + 2y(x)^2 x^2 + \left(\dfrac{d}{dx}\,y(x)\right)^2 x^4 - 2\left(\dfrac{d}{dx}\,y(x)\right)y(x)x^3\right) = 0$$

> $dsolve(Ecuacion)$

$$-\arctan\left(\dfrac{y(x)}{\sqrt{x^2 - y(x)^2}}\right) + \ln(x) - \_C1 = 0 \tag{33}$$

> $SolFinalDos;$

$$\ln(x) + \arcsin\left(\dfrac{y(x)}{x}\right) = C1 \tag{34}$$

> $restart$

> $Ecua := diff(y(x), x) = \dfrac{2\cdot x\cdot y(x)}{3\cdot x\cdot\cdot 2 - y(x)\cdot\cdot 2}$

$$\tag{35}$$

$$Ecua := \frac{d}{dx} y(x) = \frac{2 x y(x)}{3 x^2 - y(x)^2} \tag{35}$$

> $with(DEtools);$

$[AreSimilar, DEnormal, DEplot, DEplot3d, DEplot\_polygon, DFactor, DFactorLCLM,$ \hfill (36)
  $DFactorsols, Dchangevar, FunctionDecomposition, GCRD, Gosper, Heunsols,$
  $Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols,$
  $MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm,$
  $RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge,$
  $Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot,$
  $casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys,$
  $dalembertsol, dcoeffs, de2diffop, dfieldplot, diff\_table, diffop2de, dperiodic\_sols, dpolyform,$
  $dsubs, eigenring, endomorphism\_charpoly, equinv, eta\_k, eulersols, exactsol, expsols,$
  $exterior\_power, firint, firtest, formal\_sol, gen\_exp, generate\_ic, genhomosol, gensys,$
  $hamilton\_eqs, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate\_sols,$
  $intfactor, invariants, kovacicsols, leftdivision, liesol, line\_int, linearsol, matrixDE,$
  $matrix\_riccati, maxdimsystems, moser\_reduce, muchange, mult, mutest, newton\_polygon,$
  $normalG2, ode\_int\_y, ode\_y1, odeadvisor, odepde, parametricsol, particularsol,$
  $phaseportrait, poincare, polysols, power\_equivalent, rational\_equivalent, ratsols, redode,$
  $reduceOrder, reduce\_order, regular\_parts, regularsp, remove\_RootOf, riccati\_system,$
  $riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve\_group,$
  $super\_reduce, symgen, symmetric\_power, symmetric\_product, symtest, transinv, translate,$
  $untranslate, varparam, zoom]$

> $odeadvisor(Ecua)$

$$[[\_homogeneous, class A], \_rational, \_dAlembert] \tag{37}$$

> $EcuaSeparable := eval(subs(y(x) = u(x) \cdot x, Ecua))$

$$EcuaSeparable := \left(\frac{d}{dx} u(x)\right) x + u(x) = \frac{2 x^2 u(x)}{3 x^2 - u(x)^2 x^2} \tag{38}$$

> $EcuaSep := simplify(isolate(EcuaSeparable, diff(u(x), x)))$

$$EcuaSep := \frac{d}{dx} u(x) = -\frac{u(x) (-1 + u(x)^2)}{x (-3 + u(x)^2)} \tag{39}$$

> $Q(u) := \dfrac{u(-1 + u^2)}{(-3 + u^2)}$

$$Q(u) := \frac{u(-1 + u^2)}{-3 + u^2} \tag{40}$$

> $P(x) := x;$

$$P(x) := x \tag{41}$$

> $Solucion := int\left(\frac{1}{P(x)}, x\right) + int\left(\frac{1}{Q(u)}, u\right) = C1$

$$Solucion := \ln(x) + 3 \ln(u) - \ln(u - 1) - \ln(u + 1) = C1 \tag{42}$$

> $SolucionGeneral := simplify\left(subs\left(u = \frac{y}{x}, Solucion\right)\right)$

(43)

$$SolucionGeneral := \ln(x) + 3 \ln\left(\frac{y}{x}\right) - \ln\left(-\frac{-y+x}{x}\right) - \ln\left(\frac{y+x}{x}\right) = C1 \qquad \textbf{(43)}$$