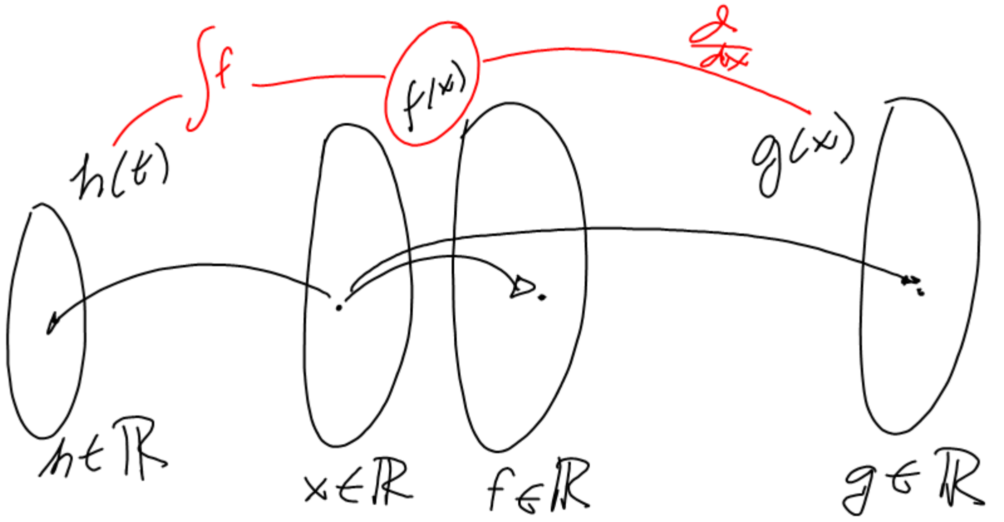
Chapter 4.- Laplace Transform as a method of solution for Ordinary Differential Equations with initial condition

¿What is a Transform?

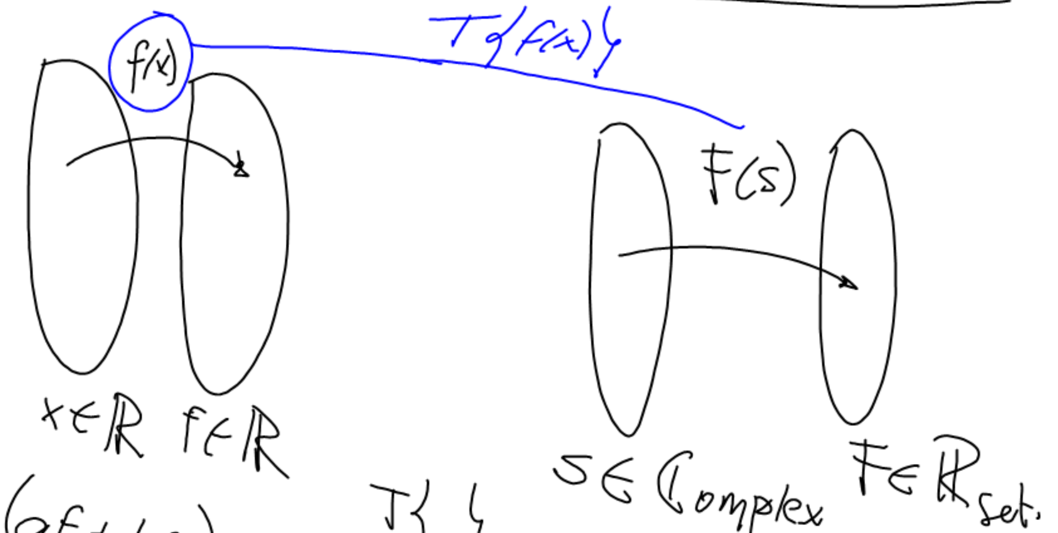
we have operators over Real funtion of real variable. like Derivates and Integers. A transform is another kind of operator over a funtion

$f(x) \quad g(x) = \frac{df}{dx} \quad h(x) = \int f \, dx$

$F(s) = T\{f(x)\}$



normal operator over a function



$a,b\in\mathbb{R}$ $\quad (af+bg) \xrightarrow{T\{\ \}} aF+bG$ Linnear

$$\frac{df}{dx} \xrightarrow{T\{\ \}} sF(s)$$

$$\int f\,dx \longrightarrow \frac{F(s)}{s}$$

Cheap

Electronic Circuits

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you use the Laplace Transform directly.

Mathematical Definition

$$T\{f(t)\} = \int_{-\infty}^{\infty} \underbrace{N(t,s)}_{\text{Nucleous}}\, \underbrace{f(t)}_{\text{Argument}}\, dt \Rightarrow F(s)$$

operator

$$\mathcal{L}\{f(t)\} \longrightarrow N(t,s) = \begin{cases} 0 \ ; & t \leq 0 \\ e^{-st} \ ; & t > 0 \end{cases}$$

$$\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

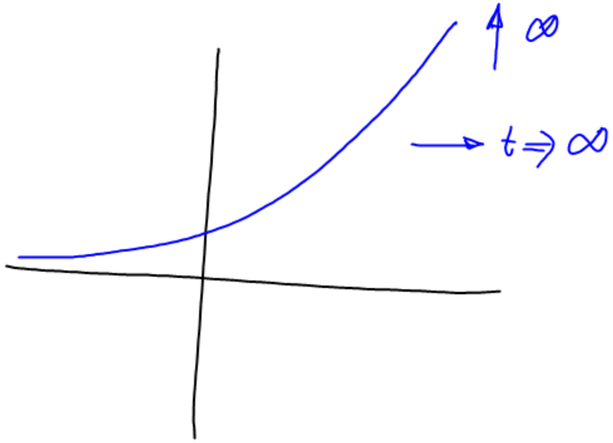
$t \in \mathbb{R}^{+}$

Example: $f(t) = 1$

$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\,dt \Rightarrow \left[\int e^{-st}\,dt\right]_0^\infty \Rightarrow \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

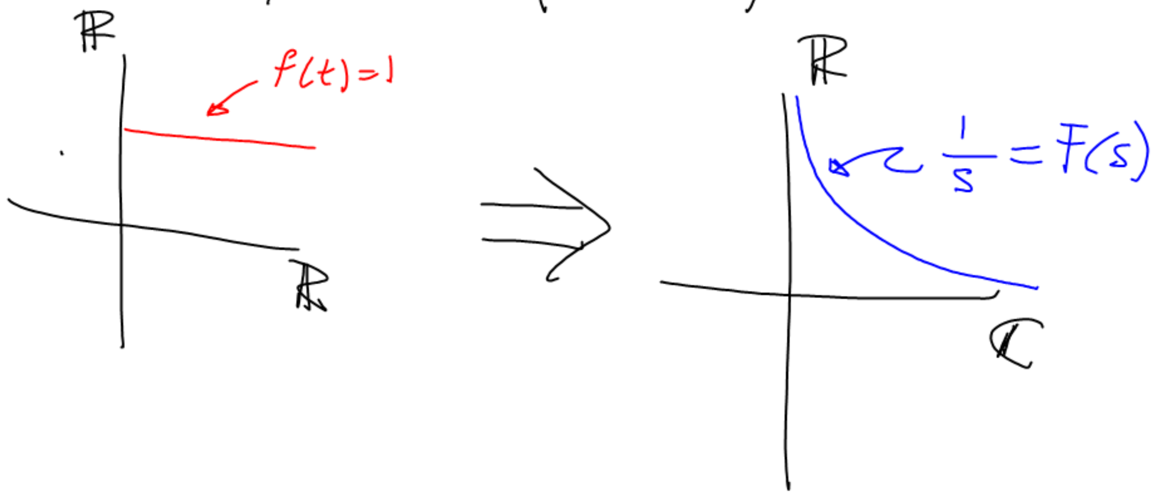
$$\mathcal{L}\{1\} = \frac{1}{-s}\left(\lim_{t\to\infty} e^{-st} - 1\right)$$

$$\lim_{t\to\infty} e^{-st} = \lim_{t\to\infty}\left(\frac{1}{e^{st}}\right) \Rightarrow \lim_{b\to\infty}\frac{1}{b} \Rightarrow 0$$



$$\mathcal{L}\{1\} = -\frac{1}{s}(0-1) = \frac{1}{s}$$

$s \in$ Comp.
$F(s) \in \mathbb{R}$



$f(t)$	$F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$ $a \in \mathbb{R}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$ $b \in \mathbb{R}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$ $b \in \mathbb{R}$