Laplace Transform

has also Inverse Laplace Transform

$$\mathcal{L}\{f(t)\} = F(s)$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

but this is not unique

$$\mathcal{L}^{-1}\{F(s)\} = \int_{a-\infty i}^{a+\infty i} e^{st} F(s)\, ds$$

$F \in \mathbb{R}$

$s \in$ Complex

$$\mathcal{L}\{f(t)\} = \int_{0}^{\infty} e^{-st} f(t)\, dt$$

$$\mathcal{L}\{f(t)\} = F(s) \iff \mathcal{L}^{-1}\{F(s)\} = f(t)$$

$$\mathcal{L}\{g(t)\} = G(s) \iff \mathcal{L}^{-1}\{G(s)\} = g(t)$$

1

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

$$a, b \in \mathbb{R}$$

$$\mathcal{L}\{5 - 2t\} = 5\mathcal{L}\{1\} - 2\mathcal{L}\{t\}$$

$$= \frac{5}{s} - \frac{2 \cdot 1!}{s^2} = \frac{5}{s} - \frac{2}{s^2}$$

$$\mathcal{L}^{-1}\left\{\frac{8}{s^3}+\frac{\sqrt{2}}{s^5}\right\}=8\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}+\sqrt{2}\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}$$

$$\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}$$

$$=\frac{8}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{s^3}\right\}+\frac{\sqrt{2}}{4!}\mathcal{L}^{-1}\left\{\frac{4!}{s^5}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{8}{s^3}+\frac{\sqrt{2}}{s^5}\right\}=4t^2+\frac{\sqrt{2}}{24}t^4$$

2

$$\mathcal{L}\{f(at)\} = \frac{1}{a} \cdot F\left(\frac{s}{a}\right) \quad a \in \mathbb{R}$$

$$\mathcal{L}\{\sin(t)\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{\sin(3t)\} = \frac{1}{3} \cdot \left(\frac{1}{\left(\frac{s}{3}\right)^2+1}\right)$$

$$= \frac{1}{3} \cdot \left(\frac{1}{\frac{s^2}{9}+1}\right)$$

$$= \frac{1}{3} \left(\frac{1}{\frac{s^2+9}{9}}\right)$$

$$= \frac{1}{3} \left(\frac{9}{s^2+9}\right)$$

$$= \frac{3}{s^2+9}$$

$$\mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}$$

③

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - f^{(n-1)}(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

ODE(1) $$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0 \qquad y(0) = 4, \quad y'(0) = -5$$

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y\right\} = \mathcal{L}\{0\}$$

$y(t)$ $$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 3\mathcal{L}\left\{\frac{dy}{dt}\right\} + 2\mathcal{L}\{y\} = (0)\mathcal{L}\{1\}$$

$$\left[s^2Y(s) - s(4) - (-5)\right] - 3\left[sY(s) - (4)\right] + 2Y(s) = 0$$

$$(s^2 - 3s + 2)Y(s) + (-4s + 17) = 0$$

$$(s^2 - 3s + 2)Y(s) = +4s - 17$$

PSLT

$$Y(s) = \frac{4s-17}{s^2-3s+2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{4s-17}{s^2-3s+2}\right\}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{13}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{-9}{s-2}\right\}$$

$$y(t) = 13\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} - 9\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}$$

$$\boxed{y(t) = 13e^{t} - 9e^{+2t}}$$ Particular Solution

$$\frac{P(t)}{S(t)} = \frac{A}{R(t)} + \frac{B}{Q(t)} + \frac{C}{N(t)} + \ldots\ldots$$

order P < order S

$$\frac{4s-17}{s^2-3s+2} = \frac{A}{(s-1)} + \frac{B}{(s-2)}$$

$$(s-1)(s-2)$$

$$4s-17 = A(s-2) + B(s-1)$$

if $s=2$

$$4(2)-17 = A(0) + B(1)$$

$$\boxed{B=-9}$$

if $s=1$

$$4(1)-17 = A(-1) + B(0)$$

$$\boxed{A=13}$$

clave : 1306

grupo : 0007   JUAN URSUL SOLANES

tipo : Teoría

Num prof : 1

Semestre : 2012-2