properties of Laplace Transform

① $\mathcal{L}\{af(t)+bg(t)\} = aF(s)+bG(s)$

$a,b \in \mathbb{R}$

② $\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$

③ $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - \left(\sum_{i=1}^{n} s^{n-i} f^{(i-1)}(0)\right)$$

④ $\mathcal{L}^{-1}\{F'(s)\} = -tf(t)$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

5 $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{F(s)}{s}$

6 $\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \dfrac{f(t)}{t}$; if $\int_s^\infty F(\sigma)\,d\sigma$ is convergent

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7 $\boxed{\mathcal{L}\left\{e^{at} f(t)\right\} = F(s-a)}$

Example

$\mathcal{L}\{\cos(4t)\} = \dfrac{s}{s^2+4^2}$

$\mathcal{L}\{e^{2t}\cos(4t)\} = \dfrac{(s-2)}{(s-2)^2+16}$

$\mathcal{L}\{t^2\} = \dfrac{2!}{s^3}$

$\mathcal{L}\{t^2 e^{3t}\} = \dfrac{2}{(s-3)^3}$

$\mathcal{L}\{1\} = \dfrac{1}{s}$

$\mathcal{L}\{e^{-5t}\} = \dfrac{1}{(s+5)}$

8 $\mathcal{L}^{-1}\left\{e^{-bs}\cdot F(s)\right\} = f(t-b)\cdot\mu(t-b)$

7 $\mathcal{L}\left\{e^{at} f(t)\right\} = F(s-a)$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+3s+3}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+3s)+3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{\left(s^2+3s+\left(\frac{3}{2}\right)^2\right)+3-\left(\frac{9}{4}\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{\left(s+\frac{3}{2}\right)^2+\left(\frac{3}{4}\right)}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{\left(s+\frac{3}{2}\right)-\frac{3}{2}}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{\left(s+\frac{3}{2}\right)}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\} - \frac{3}{2}\mathcal{L}^{-1}\left\{\frac{1}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= e^{-\frac{3}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{3}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+3s+3}\right\} = e^{-\frac{3}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \sqrt{3}\,e^{-\frac{3}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)$$

Theorem.

Existence and unity of a L.T.

$$\mathcal{L}\{f(t)\} = F(s)$$

A $f(t)$ has Laplace Transform when in "A" class funtion

A function is "A" class when:

a) has exponential order

b) sectional continuous.

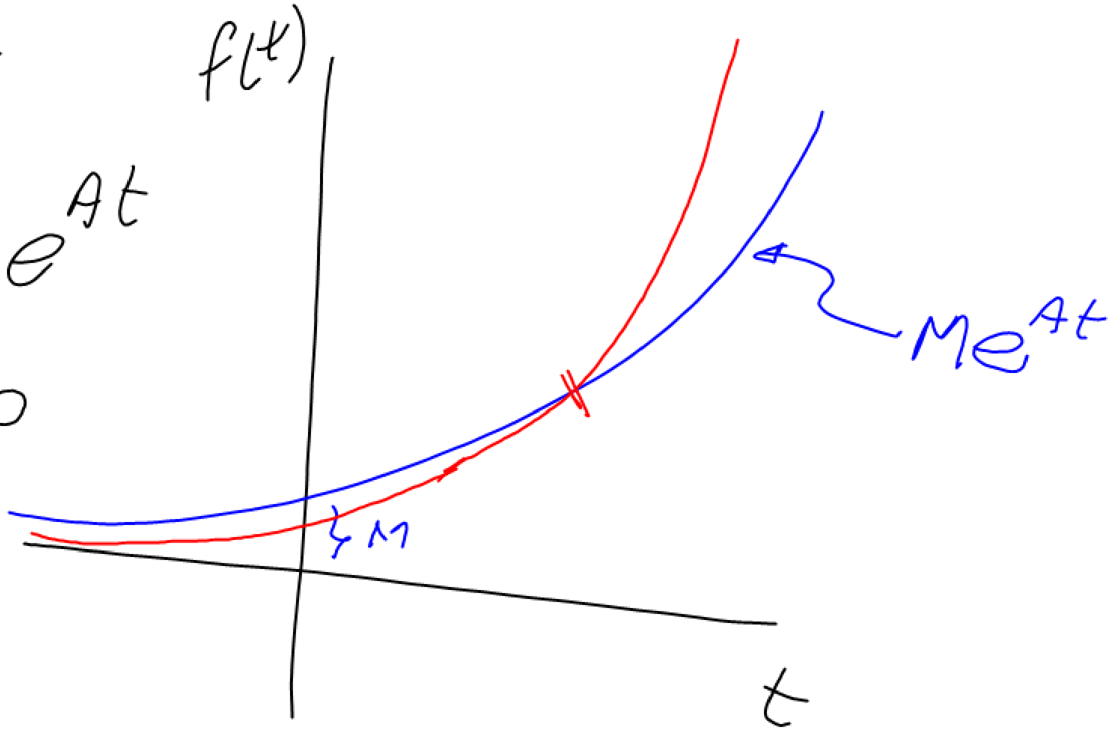
a) when

$$|f(t)| \leq Me^{At} \quad M, A \in \mathbb{R}$$

e^{t^3} has not Laplace Transform

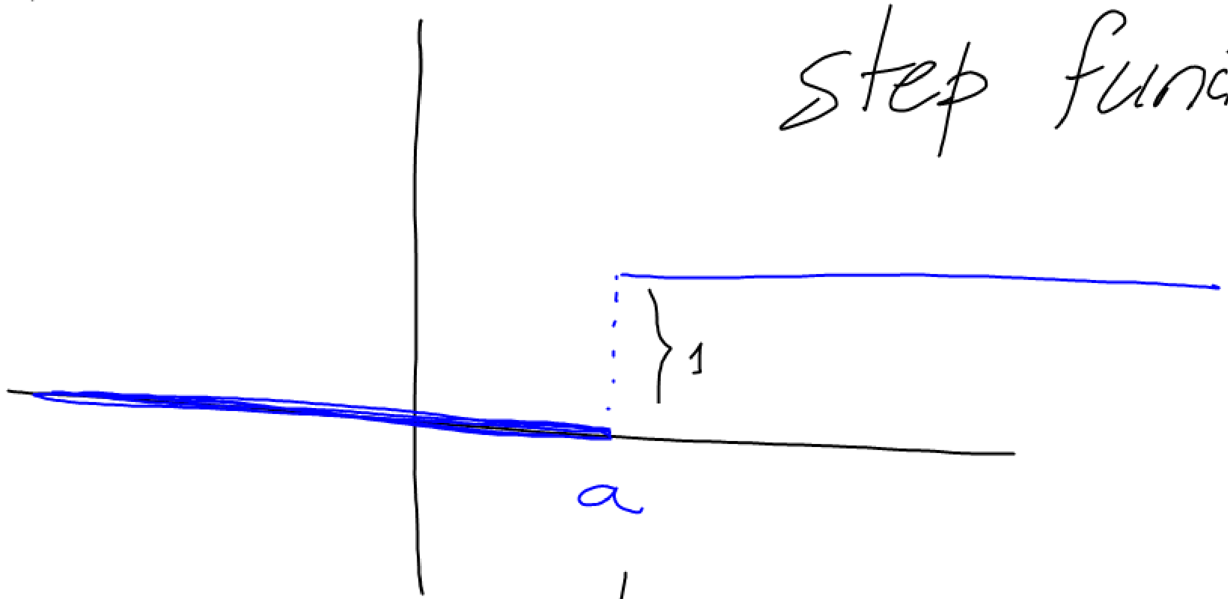
$$|f(t)| \le Me^{At}$$

$$-\infty \le t \le \infty$$



b) sectional continnous.

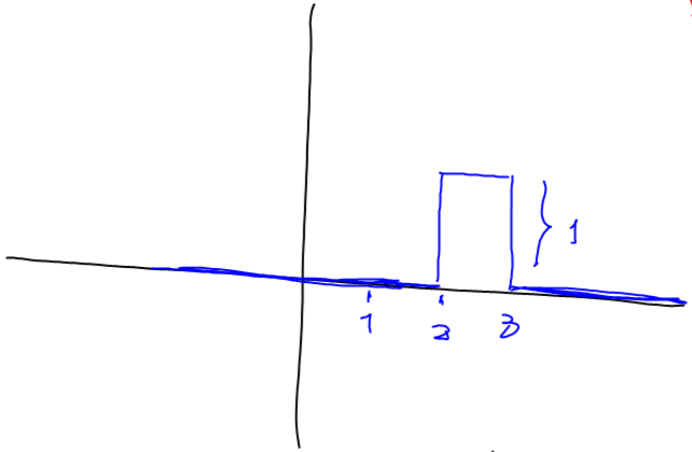
step function



$$u(t-a) = \begin{cases} 0 \; ; & t \leq a \\ 1 \; ; & t > a \end{cases}$$

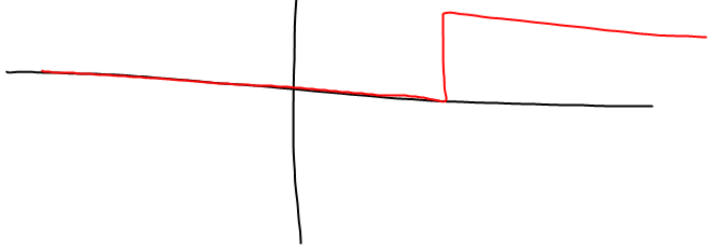
$$\text{bit} = \mu(t-2) - \mu(t-3)$$

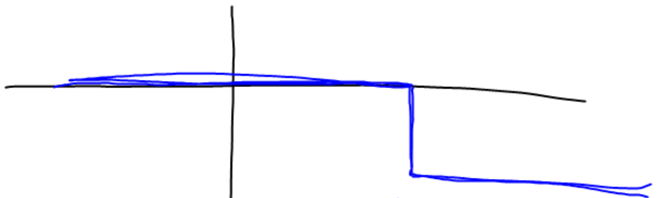
Heaviside



0 0 1 0 0

$+\mu(t-2)$





$-\mu(t-3)$

$$\mathcal{L}^{-1}\left\{\frac{e^{-5s}}{s^4}\right\}$$

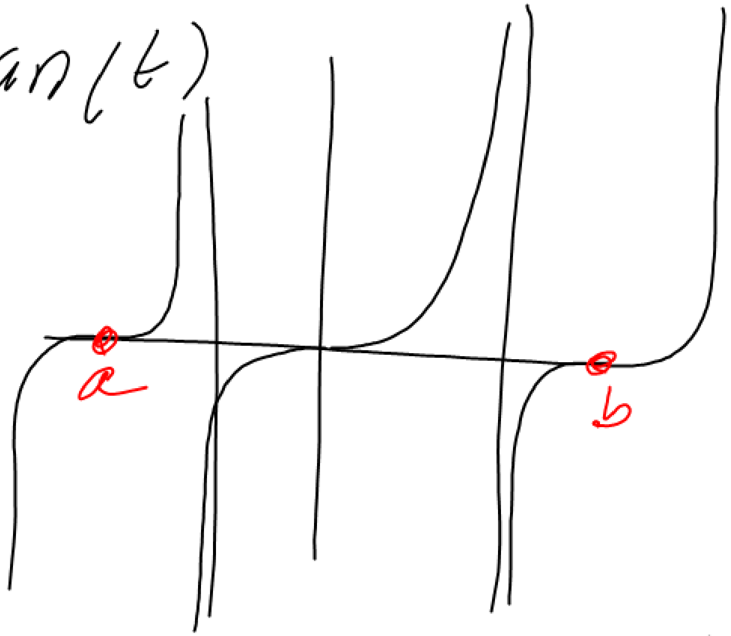
$$\mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\} = \frac{1}{3!}\mathcal{L}^{-1}\left\{\frac{3!}{s^4}\right\}$$

$$= \frac{1}{6}t^3$$

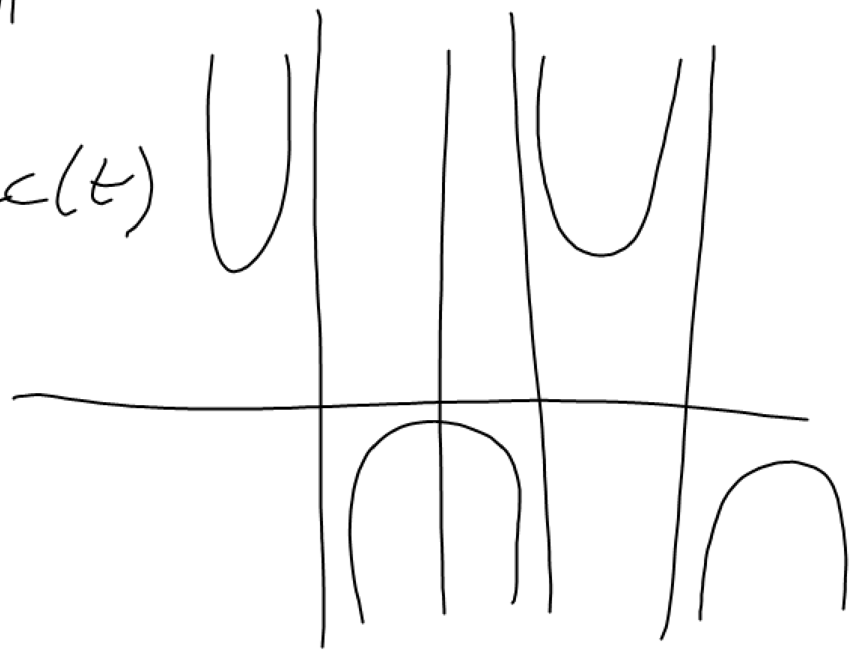
$$\mathcal{L}^{-1}\left\{\frac{e^{-5s}}{s^4}\right\} = \frac{1}{6}(t-5)^3 \cdot u(t-5)$$

$$\frac{1}{6}(t-5)^3 \cdot u(t-5) = \begin{cases} 0 & ; \ t \leq 5 \\ \frac{1}{6}(t-5)^3 & ; \ t > 5 \end{cases}$$

tan(t)

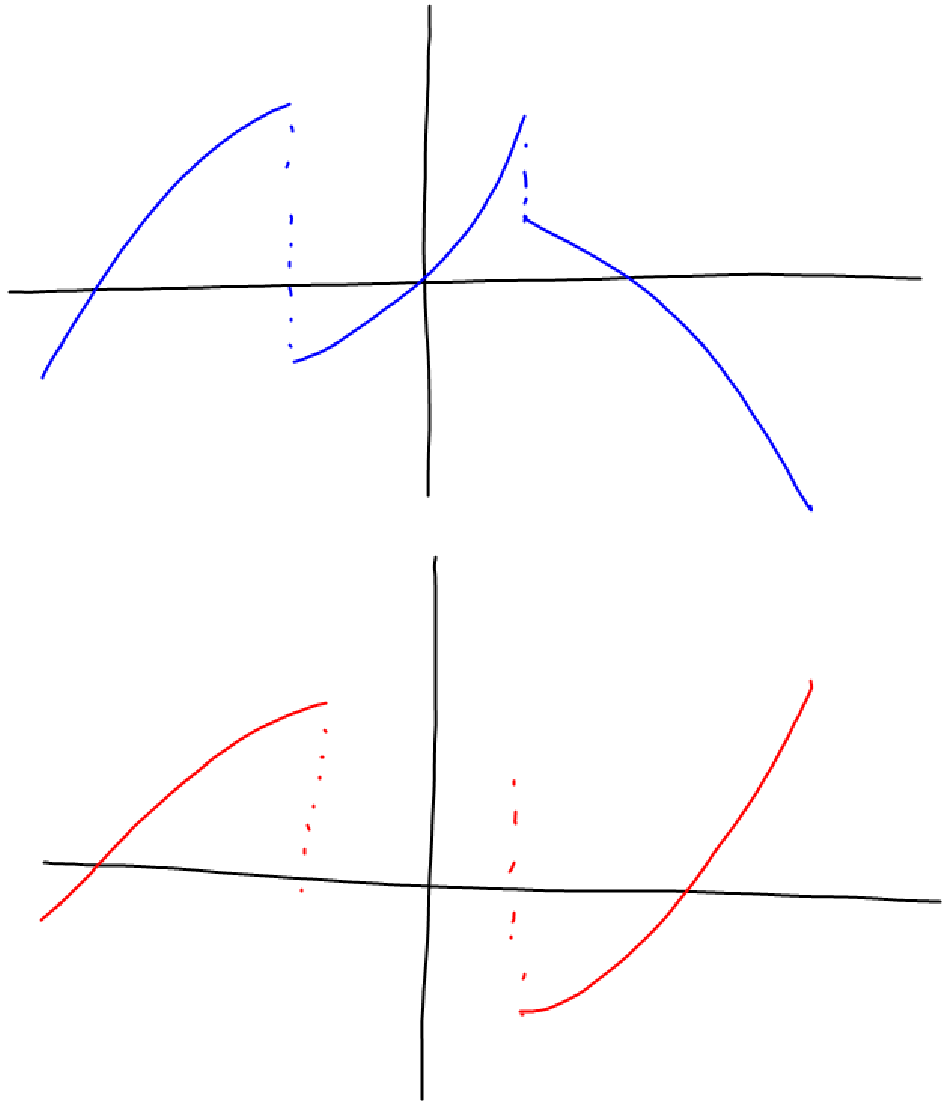


sec(t)



$a < t < b$

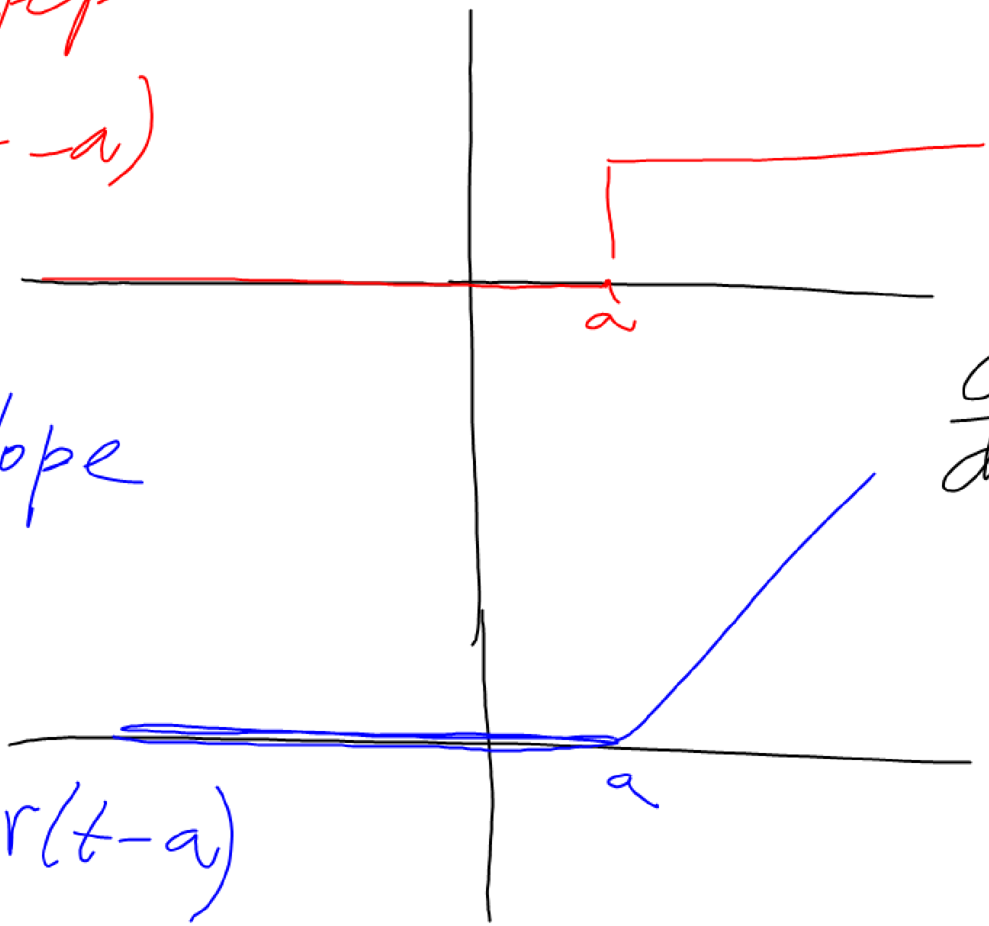
finite discont.



step

$u(t-a)$

slope



$r(t-a)$

$$\frac{d}{dt}\left\{r(t-a)\right\} = u(t-a)$$