$$x(t) + \int_0^t \tau e^{2\tau} x(t-\tau)\, d\tau = t e^{2t}$$

$$\mathcal{L}\left\{ x(t) + \int_0^t \tau e^{2\tau} x(t-\tau)\, d\tau \right\} = \mathcal{L}\left\{ t e^{2t} \right\}$$

$$X(s) + \mathcal{L}\left\{ \left(t e^{2t}\right) * x(t) \right\} = \frac{1}{(s-2)^2}$$

$$X(s) + \mathcal{L}\left\{ t e^{2t} \right\} \cdot X(s) = \frac{1}{(s-2)^2}$$

$$X(s) + \frac{1}{(s-2)^2} \cdot X(s) = \frac{1}{(s-2)^2}$$

$$X(s)\left(1 + \frac{1}{(s-2)^2}\right) = \frac{1}{(s-2)^2}$$

$$X(s) = \frac{\frac{1}{(s-2)^2}}{\left(1 + \frac{1}{(s-2)^2}\right)}.$$

$$X(s) = \frac{\frac{1}{\cancel{(s-2)^2}}}{\frac{(s-2)^2+1}{\cancel{(s-2)^2}}} \Rightarrow \frac{1}{(s-2)^2+1}$$

$$\mathcal{L}^{-1}\left\{ X(s) \right\} = \mathcal{L}^{-1}\left\{ \frac{1}{(s-2)^2+1} \right\}$$

$$x(t) = e^{2t} \operatorname{sen}(t)$$

$$\mathcal{L}^{-1}\left\{ F(s)\cdot G(s) \right\} = \int_0^t f(\tau)\cdot g(t-\tau)\, d\tau$$

$f(t) * g(t)$

$$\mathcal{L}^{-1}\left\{ \frac{b}{s^2+b^2} \right\} = \operatorname{sen}(bt)$$

$$\mathcal{L}^{-1}\left\{ \frac{b}{(s-a)^2+b^2} \right\} = e^{at}\operatorname{sen}(bt)$$

$y(x,t)$ incógnita

$$\frac{\partial^2 y}{\partial x^2} - 9\frac{\partial y}{\partial t} = 0$$

Método de Separación Variables para ED en DP.

- prueba y error
- hipótesis inicial $\quad y(x,t) = F(x)\,G(t)$ $\quad \left\{ \begin{array}{l} \frac{F}{G} \quad F+G \\ F(x)^t \quad G(t)^* \\ \mathcal{L}F \cdot \mathcal{L}G \end{array} \right.$
- sustituimos la hipótesis y sus derivadas parciales en la ED en DP.
- Separar miembro a miembro.

$$\frac{\partial^2 y}{\partial x^2} - 9\frac{\partial y}{\partial t} = 0 \qquad y(x,t) = F(x)G(t)$$

$$\frac{\partial y}{\partial x} = F'(x)G(t) \qquad \frac{\partial y}{\partial t} = F(x)G'(t)$$

$$\frac{\partial^2 y}{\partial x^2} = F''(x)G(t)$$

$$F''(x)G(t) - 9F(x)G'(t) = 0$$

$$F''(x)G(t) = 9F(x)G'(t)$$

$$\boxed{\frac{F''(x)}{9F(x)} = \frac{G'(t)}{G(t)}} \quad \text{razón (\# abstracto)}$$

$$\frac{F''(x)}{9F(x)} = \alpha \qquad \frac{G'(t)}{G(t)} = \alpha$$

$$\frac{d^2F}{dx^2} = 9\alpha F \qquad \frac{dG}{dt} = \alpha G \qquad y(x,t) = F(x)\cdot G(t)$$

EDO(2)LCCH. EDO(1)LCCH.

$$\frac{d^2F}{dx^2} - 9\alpha F = 0 \qquad \frac{dG}{dt} - \alpha G = 0$$

$\alpha = 0$

$$\frac{d^2F}{dx^2} = 0 \to \frac{dF}{dt} = k_1 \longrightarrow F(x) = k_1 x + k_2$$

$$\frac{dG}{dt} = 0 \to G(t) = C_1 \qquad \underset{g}{y}^{\alpha=0} = (k_1 x + k_2) C_1$$

$\alpha > 0 \quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{d^2F(x)}{dx^2} - 9\beta^2 F(x) = 0 \qquad \frac{dG}{dt} - \beta^2 G = 0$$

$$m^2 - 9\beta^2 = 0 \qquad \boxed{G(t) = C_1 e^{\beta^2 t}}$$

$$(m - 3\beta)(m + 3\beta) = 0$$

$$m_1 = 3\beta \quad m_2 = -3\beta \qquad \underset{g}{y}^{\alpha>0}(x,t) = \left(k_1 e^{3\beta x} + k_2 e^{-3\beta x}\right) C_1 e^{\beta^2 t}$$

$$\boxed{\underset{\alpha>0}{F(x)} = k_1 e^{3\beta x} + k_2 e^{-3\beta x}}$$

$\alpha < 0 \quad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{d^2F}{dx^2} + 9\beta^2 F = 0 \qquad \frac{dG}{dt} + \beta^2 G = 0$$

$$\boxed{G(t) = C_1 e^{-\beta^2 t}}$$

$m^2 + 9\beta^2 = 0$

$$\boxed{F(x) = k_1 \cos(3\beta x) + k_2 \operatorname{sen}(3\beta x)}$$

$\alpha < 0$

$$y(x,t) = \left(k_1 \cos(3\beta x) + k_2 \operatorname{sen}(3\beta x)\right) C_1 e^{-\beta^2 t}$$

g

$$y = C_{10} e^{-\beta^2 t} \cos(3\beta x) + C_{20} e^{-\beta^2 t} \operatorname{sen}(3\beta x)$$

$$\frac{\partial^2 y}{\partial x^2} - 9\frac{\partial y}{\partial t} = 0$$

$$\frac{\partial y}{\partial x} = \left(C_{10} e^{-\beta^2 t}\right)\left(-3\beta \operatorname{sen}(3\beta x)\right) + C_{20} e^{-\beta^2 t}\left(3\beta \cos(3\beta x)\right)$$

$$\frac{\partial y}{\partial x} = -3\beta C_{10} e^{-\beta^2 t} \operatorname{sen}(3\beta x) + 3\beta C_{20} e^{-\beta^2 t} \cos(3\beta x)$$

$$\frac{\partial^2 y}{\partial x^2} = -9\beta^2 C_{10} e^{-\beta^2 t} \cos(3\beta x) - 9\beta^2 C_{20} e^{-\beta^2 t} \operatorname{sen}(3\beta x)$$

$$-9\frac{\partial y}{\partial t} = +9 C_{10} \cos(3\beta x)\left(\beta^2 e^{-\beta^2 t}\right) + 9 C_{20} \operatorname{sen}(3\beta x)\left(\beta^2 e^{-\beta^2 t}\right)$$

$$\frac{\partial^2 y}{\partial x^2} - 9\frac{\partial y}{\partial t} = (0) e^{-\beta^2 t} \cos(3\beta x) + (0) e^{-\beta^2 t} \operatorname{sen}(3\beta x)$$

$$\frac{\partial^2 y}{\partial x^2} - 9\frac{\partial y}{\partial t} = 0.$$