> restart

> $f(x) := x\cdot\cdot 2 - 6\cdot x + 8;$

$$f(x) := x^2 - 6\,x + 8 \tag{1}$$

> $L := 4$

$$L := 4 \tag{2}$$

> $a_0 := \left(\frac{1}{L}\right)\cdot int(f(x), x = -L..L)$

$$a_0 := \frac{80}{3} \tag{3}$$

> $C := \frac{a_0}{2}$

$$C := \frac{40}{3} \tag{4}$$

> $a_n := subs\left(\sin(n\cdot\mathrm{Pi}) = 0, \cos(n\cdot\mathrm{Pi}) = (-1)\cdot\cdot n, \left(\frac{1}{L}\right)\cdot int\left(f(x)\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right), x = -L..L\right)\right)$

$$a_n := \frac{64\,(-1)^n}{n^2\,\pi^2} \tag{5}$$

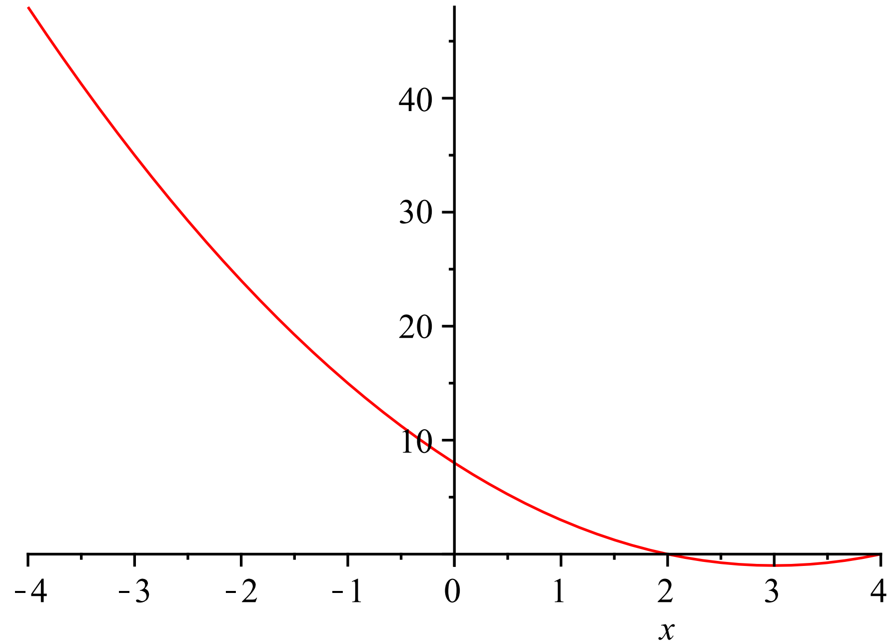
> $b_n := subs\left(\sin(n\cdot\mathrm{Pi}) = 0, \cos(n\cdot\mathrm{Pi}) = (-1)\cdot\cdot n, \left(\frac{1}{L}\right)\cdot int\left(f(x)\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right), x = -L..L\right)\right)$

$$b_n := \frac{48\,(-1)^n}{n\,\pi} \tag{6}$$

> $STF := C + Sum\left(a_n\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right) + b_n\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot x}{L}\right), n = 1..infinity\right)$

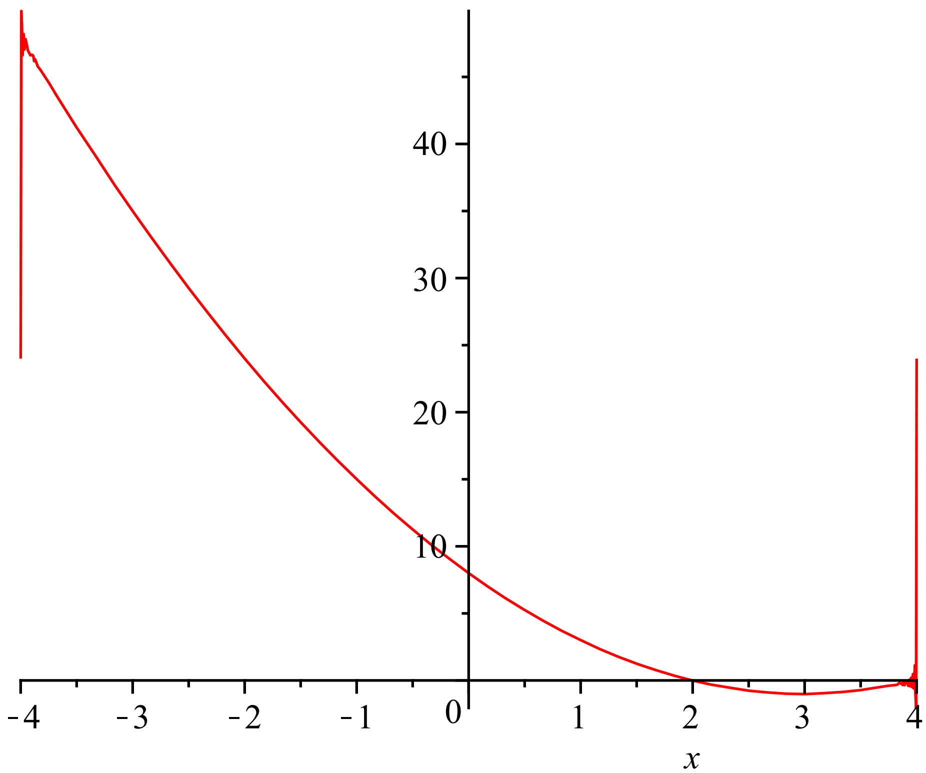
$$STF := \frac{40}{3} + \sum_{n=1}^{\infty}\left(\frac{64\,(-1)^n\cos\left(\frac{1}{4}\,n\,\pi\,x\right)}{n^2\,\pi^2} + \frac{48\,(-1)^n\sin\left(\frac{1}{4}\,n\,\pi\,x\right)}{n\,\pi}\right) \tag{7}$$

> $plot(f(x), x = -L..L)$

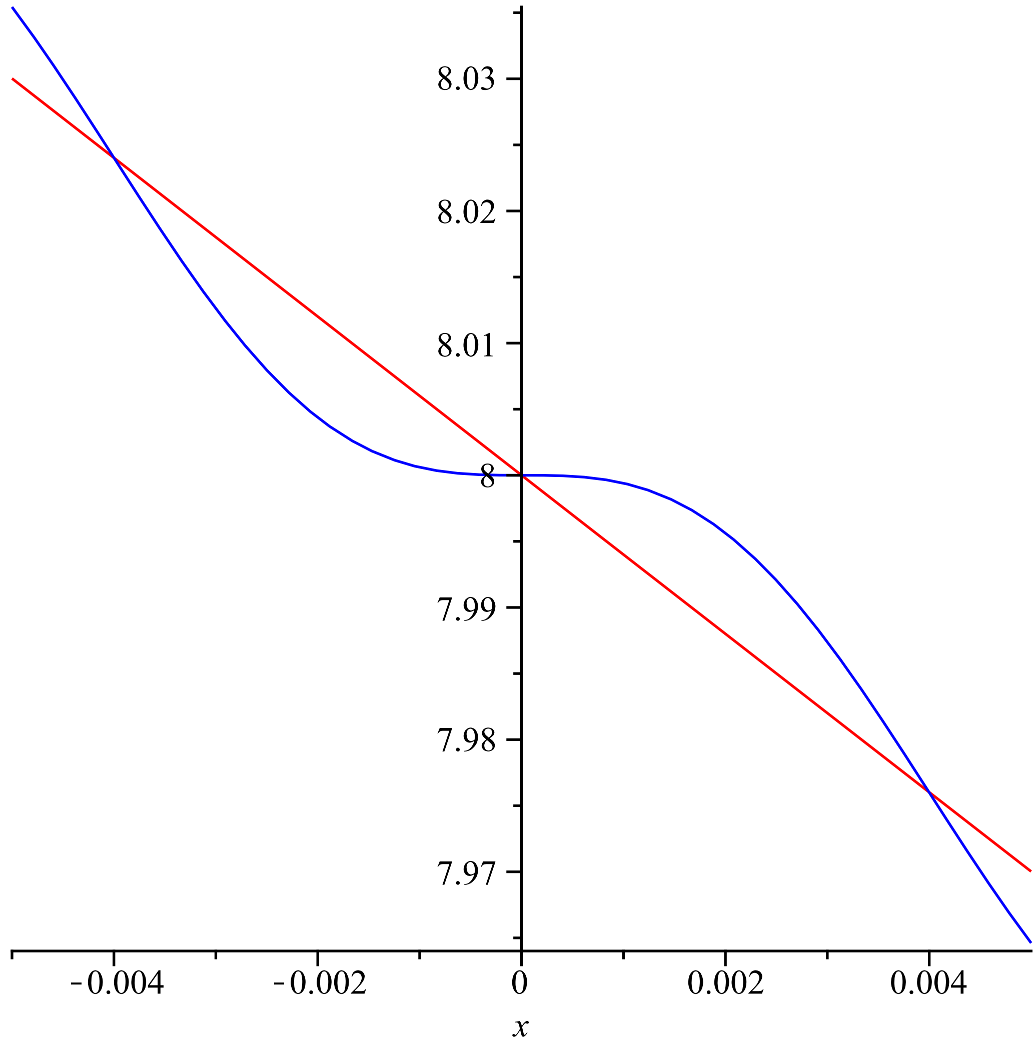


> $STF_{1000} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot x}{L}\right), n = 1 ..1000\right)$:

> $plot(STF_{1000}, x = -L ..L)$



> $plot([f(x), STF_{1000}], x = -0.005 ..0.005, color = [red, blue])$



> *restart*

> $f(t) := \exp(2\,t)$

$$f(t) := e^{2t} \tag{8}$$

> $L := 1$

$$L := 1 \tag{9}$$

> $a_0 := \left(\frac{1}{L}\right) \cdot int(f(t), t = -L..L);\ evalf(\%)$

$$a_0 := -\frac{1}{2}\,e^{-2} + \frac{1}{2}\,e^{2}$$
$$3.626860408 \tag{10}$$

> $C := \frac{a_0}{2};\ evalf(\%)$

$$C := -\frac{1}{4}\,e^{-2} + \frac{1}{4}\,e^{2}$$
$$1.813430204 \tag{11}$$

> $a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1) \cdot \cdot n, \left(\frac{1}{L}\right) \cdot int\left(f(t) \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$

(12)

$$a_n := \frac{-2\,e^{-2}\,(-1)^n + 2\,e^{2}\,(-1)^n}{4 + n^2\,\pi^2} \quad \textbf{(12)}$$

> $b_n := subs\left(\sin(n\cdot\mathrm{Pi}) = 0, \cos(n\cdot\mathrm{Pi}) = (-1)\cdot\cdot n, \left(\frac{1}{L}\right)\cdot int\left(f(t)\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), t = -L..L\right)\right)$

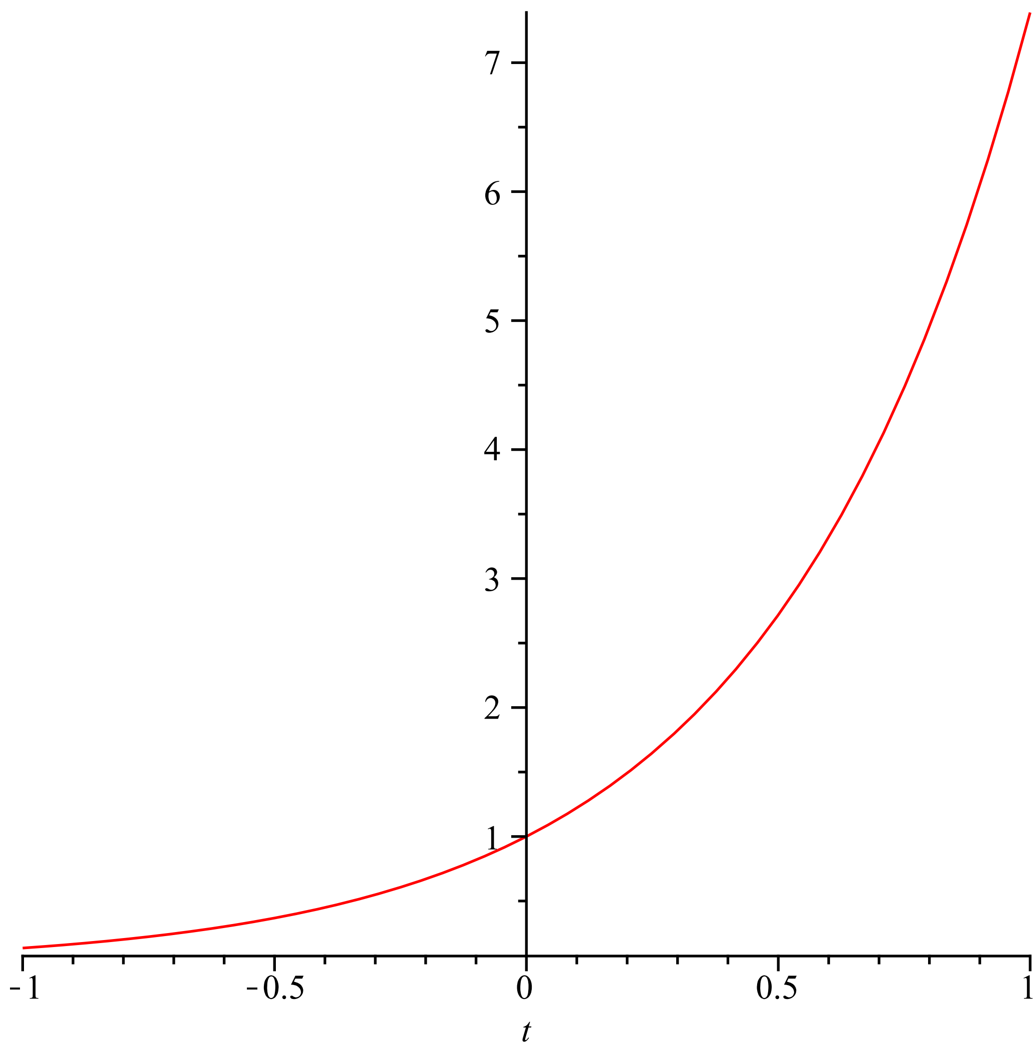
$$b_n := \frac{e^{-2}\,n\,\pi\,(-1)^n - e^{2}\,n\,\pi\,(-1)^n}{4 + n^2\,\pi^2} \quad \textbf{(13)}$$

> $STF := C + Sum\left(a_n\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right) + b_n\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), n = 1..infinity\right)$

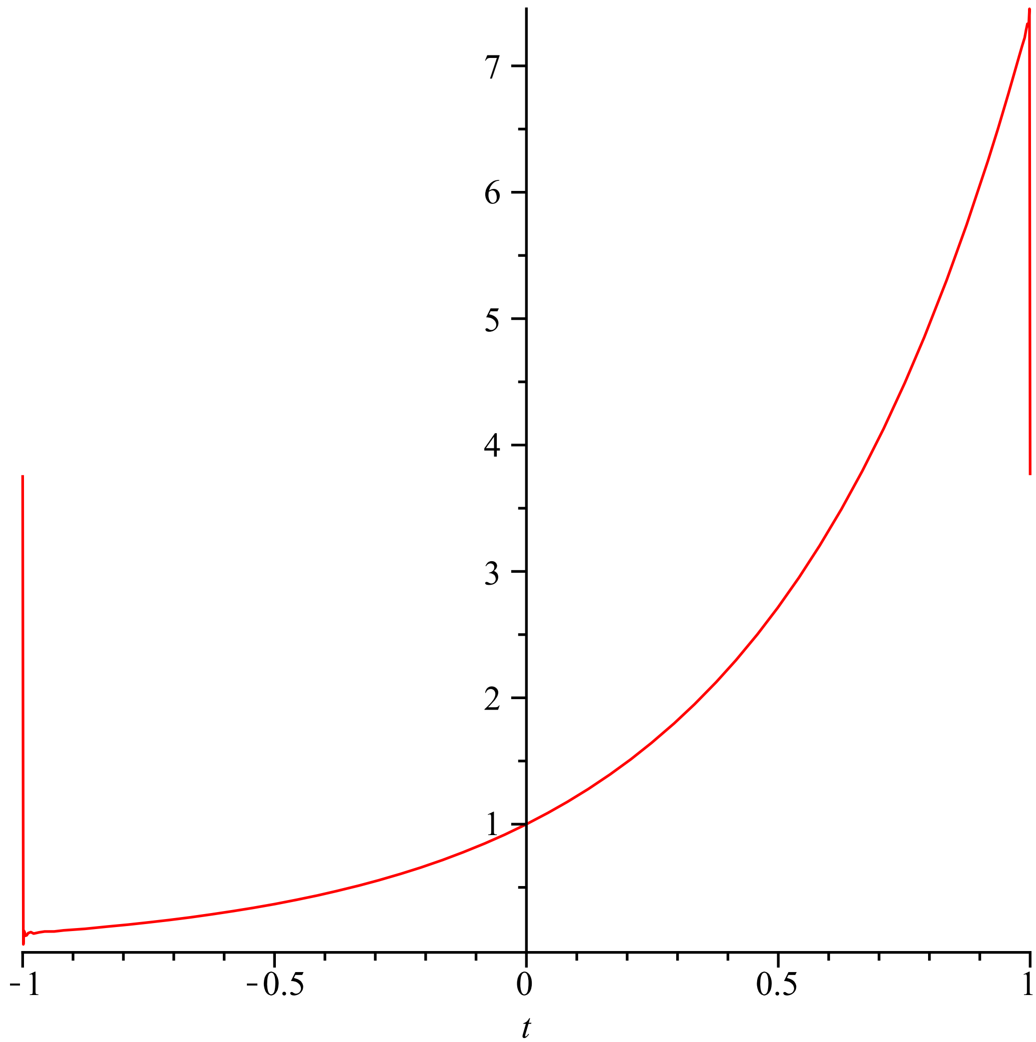
$$STF := -\frac{1}{4}\,e^{-2} + \frac{1}{4}\,e^{2} + \sum_{n=1}^{\infty}\left(\frac{\left(-2\,e^{-2}\,(-1)^n + 2\,e^{2}\,(-1)^n\right)\cos(n\,\pi\,t)}{4 + n^2\,\pi^2} + \frac{\left(e^{-2}\,n\,\pi\,(-1)^n - e^{2}\,n\,\pi\,(-1)^n\right)\sin(n\,\pi\,t)}{4 + n^2\,\pi^2}\right) \quad \textbf{(14)}$$

> $STF_{5000} := C + sum\left(a_n\cdot\cos\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right) + b_n\cdot\sin\left(\frac{n\cdot\mathrm{Pi}\cdot t}{L}\right), n = 1..5000\right):$

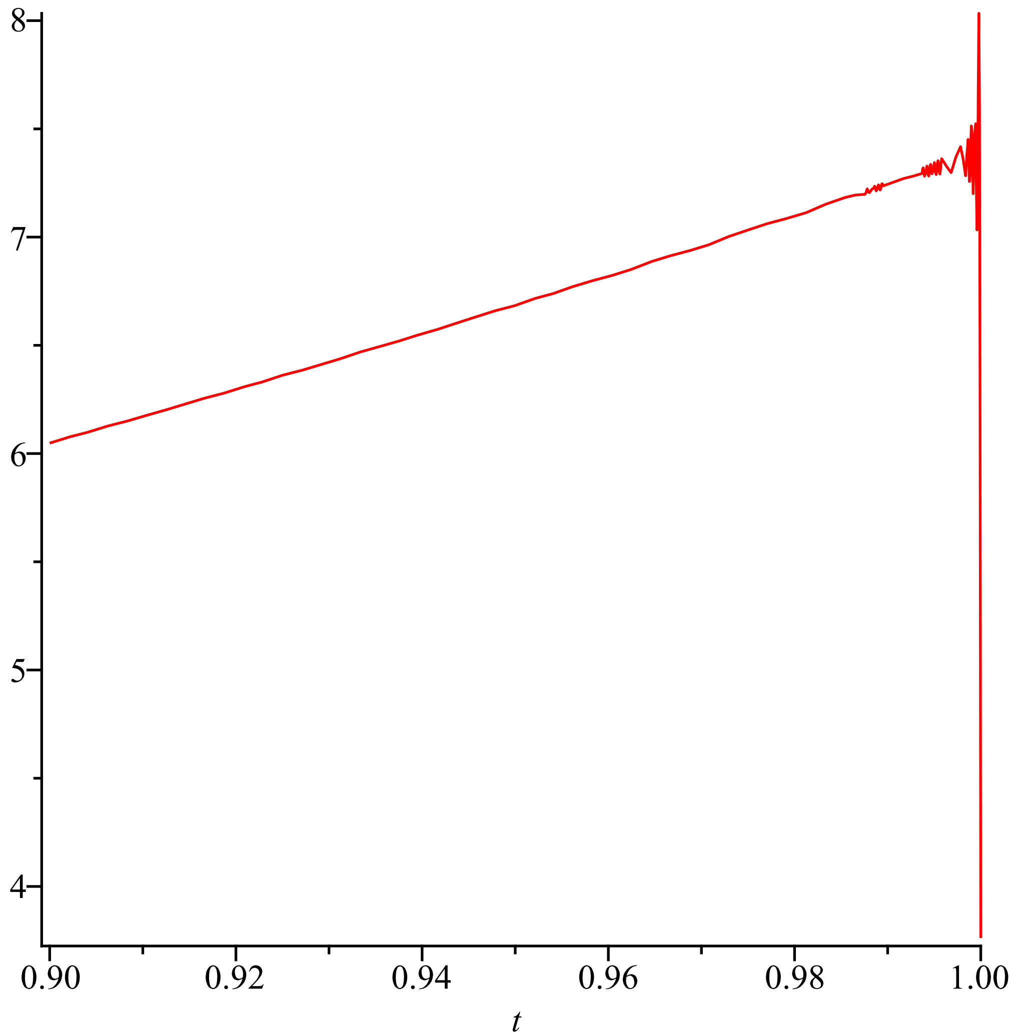
> $plot(f(t), t = -L..L)$



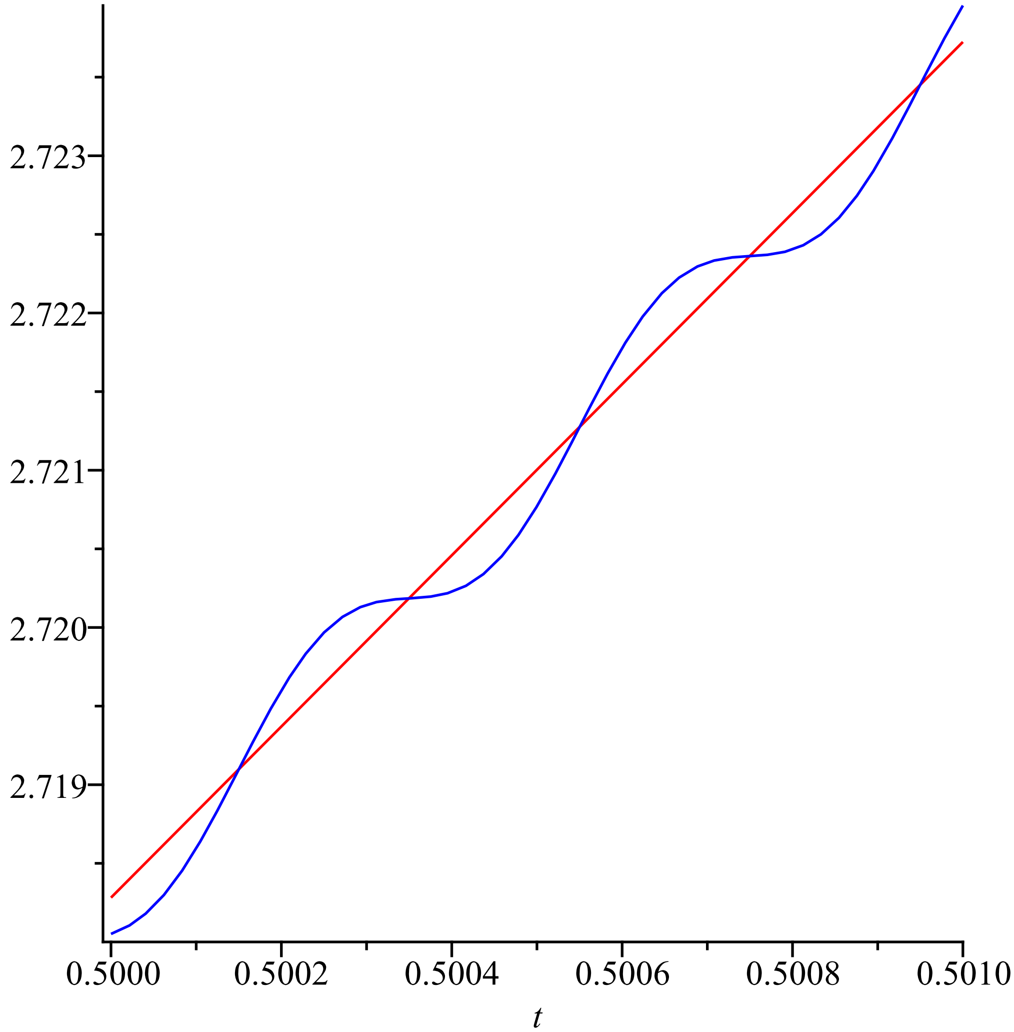
> $plot\left(STF_{5000}, t=-L..L\right)$



> $plot\left(STF_{5000}, t=0.9..1\right)$



> $plot\left(\left[f(t), STF_{5000}\right], t = 0.5\,..0.501, color = [red, blue]\right)$

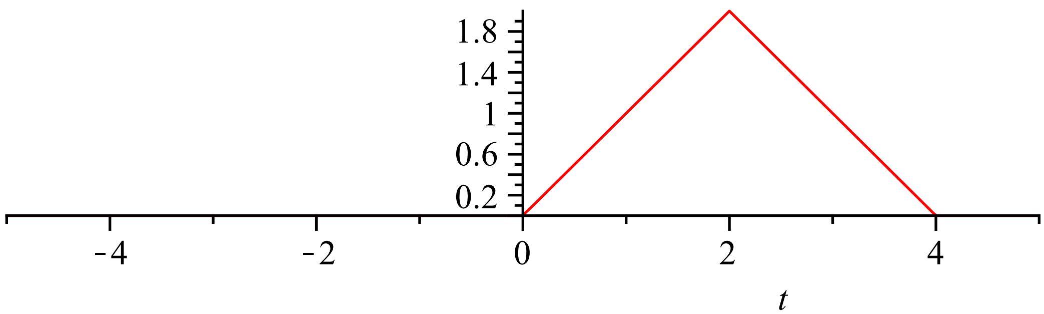


> *restart*

> $f(t) := t\cdot\text{Heaviside}(t) - 2\cdot(t-2)\cdot\text{Heaviside}(t-2) + (t-4)\cdot\text{Heaviside}(t-4);$

$$f(t) := t\,\text{Heaviside}(t) - 2\,(t-2)\,\text{Heaviside}(t-2) + (t-4)\,\text{Heaviside}(t-4) \tag{15}$$

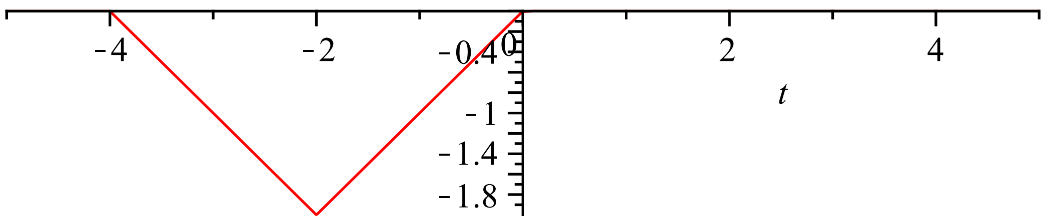
> $plot(f(t), t=-5..5, scaling=CONSTRAINED)$



> $g(t) := -(t+4)\cdot\text{Heaviside}(t+4) + 2\cdot(t+2)\cdot\text{Heaviside}(t+2) - t\cdot\text{Heaviside}(t);$

$$g(t) := -(t+4)\ \text{Heaviside}(t+4) + 2\ (t+2)\ \text{Heaviside}(t+2) - t\ \text{Heaviside}(t) \tag{16}$$

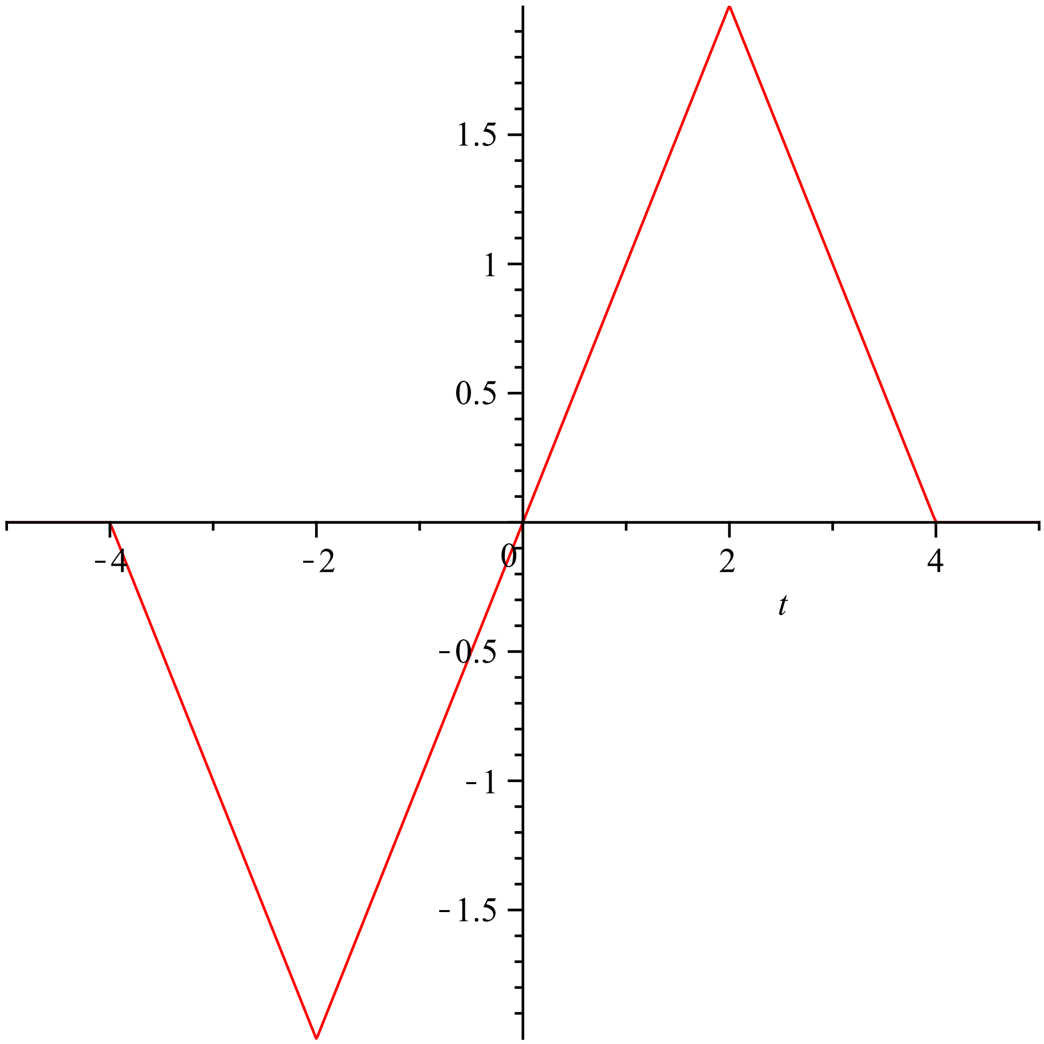
> $plot(g(t), t=-5\,..5, scaling = CONSTRAINED)$



> $h(t) := f(t) + g(t)$

$$h(t) := -2\ (t-2)\ \mathrm{Heaviside}(t-2) + (t-4)\ \mathrm{Heaviside}(t-4) - (t+4)\ \mathrm{Heaviside}(t+4) + 2\ (t+2)\ \mathrm{Heaviside}(t+2) \quad \textbf{(17)}$$

> $plot(h(t), t=-5..5)$



> $L := 5$

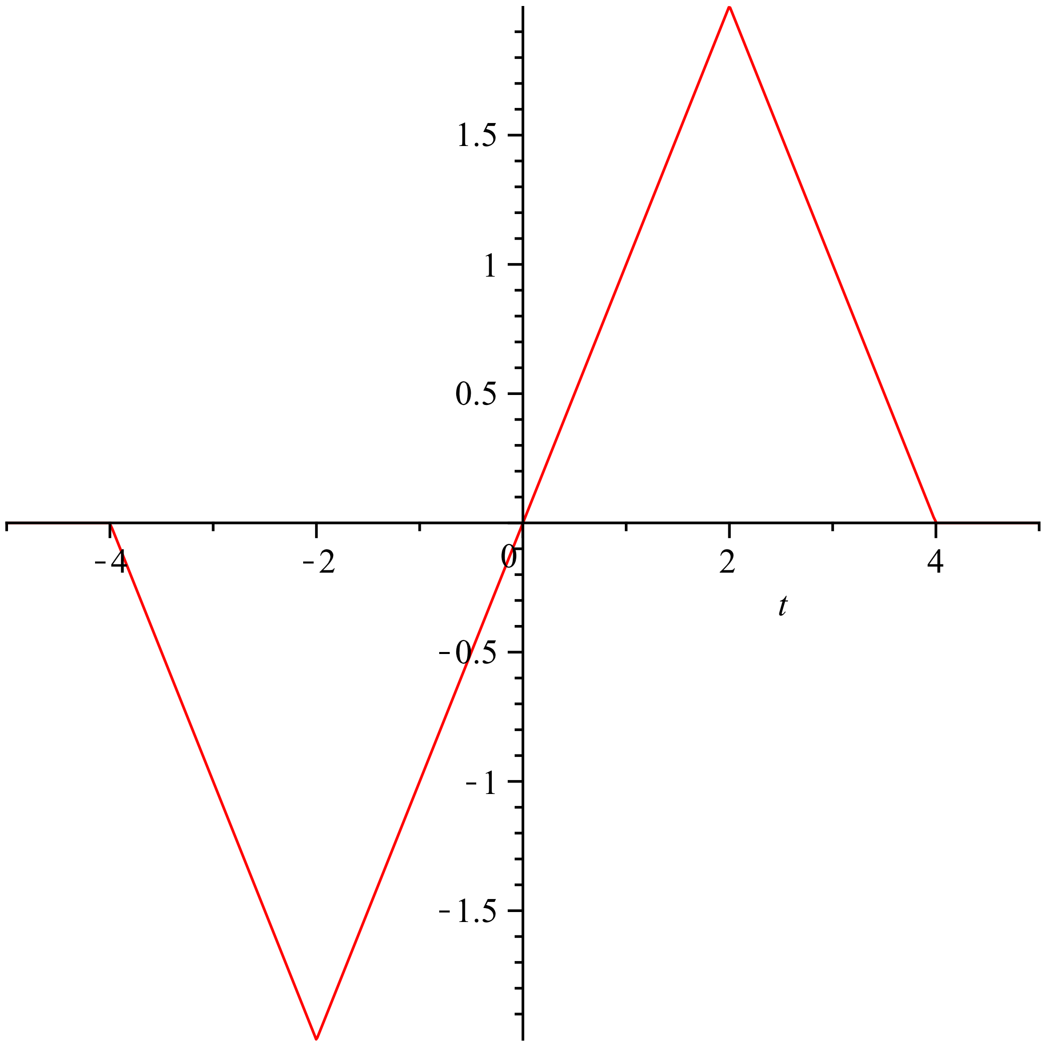
$$L := 5 \tag{18}$$

> $b_n := \left(\frac{1}{L}\right) \cdot int\left(h(t) \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t=-L..L\right)$

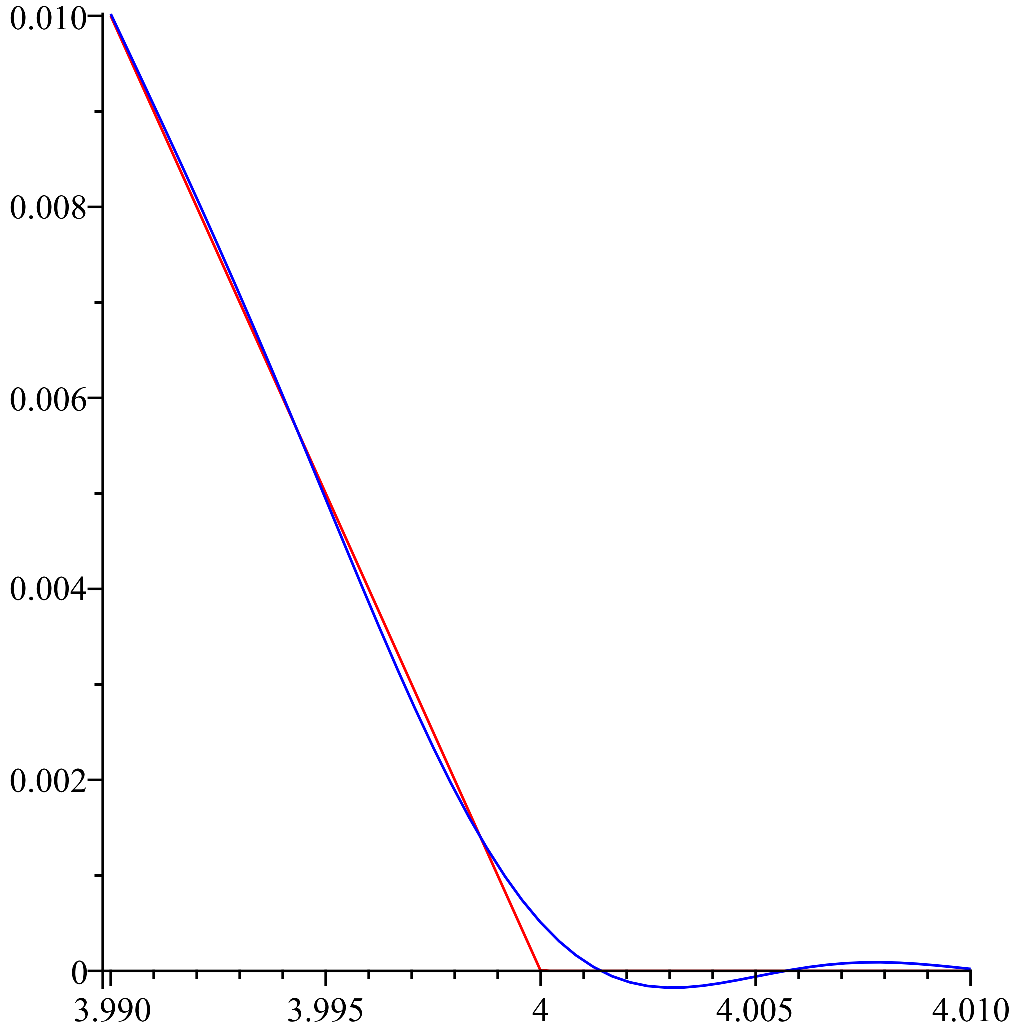
$$b_n := \frac{10\left(\sin\left(\frac{2}{5}\,n\,\pi\right) - \frac{2}{5}\,n\,\pi\cos\left(\frac{2}{5}\,n\,\pi\right)\right)}{n^2\,\pi^2} + \frac{8\cos\left(\frac{2}{5}\,n\,\pi\right)}{n\,\pi} - \frac{5\left(\sin\left(\frac{4}{5}\,n\,\pi\right) - \frac{4}{5}\,n\,\pi\cos\left(\frac{4}{5}\,n\,\pi\right)\right)}{n^2\,\pi^2} - \frac{8\cos\left(\frac{4}{5}\,n\,\pi\right)}{n\,\pi} + \frac{5\left(-\sin\left(\frac{4}{5}\,n\,\pi\right) + \frac{4}{5}\,n\,\pi\cos\left(\frac{4}{5}\,n\,\pi\right)\right)}{n^2\,\pi^2} - \frac{10\left(-\sin\left(\frac{2}{5}\,n\,\pi\right) + \frac{2}{5}\,n\,\pi\cos\left(\frac{2}{5}\,n\,\pi\right)\right)}{n^2\,\pi^2} \tag{19}$$

$> \; STF_{1000} := sum\left(b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..1000\right):$

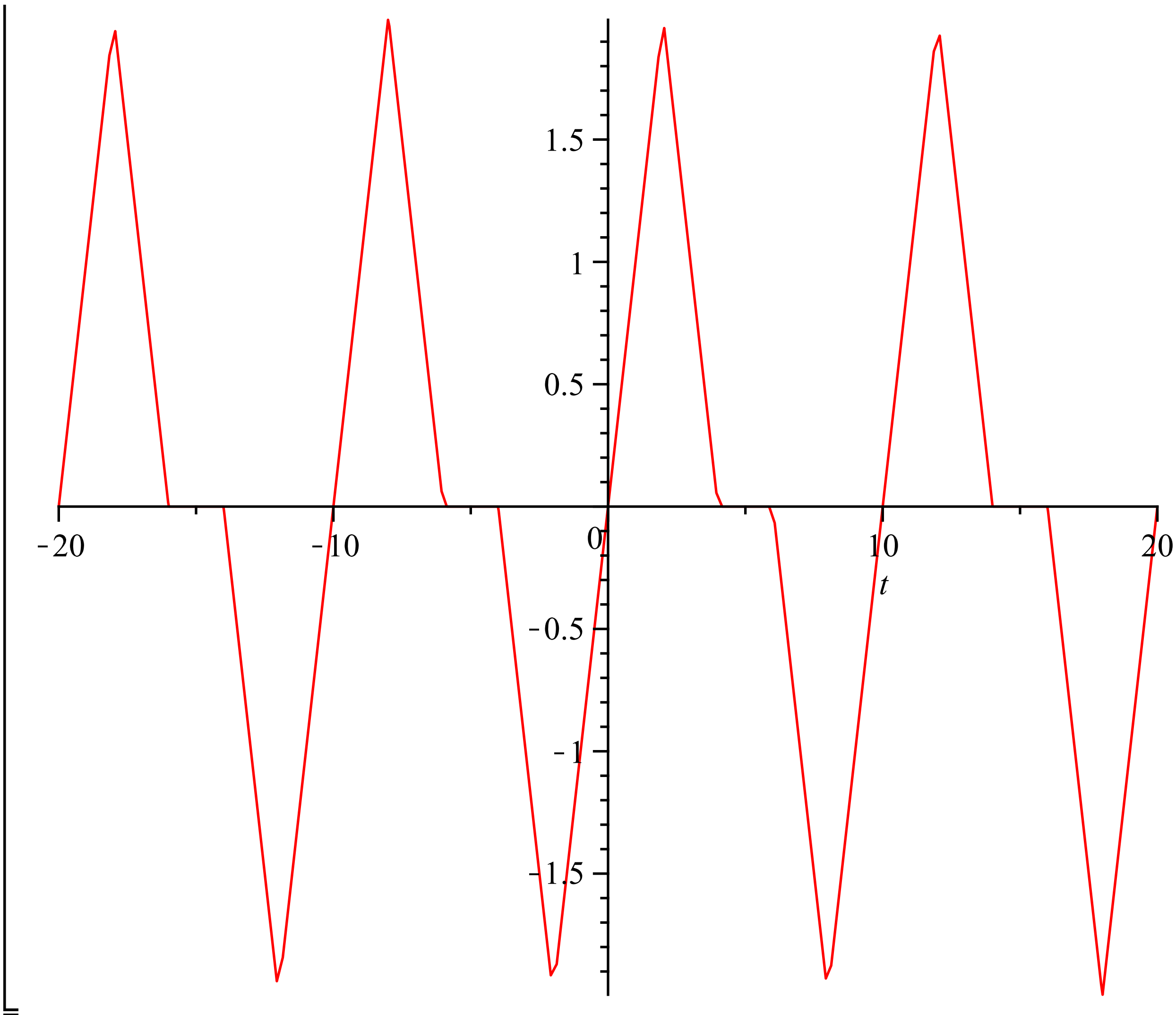
$> \; plot(STF_{1000}, t = -5..5)$



$> \; plot([h(t), STF_{1000}], t = 3.99..4.01, color = [red, blue])$



> $plot(STF_{1000}, t = -20..20)$



1.5
1
0.5
-20
-10
0
10
20
t
-0.5
-1
-1.5

>

>

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