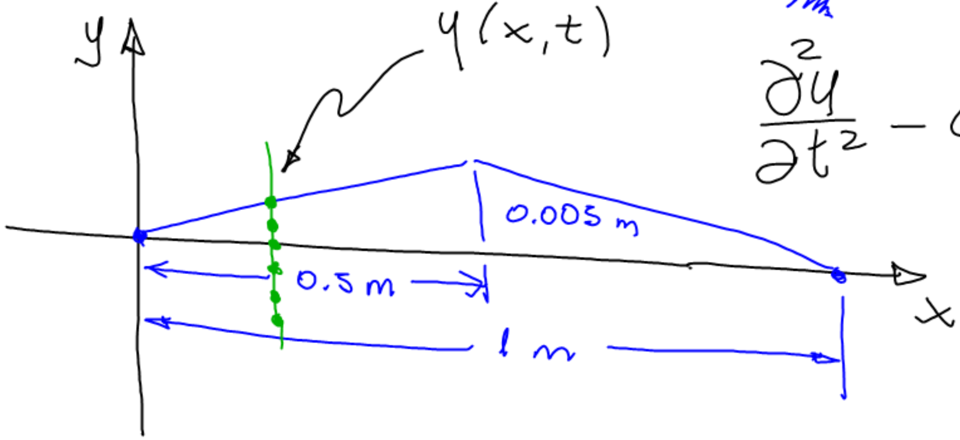
Problema de la cuerda



$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} = 0$$

$$c^2 = \frac{\sigma}{\rho}$$

Condiciones de frontera

$$y(0,t) = 0 \quad \forall t \in \mathbb{R}^+$$

$$y(1,t) = 0$$

Condiciones iniciales en el tiempo

$$y(x,0) \Rightarrow f(x) = \begin{cases} \dfrac{0.005}{0.5} x & ; \ 0 \leq x < 0.5 \\ -\dfrac{0.005}{0.5} x + 0.01 & ; \ 0.5 \leq x \leq 1 \end{cases}$$

$$\left. \frac{\partial y}{\partial t} \right|_{t=0} = 0$$

$$\left.\begin{array}{l} y(0,t)=0 \\ y(1,t)=0 \end{array}\right\} \; y(x,t)=F(x)G(t)$$

$$\left.\begin{array}{l} F(0)\,G(t)=0 \\ F(1)\,G(t)=0 \end{array}\right\} \; G(t)\neq 0$$

$$\left.\begin{array}{l} F(0)=0 \\ F(1)=0 \end{array}\right\} \; F(x) \underset{\alpha=0}{=} C_1 x + C_2 \qquad F(x)=C_1 x$$

$$C_1(0)+C_2=0 \longrightarrow C_2=0$$

$$C_1(1)=0 \longrightarrow C_1=0 \qquad F(x)=0$$

$$F(x) = C_1 \cos\left(\frac{\beta x}{c}\right) + C_2 \operatorname{sen}\left(\frac{\beta x}{c}\right)$$

$F(0) = 0 \qquad F(1) = 0 \qquad \boxed{\alpha = -\beta^2}$

$$C_1 \cos\left(\frac{\beta(0)}{c}\right) + C_2 \operatorname{sen}\left(\frac{\beta(0)}{c}\right) = 0$$

$C_1(1) = 0 \qquad \boxed{C_1 = 0}$

$$F(x) = C_2 \operatorname{sen}\left(\frac{\beta x}{c}\right)$$

$\operatorname{sen}(n\pi) = 0$

$$C_2 \operatorname{sen}\left(\frac{\beta(1)}{c}\right) = 0$$

$\frac{\beta}{c} = n\pi \qquad C_2 \neq 0$

$$F(x) = C_2 \operatorname{sen}(n\pi x)$$

$\boxed{\beta = n\pi c} \quad \boxed{\alpha = -n^2\pi^2c^2}$