> *restart*

> $Ecuacion := diff(y(x,t), t\$2) - c\cdot\cdot 2\cdot diff(y(x,t), x\$2) = 0$

$$Ecuacion := \frac{\partial^2}{\partial t^2} y(x,t) - c^2 \left(\frac{\partial^2}{\partial x^2} y(x,t)\right) = 0 \tag{1}$$

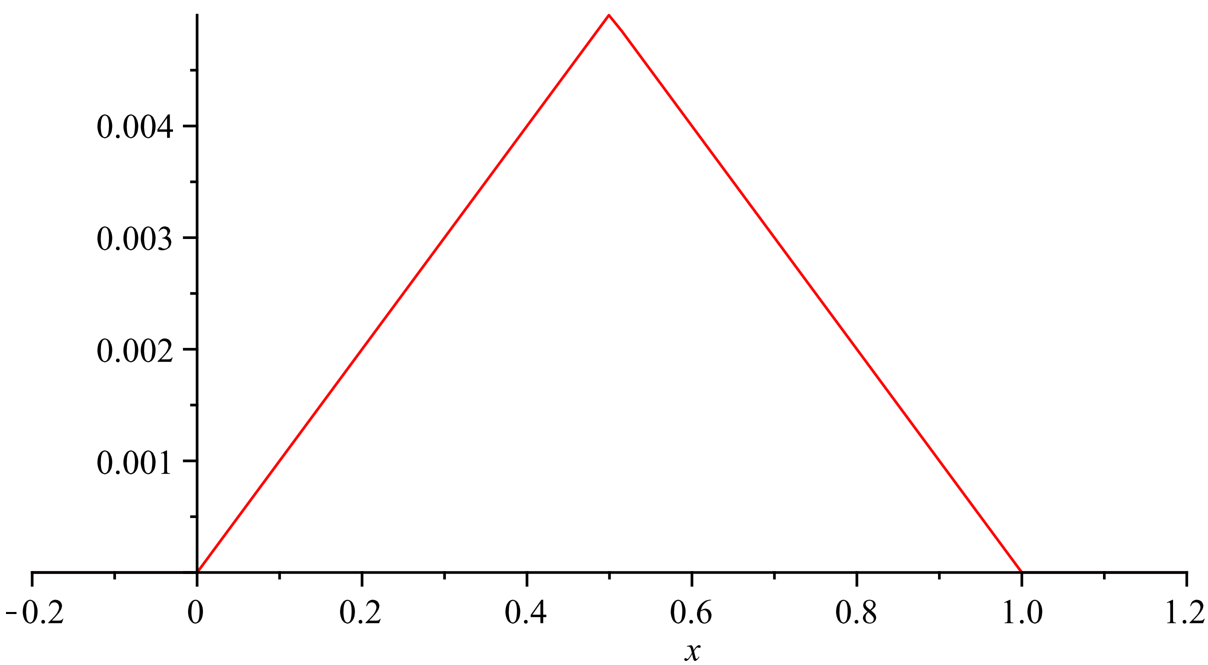
> $CondicionesFrontera := y(0,t) = 0, y(1,t) = 0$

$$CondicionesFrontera := y(0,t) = 0, y(1,t) = 0 \tag{2}$$

> $CondicionIncialTrayectoria := f(x) = \frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)}\cdot x\cdot \text{Heaviside}(x) - 2\cdot\left(\frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)}\right)\cdot\left(x - \frac{5}{10}\right)\cdot\text{Heaviside}\left(x - \frac{5}{10}\right) + \left(\frac{\left(\frac{5}{1000}\right)}{\left(\frac{5}{10}\right)}\right)\cdot(x-1)\cdot\text{Heaviside}(x-1);$

$plot(rhs(CondicionIncialTrayectoria), x = -0.2..1.2)$

$$CondicionIncialTrayectoria := f(x) = \frac{1}{100}\, x\,\text{Heaviside}(x) - \frac{1}{50}\left(x - \frac{1}{2}\right)\text{Heaviside}\left(x - \frac{1}{2}\right) + \frac{1}{100}(x-1)\,\text{Heaviside}(x-1)$$



> $CondicionIncialVelocidad := DerYcero = 0;$

$$CondicionIncialVelocidad := DerYcero = 0 \tag{3}$$

>

MÉTODO DE SEPARACION DE VARIABLES

> $EcuacionSeparable := eval(subs(y(x,t) = F(x)\cdot G(t), Ecuacion))$

$$EcuacionSeparable := F(x)\left(\frac{d^2}{dt^2} G(t)\right) - c^2\left(\frac{d^2}{dx^2} F(x)\right) G(t) = 0 \tag{4}$$

$$> EcuacionSeparada := \frac{\left(lhs(EcuacionSeparable) + c^2 \left(\frac{d^2}{dx^2} F(x)\right) G(t)\right)}{F(x) \cdot G(t)} = \frac{\left(rhs(EcuacionSeparable) + c^2 \left(\frac{d^2}{dx^2} F(x)\right) G(t)\right)}{F(x) \cdot G(t)}$$

$$EcuacionSeparada := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \frac{c^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} \quad (5)$$

$$> EcuacionX := rhs(EcuacionSeparada) = \text{alpha}; EcuacionT := lhs(EcuacionSeparada) = \text{alpha};$$

$$EcuacionX := \frac{c^2 \left(\frac{d^2}{dx^2} F(x)\right)}{F(x)} = \alpha$$

$$EcuacionT := \frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad (6)$$

$$> CondicionesF := F(0) = 0, F(1) = 0;$$

$$CondicionesF := F(0) = 0, F(1) = 0 \quad (7)$$

$$> SolucionXcero := dsolve(subs(\text{alpha} = 0, EcuacionX))$$

$$SolucionXcero := F(x) = \_C1\, x + \_C2 \quad (8)$$

$$> SolucionParticularXcero := dsolve(\{subs(\text{alpha} = 0, EcuacionX), CondicionesF\})$$

$$SolucionParticularXcero := F(x) = 0 \quad (9)$$

$$> SolucionXpos := dsolve(subs(\text{alpha} = \text{beta} \cdot\cdot 2, EcuacionX))$$

$$SolucionXpos := F(x) = \_C1\, e^{\frac{\beta x}{c}} + \_C2\, e^{-\frac{\beta x}{c}} \quad (10)$$

$$> SolucionParticularXpos := dsolve(\{subs(\text{alpha} = \text{beta} \cdot\cdot 2, EcuacionX), CondicionesF\})$$

$$SolucionParticularXpos := F(x) = 0 \quad (11)$$

$$> SolucionXneg := dsolve(subs(\text{alpha} = -n \cdot\cdot 2 \cdot \text{Pi} \cdot\cdot 2 \cdot c \cdot\cdot 2, EcuacionX))$$

$$SolucionXneg := F(x) = \_C1 \sin(\pi\, n\, x) + \_C2 \cos(\pi\, n\, x) \quad (12)$$

$$> SolucionParticularXneg := F(x) = \sin(\text{Pi} \cdot n \cdot x)$$

$$SolucionParticularXneg := F(x) = \sin(\pi\, n\, x) \quad (13)$$

$$> SolucionTneg := dsolve(subs(\text{alpha} = -n \cdot\cdot 2 \cdot \text{Pi} \cdot\cdot 2 \cdot c \cdot\cdot 2, EcuacionT))$$

$$SolucionTneg := G(t) = \_C1 \sin(\pi\, c\, n\, t) + \_C2 \cos(\pi\, c\, n\, t) \quad (14)$$

$$> SolucionParticularNeg := y(x, t) = rhs(SolucionParticularXneg) \cdot rhs(SolucionTneg)$$

$$SolucionParticularNeg := y(x, t) = \sin(\pi\, n\, x) \left(\_C1 \sin(\pi\, c\, n\, t) + \_C2 \cos(\pi\, c\, n\, t)\right) \quad (15)$$

$$> SolucionGeneral := y(x, t) = Sum\left(\sin(\text{Pi} \cdot n \cdot x) \cdot \left(b_n \cdot \cos(\text{Pi} \cdot c \cdot n \cdot t) + a_n \cdot \sin(\text{Pi} \cdot c \cdot n \cdot t)\right), n = 1 ..infinity\right)$$

$$SolucionGeneral := y(x, t) = \sum_{n=1}^{\infty} \sin(\pi\, n\, x) \left(b_n \cos(\pi\, c\, n\, t) + a_n \sin(\pi\, c\, n\, t)\right) \quad (16)$$

$$> SolucionParticularInicial := eval(subs(t = 0, SolucionGeneral))$$

$$SolucionParticularInicial := y(x, 0) = \sum_{n=1}^{\infty} \sin(\pi\, n\, x)\, b_n \tag{17}$$

> $b_n := \left(\dfrac{1}{\left(\frac{5}{10}\right)}\right) \cdot int\left(rhs(CondicionIncialTrayectoria) \cdot \sin\left(\dfrac{n \cdot \mathrm{Pi} \cdot x}{1}\right), x = 0..1\right)$

$$b_n := \frac{1}{50}\, \frac{-\sin(\pi\, n) + 2\sin\left(\frac{1}{2}\,\pi\, n\right)}{n^2\, \pi^2} \tag{18}$$

> $a_n := 0;$

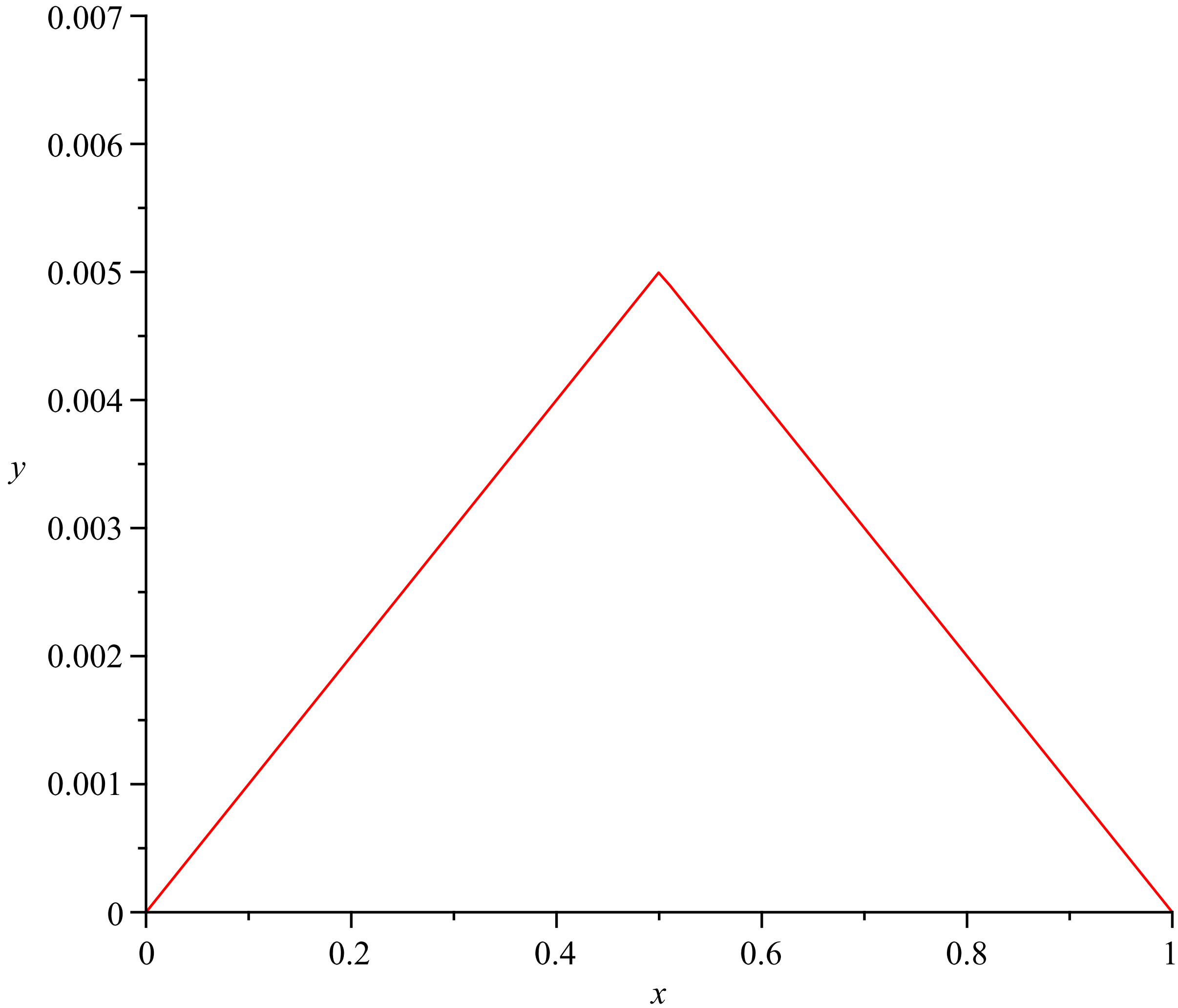
$$a_n := 0 \tag{19}$$

> $SolucionGeneral;$

$$y(x, t) = \sum_{n=1}^{\infty} \frac{1}{50}\, \frac{\sin(\pi\, n\, x)\left(-\sin(\pi\, n) + 2\sin\left(\frac{1}{2}\,\pi\, n\right)\right)\cos(\pi\, c\, n\, t)}{n^2\, \pi^2} \tag{20}$$

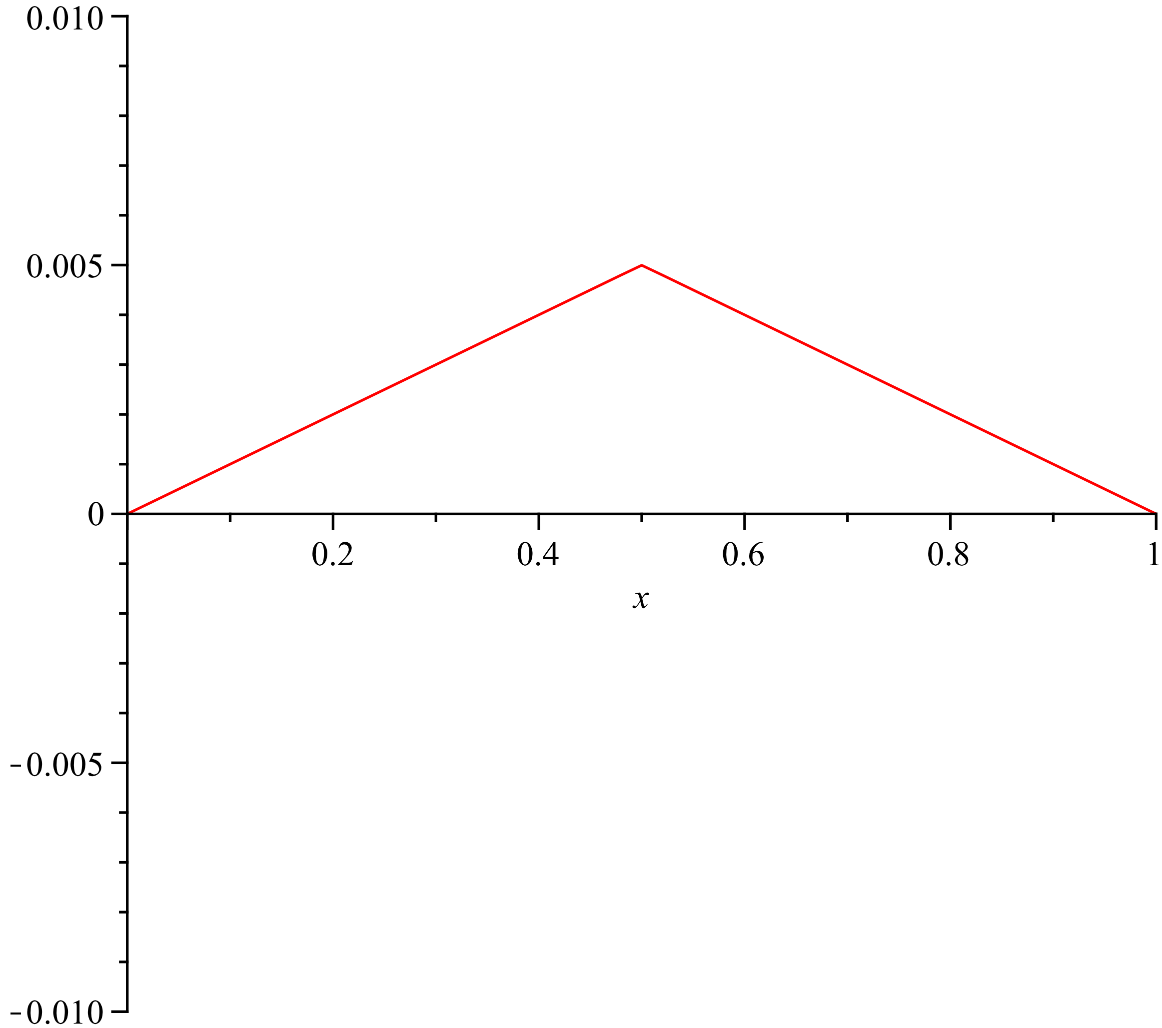
> $SolucionParticular_{500} := y(x, t) = \displaystyle\sum_{n=1}^{500} \frac{1}{50}\, \frac{\sin(\pi\, n\, x)\left(-\sin(\pi\, n) + 2\sin\left(\frac{1}{2}\,\pi\, n\right)\right)\cos(\pi\, c\, n\, t)}{n^2\, \pi^2} :$

> $plot\left(rhs\left(subs\left(c = 1, t = 0, SolucionParticular_{500}\right)\right), x = 0..1, y = 0..0.007\right)$



> $with(plots):$

> $animate\left(rhs\left(subs\left(c=1, SolucionParticular_{500}\right)\right), x=0..1, t=0..4, frames=150, view=[0..1, -0.01..0.01]\right)$



0.010
0.005
0
-0.005
-0.010
0.2
0.4
0.6
0.8
1
x

>