$$\mathcal{L}^{-1}\left\{F(s)\cdot G(s)\right\} = f(t) * g(t)$$

$$f(t) * g(t) = \int^{t} f(t-z)\cdot g(z)\,dz.$$

$$f(t) := \frac{2}{3}\,\mathrm{Heaviside}(t-3)\sqrt{3}\,e^{-\frac{3}{2}t+\frac{9}{2}}\sin\left(\frac{1}{2}\sqrt{3}\,(t-3)\right)$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}(4s+7)}{(s^2+3s+3)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+3s+3}\cdot\frac{4s+7}{s^2+3s+3}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+3s+3}\right\} = \frac{2}{\sqrt{3}}e^{-\frac{3}{2}(t-3)}\operatorname{sen}\left(\frac{\sqrt{3}}{2}(t-3)\right)u(t-3)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s+3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2+3s)+3}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{\left(s^2+3s+\frac{9}{4}\right)+3-\frac{9}{4}}\right\}$$

$$= \frac{2}{\sqrt{3}}\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= \frac{2}{\sqrt{3}}e^{-\frac{3}{2}t}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$$

$$\mathcal{L}^{-1}\left\{\frac{4s+7}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\} = 4\mathcal{L}^{-1}\left\{\frac{s+\frac{7}{4}}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= 4\mathcal{L}^{-1}\left\{\frac{\left(s+\frac{3}{2}\right)+\left(\frac{7}{4}-\frac{3}{2}\right)}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\}$$

$$= 4\mathcal{L}^{-1}\left\{\frac{s+\frac{3}{2}}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right\} + 4\left(\frac{1}{4}\cdot\frac{2}{\sqrt{3}}\right)\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{(\ )^2+(\ )^2}\right\}$$

$$= 4e^{-\frac{3}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{2}{\sqrt{3}}e^{-\frac{3}{2}t}\operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$$

$$g(t) := \frac{2}{3}\,e^{-\frac{3}{2}t}\left(6\cos\left(\frac{1}{2}\sqrt{3}\,t\right)+\sqrt{3}\,\sin\left(\frac{1}{2}\sqrt{3}\,t\right)\right)$$

Sistemas

$$\frac{dx_1(t)}{dt} = 3x_1(t) + 2x_2(t) - 4x_3(t) + e^{2t} + t^2$$

$$\frac{dx_2(t)}{dt} = -5x_2(t) + 6x_3(t) + 2t + 4$$

$$\frac{dx_3(t)}{dt} = 3x_1 + 4x_2(t) + t^2 + 3e^{2t}$$

$$x_1(0) = 5 \quad x_2(0) = -3 \quad x_3(0) = 4$$

$$A = \begin{bmatrix} 3 & 2 & -4 \\ 0 & -5 & 6 \\ 3 & 4 & 0 \end{bmatrix} \qquad \bar{x}(0) = \begin{bmatrix} 5 \\ -3 \\ 4 \end{bmatrix}$$

$$e^{At} = \beta_0(t) I + \beta_1(t) A + \beta_2(t) A^2$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} 3-\lambda & 2 & -4 \\ 0 & -5-\lambda & 6 \\ 3 & 4 & -\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\begin{vmatrix} -5-\lambda & 6 \\ 4 & -\lambda \end{vmatrix} - 2\begin{vmatrix} 0 & 6 \\ 3 & -\lambda \end{vmatrix} + (-4)\begin{vmatrix} 0 & -5-\lambda \\ 3 & 4 \end{vmatrix} = 0$$

$$(3-\lambda)(5\lambda + \lambda^2 - 24) - 2(-18) - 4(+15 + 3\lambda) = 0$$

$$-\lambda^3 - 2\lambda^2 + 39\lambda - 72 + 36 - 60 - 12\lambda = 0$$

$$-\lambda^3 - 2\lambda^2 + 27\lambda - 96 = 0$$

$$-1 \quad -2 \quad +27 \quad -96 \; \underline{|\, -}$$

$$-7.4$$

$$2.7 \pm 2.38i$$

$$e^{-7.4t} = \beta_0(t) - 7.4\beta_1(t) + (-7.4)^2 \beta_2(t)$$

$$e^{(2.7+2.38i)t} = \beta_0(t) + (2.7+2.38i)\beta_1(t) + (2.7+2.38i)^2 \beta_2(t)$$

$$e^{(2.7-2.38i)t} = \beta_0(t) + (2.7-2.38i)\beta_1(t) + (2.7-2.38i)^2 \beta_2(t)$$

$$\bar{b}(t) = \begin{bmatrix} e^{2t} + t^2 \\ 2t + 4 \\ t^2 + 3e^{2t} \end{bmatrix} \qquad \frac{d\bar{x}}{dt} = A\bar{x} + \bar{b}(t)$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t e^{A(t-\tau)}\bar{b}(\tau)\,d\tau$$