# EDOL (2) CC H.

normalizada

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

Ec. CARACTERÍSTICA

$$m^2 + a_1 m + a_2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases}$$

$$y_1 = e^{m_1 x} \qquad y_2 = e^{m_2 x}$$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x} \qquad m_1 \neq m_2$$

$$m^2 + a_1 m + a_2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases}$$

<u>CASO</u> I.- cuando $m_1, m_2 \in \mathbb{R}$ $\quad m_1 \neq m_2$

CASO II.- cuando $m_1, m_2 \in \mathbb{R}$ $\quad m_1 = m_2$

CASO III.- cuando $m_1, m_2 \in \mathbb{C}$ $\quad m_1 = a + bi$

$m_2 = a - bi$

CASO III bis - Cuando $m_1, m_2 \in \mathbb{C}$ $\quad m_1 = bi$

$m_2 = -bi$

CASO II.- Raíces reales e iguales.

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2$$

$$(m - m_1)^2 = 0 \qquad y_1 = e^{m_1 x} \quad y_2 = e^{m_1 x}$$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_1 x}$$

$$y_g = (C_1 + C_2) e^{m_1 x}$$

$$y_g = C_{10} e^{m_1 x} + C_{20} y_2$$

$$m_1 = m_2$$

$$(m_1)^2 + a_1(m_1) + a_2 \equiv 0$$

$$\frac{d}{dm}(m^2 + a_1 m + a_2) = 0$$

$$2m + a_1 = 0$$

$$2(m_1) + a_1 \equiv 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2 \qquad y_1 = e^{m_1 x}$$

Sol. part. fund.

$$\begin{tikzcd} e^{mx} \arrow[r, "m_1"] \arrow[d, bend right, "\frac{d}{dm}"'] & e^{m_1 x} \\ xe^{mx} \arrow[r, "m_1"] & xe^{m_1 x} \end{tikzcd}$$

$$W = \begin{vmatrix} e^{m_1 x} & xe^{m_1 x} \\ m_1 e^{m_1 x} & m_1 x e^{m_1 x} + e^{m_1 x} \end{vmatrix} \neq 0$$

indep. lineal.

$$e^{m_1 x}\left(m_1 x e^{m_1 x} + e^{m_1 x}\right) - e^{m_1 x} m_1 \left(x e^{m_1 x}\right) \neq 0$$

$$\cancel{m_1 x e^{m_1 x}} e^{m_1 x} + e^{m_1 x} e^{m_1 x} - \cancel{m_1 x e^{m_1 x}} e^{m_1 x} \neq 0$$

$$e^{m_1 x} e^{m_1 x} \neq 0$$

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \quad \text{EDOL}(2)\ \text{cc H.}$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = m_2 \qquad y_1 = e^{m_1 x}$$

$$y_2 = xe^{m_1 x}$$

$$\frac{dy_2}{dx} = m_1 x e^{m_1 x} + e^{m_1 x}$$

$$\frac{d^2y_2}{dx^2} = m_1\left(m_1 x e^{m_1 x} + e^{m_1 x}\right) + m_1 e^{m_1 x}$$

$$= m_1^2 x e^{m_1 x} + 2m_1 e^{m_1 x}$$

$$\left[m_1^2 \cancel{x} e^{m_1 x} + 2m_1 \cancel{e^{m_1 x}}\right) + a_1\left(m_1 \cancel{x} e^{m_1 x} + \cancel{e^{m_1 x}}\right) + a_2\left(x\cancel{e^{m_1 x}}\right) = 0$$

$$\left(m_1^2 + a_1 m_1 + a_2\right) x e^{m_1 x} + \left(2m_1 + a_1\right) e^{m_1 x} = 0$$

$$y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x} \qquad \text{CASO II.}$$

$$\frac{d^2y}{dx^2} = 0 \qquad \frac{dy}{dx} = k_1$$

$$dy = k_1\,dx$$

$$\int dy = k_1 \int dx$$

$$y + k_2 = k_1(x + k_3)$$

$$y = k_1 x + (k_1 k_3 - k_2)$$

$$\boxed{y = c_1 x + c_2}$$

$$m^2 = 0 \qquad m_1 = 0 \quad m_2 = 0$$

$$m_1 = m_2$$

$$y = c_1 e^{(0)x} + c_2 x e^{(0)x}$$

$$y = c_1 + c_2 x$$

$$y_1 = 1 \qquad y_2 = x$$

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0 \qquad m_1 = 2 \qquad m_2 = 2 \qquad m_1 = m_2$$

$$y_1 = e^{2x} \qquad y_2 = xe^{2x}$$

$$y_g = c_1 e^{2x} + c_2 x e^{2x}$$

# CASO III.- raíces complejas.

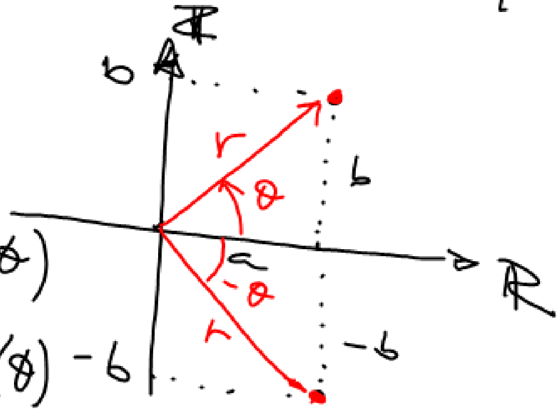
$$m^2 + a_1 m + a_2 = 0 \qquad m_1 = a + bi \quad m_1 \neq m_2$$
$$m_2 = a - bi$$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x} \qquad x \in \mathbb{R},\ y \in \mathbb{R}$$

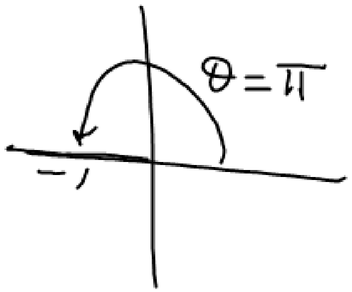
Euler

$$e^{\pi i} + 1 = 0$$

$$re^{\theta i} = r\cos(\theta) + i\, r\,\text{sen}(\theta)$$
$$re^{-\theta i} = r\cos(\theta) - i\, r\,\text{sen}(\theta)$$




$$e^{\theta i} = \cos(\theta) + i\,\text{sen}(\theta)$$
$$e^{-\theta i} = \cos(\theta) - i\,\text{sen}(\theta)$$




$$\theta = \pi \ [\text{rad}] \qquad e^{\pi i} = -1$$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y_g = C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bxi}$$

$$y_g = e^{ax}\left(C_1 e^{bxi} + C_2 e^{-bxi}\right)$$

$$y_g = e^{ax}\left(C_1\left[\cos(bx) + i\,\operatorname{sen}(bx)\right] + C_2\left[\cos(bx) - i\,\operatorname{sen}(bx)\right]\right)$$

$$y_g = e^{ax}\left((C_1 + C_2)\cos(bx) + (C_1 i - C_2 i)\operatorname{sen}(bx)\right)$$

$$y_g = e^{ax}\left(C_{10}\cos(bx) + C_{20}\operatorname{Sen}(bx)\right)$$

$$y_g = C_{10} e^{ax}\cos(bx) + C_{20} e^{ax}\operatorname{sen}(bx) \qquad x \in \mathbb{R}$$

$$y \in \mathbb{R}$$