

[>
Método de los parámetros variables

> restart
> Ecuacion := $y'' - 5y' + 6y = 8 \exp(x) - 4 \exp(-x)$

$$Ecuacion := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 8 e^x - 4 e^{-x} \quad (1)$$

> EcuacionHom := lhs(Ecuacion) = 0

$$EcuacionHom := \frac{d^2}{dx^2} y(x) - 5 \left(\frac{d}{dx} y(x) \right) + 6 y(x) = 0 \quad (2)$$

> Q := rhs(Ecuacion)

$$Q := 8 e^x - 4 e^{-x} \quad (3)$$

> EcuacionCarac := $m \cdot 2 - 5 \cdot m + 6 = 0$

$$EcuacionCarac := m^2 - 5m + 6 = 0 \quad (4)$$

> Raiz := solve(EcuacionCarac)

$$Raiz := 3, 2 \quad (5)$$

Caso I: raíces reales y distintas

> $Sol_1 := y(x) = \exp(Raiz_1 \cdot x); Sol_2 := y(x) = \exp(Raiz_2 \cdot x)$

$$Sol_1 := y(x) = e^{3x}$$

$$Sol_2 := y(x) = e^{2x} \quad (6)$$

> SolucionHomogenea := $y(x) = C_1 \cdot rhs(Sol_1) + C_2 \cdot rhs(Sol_2)$

$$SolucionHomogenea := y(x) = C_1 e^{3x} + C_2 e^{2x} \quad (7)$$

> SolucionNoHomogenea := $y(x) = A \cdot rhs(Sol_1) + B \cdot rhs(Sol_2)$

$$SolucionNoHomogenea := y(x) = A e^{3x} + B e^{2x} \quad (8)$$

> with(linalg) :

> WW := wronskian([rhs(Sol_1), rhs(Sol_2)], x)

$$WW := \begin{bmatrix} e^{3x} & e^{2x} \\ 3 e^{3x} & 2 e^{2x} \end{bmatrix} \quad (9)$$

> BB := array([0, Q])

$$BB := \begin{bmatrix} 0 & 8 e^x - 4 e^{-x} \end{bmatrix} \quad (10)$$

> SOL := linsolve(WW, BB) : Aprima := SOL₁; Bprima := SOL₂;

$$Aprima := \frac{4(2e^x - e^{-x})}{e^{3x}}$$

$$Bprima := -\frac{4(2e^x - e^{-x})}{e^{2x}} \quad (11)$$

> A := int(Aprima, x) + C₁; B := int(Bprima, x) + C₂

$$A := -\frac{4}{(e^x)^2} + \frac{1}{(e^x)^4} + C_1$$

$$B := \frac{8}{e^x} - \frac{4}{3(e^x)^3} + C_2 \quad (12)$$

> *simplify(SolucionNoHomogenea);*

$$y(x) = 4 e^x - \frac{1}{3} e^{-x} + C_1 e^{3x} + C_2 e^{2x} \quad (13)$$

> *SolGral := simplify(dsolve(Ecuacion))*

$$SolGral := y(x) = e^{2x} _C2 + e^{3x} _C1 + 4 e^x - \frac{1}{3} e^{-x} \quad (14)$$

EJEMPLO DE USO DE *linsolve(linalg)*

> *Sistemit1 := 2 x + 3 y = 5, x + 4 y = -3 : Sistemit1; Sistemit2*

$$\begin{aligned} 2x + 3y &= 5 \\ x + 4y &= -3 \end{aligned} \quad (15)$$

> *MM := array([[2, 3], [1, 4]]); NN := array([5, -3])*

$$MM := \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$NN := \begin{bmatrix} 5 & -3 \end{bmatrix} \quad (16)$$

> *Soluc := solve({Sistemit1})*

$$Soluc := \left\{ x = \frac{29}{5}, y = -\frac{11}{5} \right\} \quad (17)$$

> *Solucion := linsolve(MM, NN)*

$$Solucion := \begin{bmatrix} \frac{29}{5} & -\frac{11}{5} \end{bmatrix} \quad (18)$$

FIN DEL EJEMPLO

> *restart*

> *Ecuacion := diff(y(t), t\$2) = -g*

$$Ecuacion := \frac{d^2}{dt^2} y(t) = -g \quad (19)$$

> *EcuaHom := lhs(Ecuacion) = 0*

$$EcuaHom := \frac{d^2}{dt^2} y(t) = 0 \quad (20)$$

> *Q := rhs(Ecuacion)*

$$Q := -g \quad (21)$$

> *EcuaCarac := m \cdot 2 = 0*

$$EcuaCarac := m^2 = 0 \quad (22)$$

> *Raiz := solve(EcuaCarac)*

$$Raiz := 0, 0 \quad (23)$$

Caso II: Raíces reales e iguales

> *Sol1 := y(t) = exp(Raiz1 \cdot t); Sol2 := y(t) = t \cdot exp(Raiz1 \cdot t)*

$$\begin{aligned} \text{Sol}_1 &:= y(t) = 1 \\ \text{Sol}_2 &:= y(t) = t \end{aligned} \quad (24)$$

$$\begin{aligned} > \text{SolHom} := y(t) = C_1 \text{rhs}(\text{Sol}_1) + C_2 \cdot \text{rhs}(\text{Sol}_2) \\ \text{SolHom} &:= y(t) = C_1 + C_2 t \end{aligned} \quad (25)$$

$$\begin{aligned} > \text{SolNoHom} := y(t) = A \cdot \text{rhs}(\text{Sol}_1) + B \cdot \text{rhs}(\text{Sol}_2) \\ \text{SolNoHom} &:= y(t) = A + B t \end{aligned} \quad (26)$$

> with(linalg) :

$$\begin{aligned} > WW := \text{wronskian}([\text{rhs}(\text{Sol}_1), \text{rhs}(\text{Sol}_2)], t) \\ WW &:= \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \end{aligned} \quad (27)$$

$$\begin{aligned} > BB := \text{array}([0, Q]) \\ BB &:= \begin{bmatrix} 0 & -g \end{bmatrix} \end{aligned} \quad (28)$$

$$\begin{aligned} > SOL := \text{linsolve}(WW, BB) : \text{Aprima} := \text{SOL}_1; \text{Bprima} := \text{SOL}_2; \\ \text{Aprima} &:= t g \\ \text{Bprima} &:= -g \end{aligned} \quad (29)$$

$$\begin{aligned} > A := \text{int}(\text{Aprima}, t) + C_1; B := \text{int}(\text{Bprima}, t) + C_2 \\ A &:= \frac{1}{2} t^2 g + C_1 \\ B &:= -t g + C_2 \end{aligned} \quad (30)$$

$$\begin{aligned} > \text{simplify}(\text{SolNoHom}); \\ y(t) &= -\frac{1}{2} t^2 g + C_1 + C_2 t \end{aligned} \quad (31)$$

$$\begin{aligned} > \text{SolGral} := \text{simplify}(\text{dsolve}(\text{Ecuacion})) \\ \text{SolGral} &:= y(t) = -\frac{1}{2} t^2 g + _C1 t + _C2 \end{aligned} \quad (32)$$

$$\begin{aligned} > \text{EcuacionDos} := \text{diff}(x(t), t) = V_0 \cdot \cos\left(\frac{\text{Pi}}{4}\right) \\ \text{EcuacionDos} &:= \frac{d}{dt} x(t) = \frac{1}{2} V_0 \sqrt{2} \end{aligned} \quad (33)$$

$$\begin{aligned} > \text{EcuadDosHom} := \text{lhs}(\text{EcuacionDos}) = 0 \\ \text{EcuadDosHom} &:= \frac{d}{dt} x(t) = 0 \end{aligned} \quad (34)$$

$$\begin{aligned} > QQ := \text{rhs}(\text{EcuacionDos}) \\ QQ &:= \frac{1}{2} V_0 \sqrt{2} \end{aligned} \quad (35)$$

$$\begin{aligned} > \text{EcuacaracDos} := m = 0 \\ \text{EcuacaracDos} &:= m = 0 \end{aligned} \quad (36)$$

$$\begin{aligned} > \text{RaizDos} := \text{solve}(\text{EcuacaracDos}) \\ \text{RaizDos} &:= 0 \end{aligned} \quad (37)$$

$$> \text{Sol}_3 := x(t) = \exp(\text{RaizDos}_1 \cdot t)$$

$$Sol_3 := x(t) = 1 \quad (38)$$

$$\begin{aligned} > SolDosHom := x(t) = C_3 \cdot rhs(Sol_3) \\ SolDosHom := x(t) = C_3 \end{aligned} \quad (39)$$

$$\begin{aligned} > SolDosNoHom := x(t) = D \cdot rhs(Sol_3) \\ SolDosNoHom := x(t) = D \end{aligned} \quad (40)$$

$$\begin{aligned} > Dprima := QQ \\ Dprima := \frac{1}{2} V_0 \sqrt{2} \end{aligned} \quad (41)$$

$$\begin{aligned} > SolucionDosNoHom := x(t) = (int(Dprima, t) + C_1) rhs(Sol_3) \\ SolucionDosNoHom := x(t) = \frac{1}{2} V_0 \sqrt{2} t + C_1 \end{aligned} \quad (42)$$

> restart

Caso de construccion simple bajo sismo

$$\begin{aligned} > Ecuacion := diff(x(t), t^2) + 9 \cdot x(t) = 60 \cdot \sin(9 t) \\ Ecuacion := \frac{d^2}{dt^2} x(t) + 9 x(t) = 60 \sin(9 t) \end{aligned} \quad (43)$$

$$\begin{aligned} > Condiciones := x(0) = 0, D(x)(0) = 0; \\ Condiciones := x(0) = 0, D(x)(0) = 0 \end{aligned} \quad (44)$$

$$\begin{aligned} > EcuacionHom := lhs(Ecuacion) = 0 \\ EcuacionHom := \frac{d^2}{dt^2} x(t) + 9 x(t) = 0 \end{aligned} \quad (45)$$

$$\begin{aligned} > Q := rhs(Ecuacion) \\ Q := 60 \sin(9 t) \end{aligned} \quad (46)$$

$$\begin{aligned} > EcuaCarac := m \cdot 2 + 9 = 0 \\ EcuaCarac := m^2 + 9 = 0 \end{aligned} \quad (47)$$

$$\begin{aligned} > Raiz := solve(EcuaCarac) \\ Raiz := 3 I, -3 I \end{aligned} \quad (48)$$

Caso IIIbis: Par de raíces imaginarias

$$\begin{aligned} > Sol_1 := x(t) = \cos(\text{Im}(Raiz_1) \cdot t); Sol_2 := x(t) = \sin(\text{Im}(Raiz_2) \cdot t) \\ Sol_1 := x(t) = \cos(3 t) \\ Sol_2 := x(t) = -\sin(3 t) \end{aligned} \quad (49)$$

> with(linalg) :

$$\begin{aligned} > WW := wronskian([rhs(Sol_1), rhs(Sol_2)], t) \\ WW := \begin{bmatrix} \cos(3 t) & -\sin(3 t) \\ -3 \sin(3 t) & -3 \cos(3 t) \end{bmatrix} \end{aligned} \quad (50)$$

$$\begin{aligned} > BB := array([0, Q]) \\ BB := \begin{bmatrix} 0 & 60 \sin(9 t) \end{bmatrix} \end{aligned} \quad (51)$$

$$> SOL := linsolve(WW, BB) : Aprima := SOL_1; Bprima := SOL_2$$

$$\begin{aligned} A_{prima} &:= -\frac{20 \sin(3t) \sin(9t)}{\sin(3t)^2 + \cos(3t)^2} \\ B_{prima} &:= -\frac{20 \cos(3t) \sin(9t)}{\sin(3t)^2 + \cos(3t)^2} \end{aligned} \quad (52)$$

$$> A := \text{int}(A_{prima}, t) + C_1; B := \text{int}(B_{prima}, t) + C_2$$

$$A := -\frac{5}{3} \sin(6t) + \frac{5}{6} \sin(12t) + C_1$$

$$B := \frac{5}{6} \cos(12t) + \frac{5}{3} \cos(6t) + C_2 \quad (53)$$

$$> \text{SolNoHom} := x(t) = \text{simplify}(A \cdot \text{rhs}(\text{Sol}_1) + B \cdot \text{rhs}(\text{Sol}_2))$$

$$\text{SolNoHom} := x(t) = -\frac{10}{3} \sin(3t) \cos(3t)^2 - \sin(3t) C_2 + \frac{5}{6} \sin(3t) + \cos(3t) C_1 \quad (54)$$

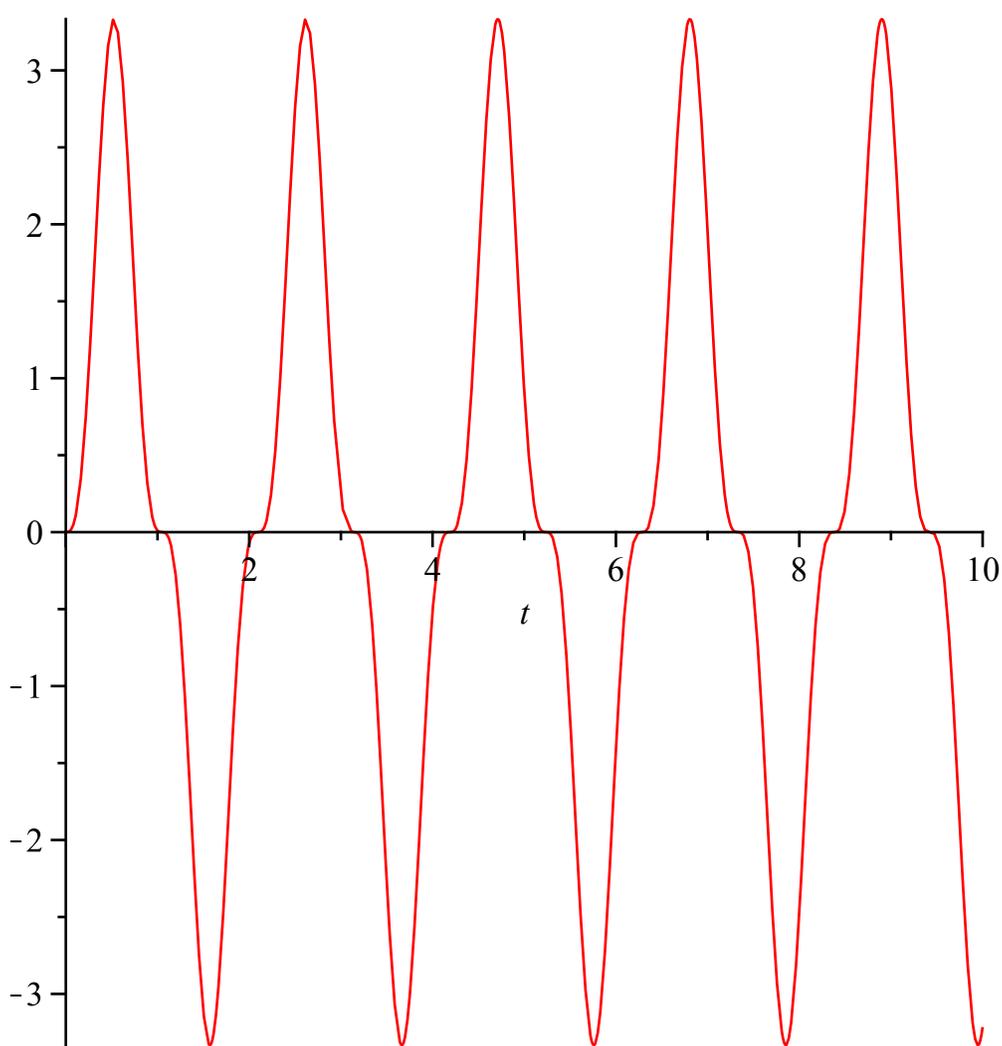
$$> \text{SolGral} := \text{dsolve}(\text{Ecuacion})$$

$$\text{SolGral} := x(t) = \sin(3t) _C2 + \cos(3t) _C1 - \frac{5}{6} \sin(9t) \quad (55)$$

$$> \text{SolPart} := \text{dsolve}(\{\text{Ecuacion}, \text{Condiciones}\})$$

$$\text{SolPart} := x(t) = \frac{5}{2} \sin(3t) - \frac{5}{6} \sin(9t) \quad (56)$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), t=0..10)$$



> restart

Caso de construcción simple bajo sismo con mediana resonancia

> Ecuacion := diff(x(t), t\$2) + 27·x(t) = 60·sin(9 t)

$$Ecuacion := \frac{d^2}{dt^2} x(t) + 27 x(t) = 60 \sin(9 t) \quad (57)$$

> Condiciones := x(0) = 0, D(x)(0) = 0;

$$Condiciones := x(0) = 0, D(x)(0) = 0 \quad (58)$$

> EcuacionHom := lhs(Ecuacion) = 0

$$EcuacionHom := \frac{d^2}{dt^2} x(t) + 27 x(t) = 0 \quad (59)$$

> Q := rhs(Ecuacion)

$$Q := 60 \sin(9 t) \quad (60)$$

> EcuaCarac := m·2 + 9 = 0

$$EcuaCarac := m^2 + 9 = 0 \quad (61)$$

> Raiz := solve(EcuaCarac)

$$Raiz := 3 I, -3 I \quad (62)$$

Caso IIIbis: Par de raíces imaginarias

$$\begin{aligned}
> \text{Sol}_1 &:= x(t) = \cos(\text{Im}(\text{Raiz}_1) \cdot t); \text{Sol}_2 := x(t) = \sin(\text{Im}(\text{Raiz}_2) \cdot t) \\
&\text{Sol}_1 := x(t) = \cos(3 t) \\
&\text{Sol}_2 := x(t) = -\sin(3 t)
\end{aligned} \tag{63}$$

$$\begin{aligned}
> \text{with}(\text{linalg}) : \\
> \text{WW} := \text{wronskian}([\text{rhs}(\text{Sol}_1), \text{rhs}(\text{Sol}_2)], t) \\
&\text{WW} := \begin{bmatrix} \cos(3 t) & -\sin(3 t) \\ -3 \sin(3 t) & -3 \cos(3 t) \end{bmatrix}
\end{aligned} \tag{64}$$

$$\begin{aligned}
> \text{BB} := \text{array}([0, Q]) \\
&\text{BB} := [0 \quad 60 \sin(9 t)]
\end{aligned} \tag{65}$$

$$\begin{aligned}
> \text{SOL} := \text{linsolve}(\text{WW}, \text{BB}) : \text{Aprima} := \text{SOL}_1; \text{Bprima} := \text{SOL}_2 \\
&\text{Aprima} := -\frac{20 \sin(3 t) \sin(9 t)}{\sin(3 t)^2 + \cos(3 t)^2} \\
&\text{Bprima} := -\frac{20 \cos(3 t) \sin(9 t)}{\sin(3 t)^2 + \cos(3 t)^2}
\end{aligned} \tag{66}$$

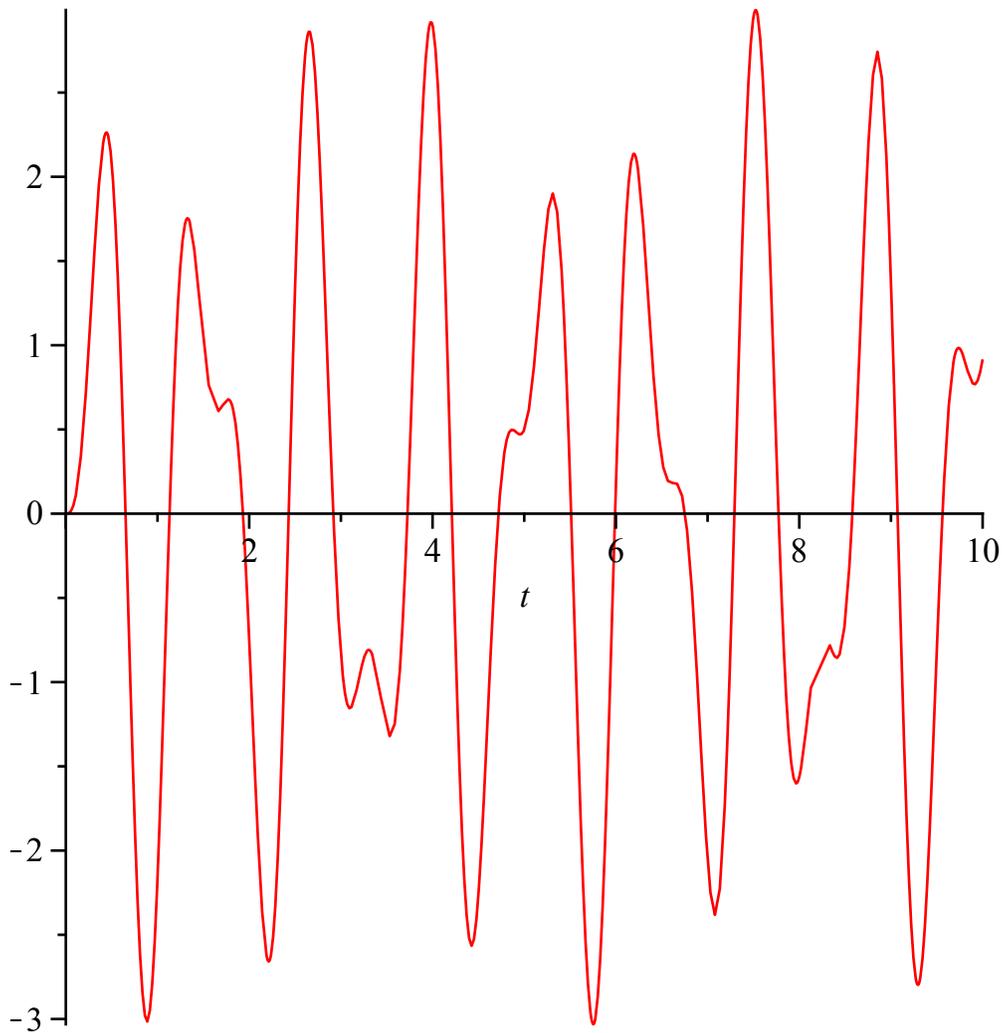
$$\begin{aligned}
> \text{A} := \text{int}(\text{Aprima}, t) + C_1; \text{B} := \text{int}(\text{Bprima}, t) + C_2 \\
&\text{A} := -\frac{5}{3} \sin(6 t) + \frac{5}{6} \sin(12 t) + C_1 \\
&\text{B} := \frac{5}{6} \cos(12 t) + \frac{5}{3} \cos(6 t) + C_2
\end{aligned} \tag{67}$$

$$\begin{aligned}
> \text{SolNoHom} := x(t) = \text{simplify}(\text{A} \cdot \text{rhs}(\text{Sol}_1) + \text{B} \cdot \text{rhs}(\text{Sol}_2)) \\
&\text{SolNoHom} := x(t) = -\frac{10}{3} \sin(3 t) \cos(3 t)^2 - \sin(3 t) C_2 + \frac{5}{6} \sin(3 t) + \cos(3 t) C_1
\end{aligned} \tag{68}$$

$$\begin{aligned}
> \text{SolGral} := \text{dsolve}(\text{Ecuacion}) \\
&\text{SolGral} := x(t) = \sin(3 \sqrt{3} t) _C2 + \cos(3 \sqrt{3} t) _C1 - \frac{10}{9} \sin(9 t)
\end{aligned} \tag{69}$$

$$\begin{aligned}
> \text{SolPart} := \text{dsolve}(\{ \text{Ecuacion}, \text{Condiciones} \}) \\
&\text{SolPart} := x(t) = \frac{10}{9} \sin(3 \sqrt{3} t) \sqrt{3} - \frac{10}{9} \sin(9 t)
\end{aligned} \tag{70}$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), t=0..10)$$



> restart

Caso de construcción simple bajo sismo con resonancia completa

> Ecuacion := diff(x(t), t\$2) + 81·x(t) = 60·sin(9 t)

$$Ecuacion := \frac{d^2}{dt^2} x(t) + 81 x(t) = 60 \sin(9 t) \quad (71)$$

> Condiciones := x(0) = 0, D(x)(0) = 0;

$$Condiciones := x(0) = 0, D(x)(0) = 0 \quad (72)$$

> EcuacionHom := lhs(Ecuacion) = 0

$$EcuacionHom := \frac{d^2}{dt^2} x(t) + 81 x(t) = 0 \quad (73)$$

> Q := rhs(Ecuacion)

$$Q := 60 \sin(9 t) \quad (74)$$

> EcuaCarac := m·2 + 9 = 0

$$EcuaCarac := m^2 + 9 = 0 \quad (75)$$

> Raiz := solve(EcuaCarac)

$$Raiz := 3 I, -3 I \quad (76)$$

Caso IIIbis: Par de raíces imaginarias

$$\begin{aligned}
 > \text{Sol}_1 := x(t) = \cos(\text{Im}(\text{Raiz}_1) \cdot t); \text{Sol}_2 := x(t) = \sin(\text{Im}(\text{Raiz}_2) \cdot t) \\
 & \quad \text{Sol}_1 := x(t) = \cos(3 t) \\
 & \quad \text{Sol}_2 := x(t) = -\sin(3 t)
 \end{aligned} \tag{77}$$

$$\begin{aligned}
 > \text{with}(\text{linalg}) : \\
 > \text{WW} := \text{wronskian}([\text{rhs}(\text{Sol}_1), \text{rhs}(\text{Sol}_2)], t) \\
 & \quad \text{WW} := \begin{bmatrix} \cos(3 t) & -\sin(3 t) \\ -3 \sin(3 t) & -3 \cos(3 t) \end{bmatrix}
 \end{aligned} \tag{78}$$

$$\begin{aligned}
 > \text{BB} := \text{array}([0, Q]) \\
 & \quad \text{BB} := [0 \quad 60 \sin(9 t)]
 \end{aligned} \tag{79}$$

$$\begin{aligned}
 > \text{SOL} := \text{linsolve}(\text{WW}, \text{BB}) : \text{Aprima} := \text{SOL}_1; \text{Bprima} := \text{SOL}_2 \\
 & \quad \text{Aprima} := -\frac{20 \sin(3 t) \sin(9 t)}{\sin(3 t)^2 + \cos(3 t)^2} \\
 & \quad \text{Bprima} := -\frac{20 \cos(3 t) \sin(9 t)}{\sin(3 t)^2 + \cos(3 t)^2}
 \end{aligned} \tag{80}$$

$$\begin{aligned}
 > A := \text{int}(\text{Aprima}, t) + C_1; B := \text{int}(\text{Bprima}, t) + C_2 \\
 & \quad A := -\frac{5}{3} \sin(6 t) + \frac{5}{6} \sin(12 t) + C_1 \\
 & \quad B := \frac{5}{6} \cos(12 t) + \frac{5}{3} \cos(6 t) + C_2
 \end{aligned} \tag{81}$$

$$\begin{aligned}
 > \text{SolNoHom} := x(t) = \text{simplify}(A \cdot \text{rhs}(\text{Sol}_1) + B \cdot \text{rhs}(\text{Sol}_2)) \\
 & \quad \text{SolNoHom} := x(t) = -\frac{10}{3} \sin(3 t) \cos(3 t)^2 - \sin(3 t) C_2 + \frac{5}{6} \sin(3 t) + \cos(3 t) C_1
 \end{aligned} \tag{82}$$

$$\begin{aligned}
 > \text{SolGral} := \text{dsolve}(\text{Ecuacion}) \\
 & \quad \text{SolGral} := x(t) = \sin(9 t) _C2 + \cos(9 t) _C1 - \frac{10}{3} \cos(9 t) t
 \end{aligned} \tag{83}$$

$$\begin{aligned}
 > \text{SolPart} := \text{dsolve}(\{\text{Ecuacion}, \text{Condiciones}\}) \\
 & \quad \text{SolPart} := x(t) = \frac{10}{27} \sin(9 t) - \frac{10}{3} \cos(9 t) t
 \end{aligned} \tag{84}$$

$$> \text{plot}(\text{rhs}(\text{SolPart}), t=0..10)$$

