Método de Parámetros Variables

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = Q(x)$$

1º Resolver la homogénea asociada

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases} \text{raíces.} \quad m_1 \neq m_2$$

$$y_1 = e^{m_1 x} \qquad y_2 = e^{m_2 x}$$

$$y_h = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y_h = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

2° Plantear la solución de la no-homogénea

$$y_{nh} = A(x) e^{m_1 x} + B(x) e^{m_2 x}$$

$$\frac{dy}{dx} = A(x) m_1 e^{m_1 x} + B(x) m_2 e^{m_2 x} + \underbrace{\left(A'(x) e^{m_1 x} + B'(x) e^{m_2 x}\right)}_{=0}$$

$$\frac{dy}{dx} = m_1 A(x) e^{m_1 x} + m_2 B(x) e^{m_2 x} + (0)$$

$$\frac{d^2y}{dx^2} = m_1^2 A(x) e^{m_1 x} + m_2^2 B(x) e^{m_2 x} + \underbrace{\left(m_1 A'(x) e^{m_1 x} + m_2 B'(x) e^{m_2 x}\right)}_{=Q(x)}$$

$$\frac{d^2y}{dx^2} = m_1^2 A(x) e^{m_1 x} + m_2^2 B(x) e^{m_2 x} + Q(x)$$

Esta solución y sus derivadas satisfacen la EDOL(2) CC NH.

$$A'(x)e^{m_1x} + B'(x)e^{m_2x} = 0$$

$$m_1A'(x)e^{m_1x} + m_2B'(x)e^{m_2x} = Q(x)$$

$$\underbrace{\begin{bmatrix} e^{m_1x} & e^{m_2x} \\ m_1e^{m_1x} & m_2e^{m_2x} \end{bmatrix}}_{\text{Wronstiano}} \begin{bmatrix} A'(x) \\ B'(x) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ Q(x) \end{bmatrix}}_{\text{Vector indep.}}$$

$$\left. \begin{aligned} A'(x) &= \frac{\begin{vmatrix} 0 & e^{m_2x} \\ Q(x) & m_2e^{m_2x} \end{vmatrix}}{(m_2-m_1)e^{m_1x}e^{m_2x}} \\ B'(x) &= \frac{\begin{vmatrix} e^{m_1x} & 0 \\ m_1e^{m_1x} & Q(x) \end{vmatrix}}{(m_2-m_1)e^{m_1x}e^{m_2x}} \end{aligned} \right\} \text{Resolución por el método de Kramer}$$

$$A(x) = \int A'(x)\,dx + C_1$$

$$B(x) = \int B'(x)\,dx + C_2$$

$$y_{ah} = A(x)\,e^{m_1 x} + B(x)\,e^{m_2 x}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 8e^{x} - 4e^{-x}$$

$$SolGral := y(x) = e^{2x}\_C2 + e^{3x}\_C1 + 4\,e^{x} - \frac{1}{3}\,e^{-x}$$

Otro ejemplo

$$\frac{d^2y}{dt^2} = -g$$

$$y(t) = -\frac{1}{2} t^2 g + C_1 + C_2 t$$

$$\frac{dx}{dt} = V_0 \cos\left(\frac{\pi}{4}\right)$$

$$SolucionDosNoHom := x(t) = \frac{1}{2} V_0 \sqrt{2}\, t + C_1$$

Caso del sismo.

$$\frac{d^2x}{dt^2} + 9x = 60\,\text{Sen}(9t)$$

Condiciones

$x(0) = 0$

$x'(0) = 0$

$$SolNoHom := x(t) = -\frac{10}{3}\sin(3\,t)\cos(3\,t)^2 - \sin(3\,t)\,C_2 + \frac{5}{6}\sin(3\,t) + \cos(3\,t)\,C_1$$

$$SolGral := x(t) = \sin(3\,t)\,\_C2 + \cos(3\,t)\,\_C1 - \frac{5}{6}\sin(9\,t)$$

$$SolPart := x(t) = \frac{5}{2}\sin(3\,t) - \frac{5}{6}\sin(9\,t)$$