

> restart

>

$$y''' - 6y'' + 4y' - 8y = 3e^{2x} - 4e^x \sin(2x)$$
$$y(0) = 4 \quad y'(0) = -2 \quad y''(0) = 10.$$

> EcuacionNoHom := $y''' - 6y'' + 4y' - 8y = 3 \exp(2x) - 4 \exp(x) \sin(2x)$

$$\text{EcuacionNoHom} := \frac{d^3}{dx^3} y(x) - 6 \left(\frac{d^2}{dx^2} y(x) \right) + 4 \left(\frac{d}{dx} y(x) \right) - 8 y(x) = 3 e^{2x} - 4 e^x \sin(2x) \quad (1)$$

> Condiciones := $y(0) = 4, D(y)(0) = -2, D(D(y))(0) = 10$

$$\text{Condiciones} := y(0) = 4, D(y)(0) = -2, D^{(2)}(y)(0) = 10 \quad (2)$$

APLICANDO EL MÉTODO DE LA ECUACION CARACTERÍSTICA

> EcuacionHom := lhs(EcuacionNoHom) = 0

$$\text{EcuacionHom} := \frac{d^3}{dx^3} y(x) - 6 \left(\frac{d^2}{dx^2} y(x) \right) + 4 \left(\frac{d}{dx} y(x) \right) - 8 y(x) = 0 \quad (3)$$

> Q := rhs(EcuacionNoHom)

$$Q := 3 e^{2x} - 4 e^x \sin(2x) \quad (4)$$

> EcuaCarac := $m \cdot 3 - 6m \cdot 2 + 4m - 8 = 0$

$$\text{EcuaCarac} := m^3 - 6m^2 + 4m - 8 = 0 \quad (5)$$

> Raiz := solve(EcuaCarac) : evalf(% , 2)

$$5.6, 0.3 + 1.1i, 0.3 - 1.1i \quad (6)$$

Caso III combinado con Caso I

> Sol₁ := $y(x) = \exp(\text{Raiz}_1 \cdot x) : \text{evalf}(\%, 2)$

$$y(x) = e^{5.6x} \quad (7)$$

> Sol₂ := $y(x) = \exp(\text{Re}(\text{Raiz}_2) \cdot x) \cdot \cos(\text{Im}(\text{Raiz}_2) \cdot x) : \text{evalf}(\%, 2)$

$$y(x) = e^{0.3x} \cos(1.1x) \quad (8)$$

> Sol₃ := $y(x) = \exp(\text{Re}(\text{Raiz}_2) \cdot x) \cdot \sin(\text{Im}(\text{Raiz}_2) \cdot x) : \text{evalf}(\%, 2)$

$$y(x) = e^{0.3x} \sin(1.1x) \quad (9)$$

> SolucionHom := $y(x) = C_1 \cdot \text{rhs}(\text{Sol}_1) + C_2 \cdot \text{rhs}(\text{Sol}_2) + C_3 \cdot \text{rhs}(\text{Sol}_3) : \text{evalf}(\%, 2)$

$$y(x) = C_1 e^{5.6x} + C_2 e^{0.3x} \cos(1.1x) + C_3 e^{0.3x} \sin(1.1x) \quad (10)$$

> SolucionNoHom := $y(x) = A \cdot \text{rhs}(\text{Sol}_1) + B \cdot \text{rhs}(\text{Sol}_2) + E \cdot \text{rhs}(\text{Sol}_3) : \text{evalf}(\%, 2)$

$$y(x) = A e^{5.6x} + B e^{0.3x} \cos(1.1x) + E e^{0.3x} \sin(1.1x) \quad (11)$$

OBTENER LOS PARAMETROS VARIABLES

> with(linalg) :

> $WW := \text{wronskian}([rhs(Sol_1), rhs(Sol_2), rhs(Sol_3)], x) : \text{evalf}(WW[1, 1], 2);$
 $\text{evalf}(WW[2, 2], 2); \text{evalf}(WW[3, 3], 2);$
 $e^{5.6x}$

$$\begin{aligned} & 0.3 e^{0.3x} \cos(1.1x) - 1.1 e^{0.3x} \sin(1.1x) \\ & - 1.2 e^{0.3x} \sin(1.1x) + 0.68 e^{0.3x} \cos(1.1x) \end{aligned} \quad (12)$$

> $BB := \text{array}([0, 0, Q])$

$$BB := \begin{bmatrix} 0 & 0 & 3 e^{2x} - 4 e^x \sin(2x) \end{bmatrix} \quad (13)$$

> $SOL := \text{linsolve}(WW, BB) :$

> $Aprima := SOL_1 : \text{evalf}(\%, 2); Bprima := SOL_2 : \text{evalf}(\%, 2); Eprima := SOL_3 : \text{evalf}(\%, 2)$

$$\begin{aligned} & \frac{0.031 (3. e^{2x} - 4. e^x \sin(2x))}{e^{5.4x}} \\ & - \frac{0.10 (-57. \sin(1.1x) + 12. \cos(1.1x)) (3. e^{2x} - 4. e^x \sin(2x))}{e^{0.27x} (41. \cos(1.1x)^2 + 41. \sin(1.1x)^2)} \\ & - \frac{0.10 (57. \cos(1.1x) + 12. \sin(1.1x)) (3. e^{2x} - 4. e^x \sin(2x))}{e^{0.27x} (41. \cos(1.1x)^2 + 41. \sin(1.1x)^2)} \end{aligned} \quad (14)$$

> $A := \text{simplify}(\text{int}(Aprima, x) + C_1) : \text{evalf}(\%, 2); B := \text{simplify}(\text{int}(Bprima, x) + C_2) :$
 $\text{evalf}(\%, 2); E := \text{simplify}(\text{int}(Eprima, x) + C_3) : \text{evalf}(\%, 2)$

$$\begin{aligned} & - \frac{1}{1. + \tan(x)^2} (0.0000016 (-9000. e^x - 6.1 10^5 C_1 e^{5.4x} + 16000. e^{2x} \tan(x)^2 \\ & - 6.1 10^5 C_1 e^{5.4x} \tan(x)^2 + 18000. e^{2x} - 30000. e^x \tan(x) + 6000. e^x \tan(x)^2) e^{-5.4x}) \\ & - 0.12 e^{(0.73 - 0.89I)x} - 0.081 e^{(1.7 - 1.1I)x} + C_2 + 0.08 I e^{(0.73 + 0.89I)x} + 0.06 I e^{(0.73 - 3.0I)x} \\ & - 0.08 I e^{(0.73 - 0.89I)x} - 0.08 I e^{(1.7 + 1.1I)x} + 0.083 I e^{(1.7 - 1.1I)x} - 0.059 I e^{(0.73 + 3.0I)x} \\ & - 0.006 e^{(0.73 + 3.0I)x} - 0.11 e^{(0.73 + 0.89I)x} - 0.003 e^{(0.73 - 3.0I)x} - 0.081 e^{(1.7 + 1.1I)x} \\ & - 0.09 e^{(0.73 - 0.89I)x} - 0.083 e^{(1.7 - 1.1I)x} + C_3 - 0.05 e^{(0.73 + 3.0I)x} - 0.083 e^{(0.73 + 0.89I)x} \\ & - 0.050 e^{(0.73 - 3.0I)x} + 0.003 I e^{(0.73 + 3.0I)x} - 0.003 I e^{(0.73 - 3.0I)x} - 0.12 I e^{(0.73 + 0.89I)x} \\ & + 0.080 I e^{(1.7 + 1.1I)x} - 0.080 I e^{(1.7 - 1.1I)x} + 0.11 I e^{(0.73 - 0.89I)x} - 0.083 e^{(1.7 + 1.1I)x} \end{aligned} \quad (15)$$

> $\text{simplify}(\text{SolucionNoHom}) : \text{evalf}(\%, 2)$

$$\begin{aligned} y(x) = & -0.11 e^{(0.99 + 0.89I)x} \cos(1.1x) - 0.01 e^x + e^{0.27x} \sin(1.1x) C_3 + e^{0.27x} \cos(1.1x) C_2 \\ & - 0.083 I e^{(1.9 + 1.1I)x} \cos(1.1x) + 0.083 I e^{(1.9 - 1.1I)x} \cos(1.1x) + 0.03 e^x \sin(x) \cos(x) \\ & - 0.11 e^{(0.99 - 0.89I)x} \cos(1.1x) - 0.052 e^{(0.99 - 3.0I)x} \sin(1.1x) \\ & - 0.055 e^{(0.99 + 3.0I)x} \sin(1.1x) + 0.080 I e^{(1.9 + 1.1I)x} \sin(1.1x) \\ & - 0.080 I e^{(1.9 - 1.1I)x} \sin(1.1x) - 0.11 I e^{(0.99 + 0.89I)x} \sin(1.1x) \\ & - 0.090 I e^{(0.99 - 0.89I)x} \cos(1.1x) - 0.055 I e^{(0.99 + 3.0I)x} \cos(1.1x) \\ & + 0.078 I e^{(0.99 + 0.89I)x} \cos(1.1x) + 0.055 I e^{(0.99 - 3.0I)x} \cos(1.1x) \\ & + 0.11 I e^{(0.99 - 0.89I)x} \sin(1.1x) - 0.0066 I e^{(0.99 - 3.0I)x} \sin(1.1x) \\ & + 0.0026 I e^{(0.99 + 3.0I)x} \sin(1.1x) + e^{5.4x} C_1 - 0.08 e^{(0.99 + 0.89I)x} \sin(1.1x) \end{aligned} \quad (16)$$

$$\begin{aligned}
& + 0.03 e^x \cos(x)^2 - 0.03 e^{2x} - 0.003 e^{(0.99 - 3.0 I) x} \cos(1.1 x) \\
& - 0.081 e^{(1.9 - 1.1 I) x} \cos(1.1 x) - 0.005 e^{(0.99 + 3.0 I) x} \cos(1.1 x) \\
& - 0.079 e^{(0.99 - 0.89 I) x} \sin(1.1 x) - 0.083 e^{(1.9 + 1.1 I) x} \sin(1.1 x) \\
& - 0.083 e^{(1.9 - 1.1 I) x} \sin(1.1 x) - 0.080 e^{(1.9 + 1.1 I) x} \cos(1.1 x)
\end{aligned}$$

> *Sistema* := eval(subs(x=0, rhs(*SolucionNoHom*) = rhs(*Condiciones*₁))), eval(subs(x=0, rhs(diff(*SolucionNoHom*, x)) = rhs(*Condiciones*₂))), eval(subs(x=0, rhs(diff(*SolucionNoHom*, x\$2)) = rhs(*Condiciones*₃))) : evalf(*Sistema*₁, 2); evalf(*Sistema*₂, 2); evalf(*Sistema*₃, 2)

$$-0.40 + C_1 + C_2 = 4.$$

$$5.6 C_1 + 0.3 C_2 + 1.1 C_3 - 0.70 + 0.010 I = -2.$$

$$-0.40 - 0.083 I + 33. C_1 - 1.2 C_2 + 0.68 C_3 = 10. \quad (17)$$

> *Parametro* := solve({*Sistema*}, {C₁, C₂, C₃}) : evalf(%, 2);

$$\{C_1 = 0.60, C_2 = 3.8, C_3 = -4.6\} \quad (18)$$

> *SolucionPart* := simplify(subs(C₁=rhs(*Parametro*₁), C₂=rhs(*Parametro*₂), C₃=rhs(*Parametro*₃), *SolucionNoHom*)) : evalf(%, 2)

$$y(x) = 0.027 e^x \cos(x)^2 - 0.011 e^x - 0.078 e^{(1.9 + 1.1 I) x} \cos(1.1 x) \quad (19)$$

$$\begin{aligned}
& - 0.072 e^{(1.9 - 1.1 I) x} \sin(1.1 x) - 0.10 e^{(0.99 + 0.89 I) x} \cos(1.1 x) \\
& - 0.078 e^{(1.9 - 1.1 I) x} \cos(1.1 x) - 0.078 e^{(0.99 + 0.89 I) x} \sin(1.1 x) + 3.8 e^{0.27 x} \cos(1.1 x) \\
& - 4.4 e^{0.27 x} \sin(1.1 x) + 0.044 e^x \sin(x) \cos(x) - 0.028 e^{2x} \\
& - 0.078 e^{(0.99 - 0.89 I) x} \sin(1.1 x) - 0.0014 e^{(0.99 - 3.0 I) x} \cos(1.1 x) \\
& - 0.046 e^{(0.99 + 3.0 I) x} \sin(1.1 x) - 0.11 e^{(0.99 - 0.89 I) x} \cos(1.1 x) \\
& - 0.00091 e^{(0.99 + 3.0 I) x} \cos(1.1 x) - 0.078 e^{(1.9 + 1.1 I) x} \sin(1.1 x) + 0.68 e^{5.4 x} \\
& - 0.046 e^{(0.99 - 3.0 I) x} \sin(1.1 x) - 0.046 I e^{(0.99 + 3.0 I) x} \cos(1.1 x) \\
& - 0.11 I e^{(0.99 + 0.89 I) x} \sin(1.1 x) + 0.10 I e^{(0.99 - 0.89 I) x} \sin(1.1 x) \\
& - 0.00091 I e^{(0.99 - 3.0 I) x} \sin(1.1 x) + 0.072 I e^{(0.99 + 0.89 I) x} \cos(1.1 x) \\
& + 0.040 I e^{(0.99 - 3.0 I) x} \cos(1.1 x) + 0.00091 I e^{(0.99 + 3.0 I) x} \sin(1.1 x) \\
& - 0.072 I e^{(1.9 + 1.1 I) x} \cos(1.1 x) - 0.078 I e^{(1.9 - 1.1 I) x} \sin(1.1 x) \\
& + 0.072 I e^{(1.9 - 1.1 I) x} \cos(1.1 x) + 0.078 I e^{(1.9 + 1.1 I) x} \sin(1.1 x) \\
& - 0.072 I e^{(0.99 - 0.89 I) x} \cos(1.1 x)
\end{aligned}$$

> plot([rhs(*SolucionPart*), rhs(diff(*SolucionPart*, x)), rhs(diff(*SolucionPart*, x\$2))], x=0..1, color=[red, blue, brown])

