$$A \longrightarrow e^{At}$$

$$\frac{d}{dt} e^{At} = A e^{At}$$

$$\left[e^{At} \right]_{t=0} = I$$

$$\left[\frac{d}{dt} e^{At} \right]_{t=0} = A \left[e^{At} \right]_{t=0}$$

$$= A \cdot I$$

$$= A$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \qquad \det \begin{bmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{bmatrix} = 0$$

$$\det(A - \lambda I) = 0 \qquad (2-\lambda)(4-\lambda) - (3)(1) = 0$$

$$\lambda^2 - 6\lambda + 5 = 0 \qquad (\lambda - 1)(\lambda - 5) = 0$$

$$\boxed{\lambda_1 = 1} \qquad \boxed{\lambda_2 = 5}$$

$$e^{At} = \beta_0(t) I + \beta_1(t) A$$

$$e^{\lambda_i t} = \beta_0(t)(1) + \beta_1(t)\lambda_i$$

$$\begin{cases} e^{t} = \beta_0(t) + \beta_1(t) \\ e^{5t} = \beta_0(t) + 5\beta_1(t) \end{cases} \qquad \begin{bmatrix} 1 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} \beta_0(t) \\ \beta_1(t) \end{bmatrix} = \begin{bmatrix} e^{t} \\ e^{5t} \end{bmatrix}$$

$$\beta_0(t) = \frac{\begin{vmatrix} e^t & 1 \\ e^{5t} & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix}} \Rightarrow \frac{(5e^{t} - e^{5t})}{4}$$

$$\beta_1(t) = \frac{\begin{vmatrix} 1 & e^t \\ 1 & e^{5t} \end{vmatrix}}{4} \Rightarrow \frac{e^{5t} - e^{t}}{4}$$

$$e^{At} = \left(\frac{5}{4}e^{t} - \frac{1}{4}e^{5t}\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^{t}\right) \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \frac{1}{4} \begin{bmatrix} 5-2 & -3 \\ -1 & 5-4 \end{bmatrix} e^{t} + \frac{1}{4} \begin{bmatrix} -1+2 & 3 \\ 1 & -1+4 \end{bmatrix} e^{5t}$$

$$e^{At} = \frac{1}{4} \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4} \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$e^{At} = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$\frac{d}{dt} e^{At} = \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix} e^{5t}$$

$$\left(\frac{d}{dt} e^{At}\right)_{t=0} = \frac{1}{4}\begin{bmatrix} 3+5 & -3+15 \\ -1+5 & 1+15 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 & 12 \\ 4 & 16 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\frac{d}{dt}\bar{x}(t) = A\bar{x}(t) + \bar{b}(t)$$

$$e^{At} =$$

$$\bar{x} = e^{At}\bar{x}(0) + \int_0^t \underline{e^{A(t-\tau)}}\,\underline{b(\tau)}\,d\tau.$$

$$\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \\ y_4(0) \end{bmatrix}$$

$$\frac{dx_1(t)}{dt} = 2x_1(t) + 3x_2(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) + 4x_3(t) + 5\cos(3t)$$

$$x_1(0) = -10$$

$$x_2(0) = 5$$