> restart

> $Ecuacion := diff(z(x,y), x\$2) - 6\cdot diff(z(x,y), y) = 0$

$$Ecuacion := \frac{\partial^2}{\partial x^2} z(x,y) - 6 \left(\frac{\partial}{\partial y} z(x,y)\right) = 0 \tag{1}$$

> $EcuacionDos := eval(subs(z(x,y) = F(x)\cdot G(y), Ecuacion))$

$$EcuacionDos := \left(\frac{d^2}{dx^2} F(x)\right) G(y) - 6\, F(x) \left(\frac{d}{dy} G(y)\right) = 0 \tag{2}$$

> $EcuacionTres := simplify\left(\frac{\left(lhs(EcuacionDos) + 6\, F(x) \left(\frac{d}{dy} G(y)\right)\right)}{F(x)\cdot G(y)} = \frac{\left(rhs(EcuacionDos) + 6\, F(x) \left(\frac{d}{dy} G(y)\right)\right)}{F(x)\cdot G(y)}\right)$

$$EcuacionTres := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \frac{6 \left(\frac{d}{dy} G(y)\right)}{G(y)} \tag{3}$$

> $EcuacionX := lhs(EcuacionTres) = \mathrm{alpha}; EcuacionY := rhs(EcuacionTres) = \mathrm{alpha};$

$$EcuacionX := \frac{\frac{d^2}{dx^2} F(x)}{F(x)} = \alpha$$

$$EcuacionY := \frac{6 \left(\frac{d}{dy} G(y)\right)}{G(y)} = \alpha \tag{4}$$

> $SolucionCeroX := dsolve(subs(\mathrm{alpha} = 0, EcuacionX))$

$$SolucionCeroX := F(x) = \_C1\, x + \_C2 \tag{5}$$

> $SolucionCeroY := dsolve(subs(\mathrm{alpha} = 0, EcuacionY))$

$$SolucionCeroY := G(y) = \_C1 \tag{6}$$

> $SolucionCero := z(x,y) = rhs(SolucionCeroX)\cdot subs(\_C1 = 1, rhs(SolucionCeroY))$

$$SolucionCero := z(x,y) = \_C1\, x + \_C2 \tag{7}$$

_C

> $comprobacion_1 := simplify(eval(subs(z(x,y) = rhs(SolucionCero), Ecuacion)))$

$$comprobacion_1 := 0 = 0 \tag{8}$$

> $SolucionPosX := dsolve(subs(\mathrm{alpha} = \mathrm{beta}\cdot{}^{\wedge}2, EcuacionX))$

$$SolucionPosX := F(x) = \_C1\, e^{-\beta x} + \_C2\, e^{\beta x} \tag{9}$$

> $SolucionPosY := dsolve(subs(\mathrm{alpha} = \mathrm{beta}\cdot{}^{\wedge}2, EcuacionY))$

$$SolucionPosY := G(y) = \_C1\, e^{\frac{1}{6}\beta^2 y} \tag{10}$$

> $SolucionPositiva := z(x,y) = rhs(SolucionPosX)\cdot subs(\_C1 = 1, rhs(SolucionPosY))$

$$SolucionPositiva := z(x,y) = \left(\_C1\, e^{-\beta x} + \_C2\, e^{\beta x}\right) e^{\frac{1}{6}\beta^2 y} \tag{11}$$

> $comprobacion_2 := simplify(eval(subs(z(x,y) = rhs(SolucionPositiva), Ecuacion)))$

$$comprobacion_2 := 0 = 0 \tag{12}$$

> $SolucionNegX := dsolve(subs(\text{alpha} = -\text{beta} \cdot \cdot 2, EcuacionX))$

$$SolucionNegX := F(x) = \_C1 \sin(\beta x) + \_C2 \cos(\beta x) \tag{13}$$

> $SolucionNegY := dsolve(subs(\text{alpha} = -\text{beta} \cdot \cdot 2, EcuacionY))$

$$SolucionNegY := G(y) = \_C1\, e^{-\frac{1}{6}\beta^2 y} \tag{14}$$

> $SolucionNegativa := z(x, y) = rhs(SolucionNegX) \cdot subs(\_C1 = 1, rhs(SolucionNegY))$

$$SolucionNegativa := z(x, y) = (\_C1 \sin(\beta x) + \_C2 \cos(\beta x))\, e^{-\frac{1}{6}\beta^2 y} \tag{15}$$

> $comprobacion_3 := simplify(eval(subs(z(x, y) = rhs(SolucionNegativa), Ecuacion)))$

$$comprobacion_3 := 0 = 0 \tag{16}$$

> $with(PDEtools):$

> $pdsolve(Ecuacion) : SolGral := build(\%)$

$$SolGral := z(x, y) = e^{\sqrt{\_c_1}\, x}\, \_C3\, e^{\frac{1}{6}\_c_1 y}\, \_C1 + \frac{\_C3\, e^{\frac{1}{6}\_c_1 y}\, \_C2}{e^{\sqrt{\_c_1}\, x}} \tag{17}$$

> $comprobacion_4 := simplify(eval(subs(z(x, y) = rhs(SolGral), Ecuacion)))$

$$comprobacion_4 := 0 = 0 \tag{18}$$

> $restart$

> $Ecuacion := diff(y(x, t), x) + x \cdot diff(y(x, t), t\$2) = 5 \cdot y(x, t)$

$$Ecuacion := \frac{\partial}{\partial x} y(x, t) + x \left(\frac{\partial^2}{\partial t^2} y(x, t)\right) = 5\, y(x, t) \tag{19}$$

> $EcuacionDos := simplify(eval(subs(y(x, t) = F(x) \cdot G(t), Ecuacion)))$

$$EcuacionDos := \left(\frac{d}{dx} F(x)\right) G(t) + x\, F(x) \left(\frac{d^2}{dt^2} G(t)\right) = 5\, F(x)\, G(t) \tag{20}$$

> $EcuacionTres := lhs(EcuacionDos) - 5\, F(x)\, G(t) - x\, F(x) \left(\frac{d^2}{dt^2} G(t)\right)$
> $= rhs(EcuacionDos) - 5\, F(x)\, G(t) - x\, F(x) \left(\frac{d^2}{dt^2} G(t)\right)$

$$EcuacionTres := \left(\frac{d}{dx} F(x)\right) G(t) - 5\, F(x)\, G(t) = -x\, F(x) \left(\frac{d^2}{dt^2} G(t)\right) \tag{21}$$

> $EcuacionSeparada := simplify\left(\frac{lhs(EcuacionTres)}{x \cdot F(x) \cdot G(t)} = \frac{rhs(EcuacionTres)}{x \cdot F(x) \cdot G(t)}\right)$

$$EcuacionSeparada := \frac{\frac{d}{dx} F(x) - 5\, F(x)}{x\, F(x)} = -\frac{\frac{d^2}{dt^2} G(t)}{G(t)} \tag{22}$$

> $EcuacionX := lhs(EcuacionSeparada) = \text{alpha};\ EcuacionT := rhs(EcuacionSeparada)$
> $= \text{alpha};$

$$EcuacionX := \frac{\frac{d}{dx} F(x) - 5\, F(x)}{x\, F(x)} = \alpha$$

$$EcuacionT := -\frac{\frac{d^2}{dt^2} G(t)}{G(t)} = \alpha \quad \textbf{(23)}$$

> $SolucionCeroX := dsolve(subs(\text{alpha} = 0, EcuacionX))$

$$SolucionCeroX := F(x) = \_C1\, e^{5x} \quad \textbf{(24)}$$

> $SolucionCeroT := dsolve(subs(\text{alpha} = 0, EcuacionT))$

$$SolucionCeroT := G(t) = \_C1\, t + \_C2 \quad \textbf{(25)}$$

> $SolucionCero := y(x, t) = subs(\_C1 = 1, rhs(SolucionCeroX)) \cdot rhs(SolucionCeroT)$

$$SolucionCero := y(x, t) = e^{5x} (\_C1\, t + \_C2) \quad \textbf{(26)}$$

> $Comprobacion_1 := simplify(eval(subs(y(x, t) = rhs(SolucionCero), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad \textbf{(27)}$$

> $SolucionPosX := dsolve(subs(\text{alpha} = \text{beta} \cdot{} \cdot 2, EcuacionX))$

$$SolucionPosX := F(x) = \_C1\, e^{\frac{1}{2} x (10 + \beta^2 x)} \quad \textbf{(28)}$$

> $SolucionPosT := dsolve(subs(\text{alpha} = \text{beta} \cdot{} \cdot 2, EcuacionT))$

$$SolucionPosT := G(t) = \_C1 \sin(\beta t) + \_C2 \cos(\beta t) \quad \textbf{(29)}$$

> $SolucionPositiva := y(x, t) = subs(\_C1 = 1, rhs(SolucionPosX)) \cdot rhs(SolucionPosT)$

$$SolucionPositiva := y(x, t) = e^{\frac{1}{2} x (10 + \beta^2 x)} (\_C1 \sin(\beta t) + \_C2 \cos(\beta t)) \quad \textbf{(30)}$$

> $Comprobacion_2 := simplify(eval(subs(y(x, t) = rhs(SolucionPositiva), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad \textbf{(31)}$$

> $SolucionNegX := dsolve(subs(\text{alpha} = -\text{beta} \cdot{} \cdot 2, EcuacionX))$

$$SolucionNegX := F(x) = \_C1\, e^{-\frac{1}{2} x (-10 + \beta^2 x)} \quad \textbf{(32)}$$

> $SolucionNegT := dsolve(subs(\text{alpha} = -\text{beta} \cdot{} \cdot 2, EcuacionT))$

$$SolucionNegT := G(t) = \_C1\, e^{\beta t} + \_C2\, e^{-\beta t} \quad \textbf{(33)}$$

> $SolucionNegativa := y(x, t) = subs(\_C1 = 1, rhs(SolucionNegX)) \cdot rhs(SolucionNegT)$

$$SolucionNegativa := y(x, t) = e^{-\frac{1}{2} x (-10 + \beta^2 x)} (\_C1\, e^{\beta t} + \_C2\, e^{-\beta t}) \quad \textbf{(34)}$$

> $Comprobacion_3 := simplify(eval(subs(y(x, t) = rhs(SolucionNegativa), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_3 := 0 = 0 \quad \textbf{(35)}$$

> $with(PDEtools):$

> $pdsolve(Ecuacion); SolGral := build(\%)$

$$(y(x, t) = \_F1(x)\, \_F2(t)) \;\&\text{where}\; \left[\left\{ \frac{d}{dx} \_F1(x) = -\_c_1\, x\, \_F1(x) + 5\, \_F1(x), \frac{d^2}{dt^2} \_F2(t) = \_F2(t)\, \_c_1 \right\}\right]$$

$$SolGral := y(x,t) = \_C1\,\mathrm{e}^{-\frac{1}{2}\,\_c_1\,x^2}\,\left(\mathrm{e}^{x}\right)^5\,\_C2\,\mathrm{e}^{\sqrt{\_c_1}\,t} + \frac{\_C1\,\mathrm{e}^{-\frac{1}{2}\,\_c_1\,x^2}\,\left(\mathrm{e}^{x}\right)^5\,\_C3}{\mathrm{e}^{\sqrt{\_c_1}\,t}} \tag{36}$$

> $Comprobacion_4 := simplify(eval(subs(y(x,t) = rhs(SolGral), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_4 := 0 = 0 \tag{37}$$

> $restart$

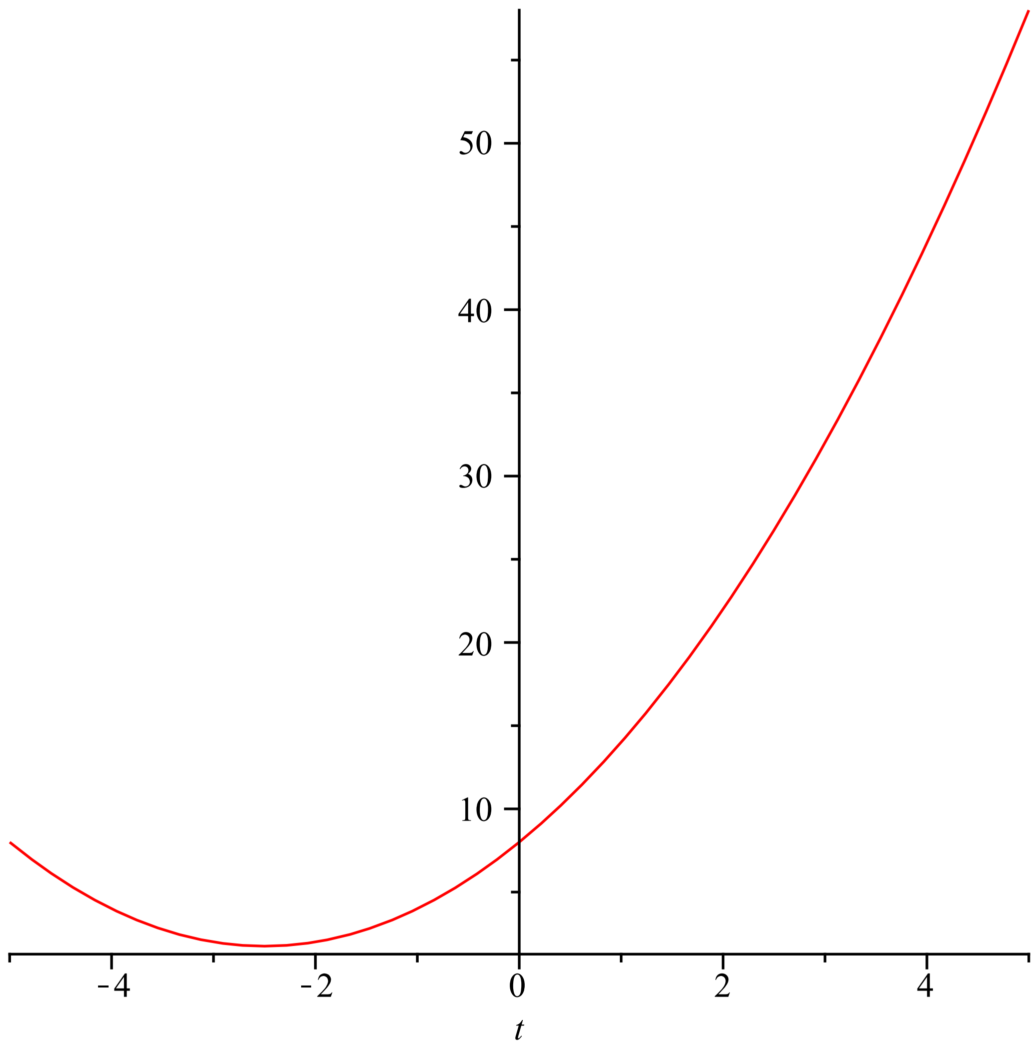
> $f := t^{\cdot\cdot}2 + 5\cdot t + 8;$

$$f := t^2 + 5\,t + 8 \tag{38}$$

> $L := 5$

$$L := 5 \tag{39}$$

> $plot(f, t = -L..L)$



> $a_0 := \left(\frac{1}{L}\right)\cdot int(f, t = -L..L)$

$$a_0 := \frac{98}{3} \tag{40}$$

$$> C := \frac{a_0}{2}$$

$$C := \frac{49}{3} \tag{41}$$

$$> a_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge} n, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$$

$$a_n := \frac{100 \, (-1)^n}{n^2 \, \pi^2} \tag{42}$$

$$> b_n := subs\left(\sin(n \cdot \mathrm{Pi}) = 0, \cos(n \cdot \mathrm{Pi}) = (-1)^{\wedge} n, \left(\frac{1}{L}\right) \cdot int\left(f \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$$

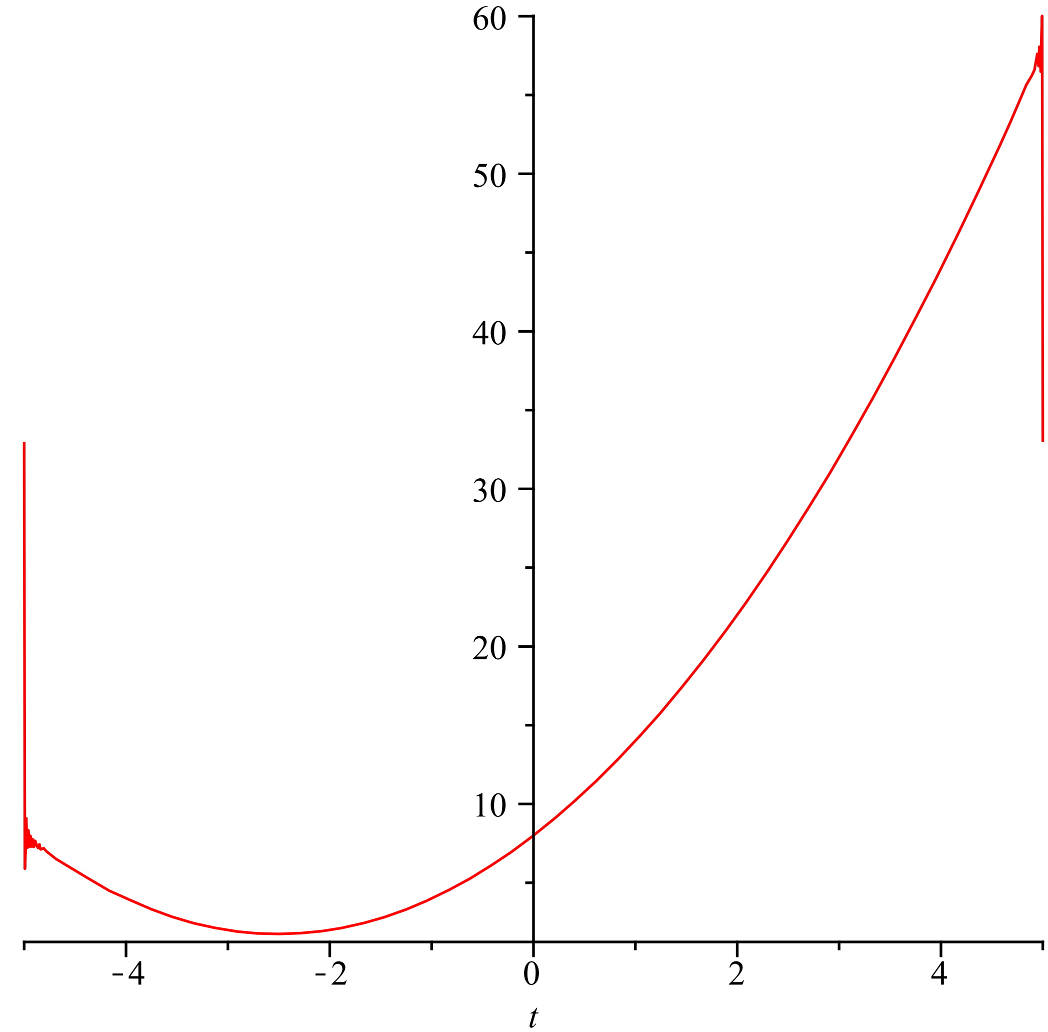
$$b_n := -\frac{50 \, (-1)^n}{n \, \pi} \tag{43}$$

$$> STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..infinity\right)$$

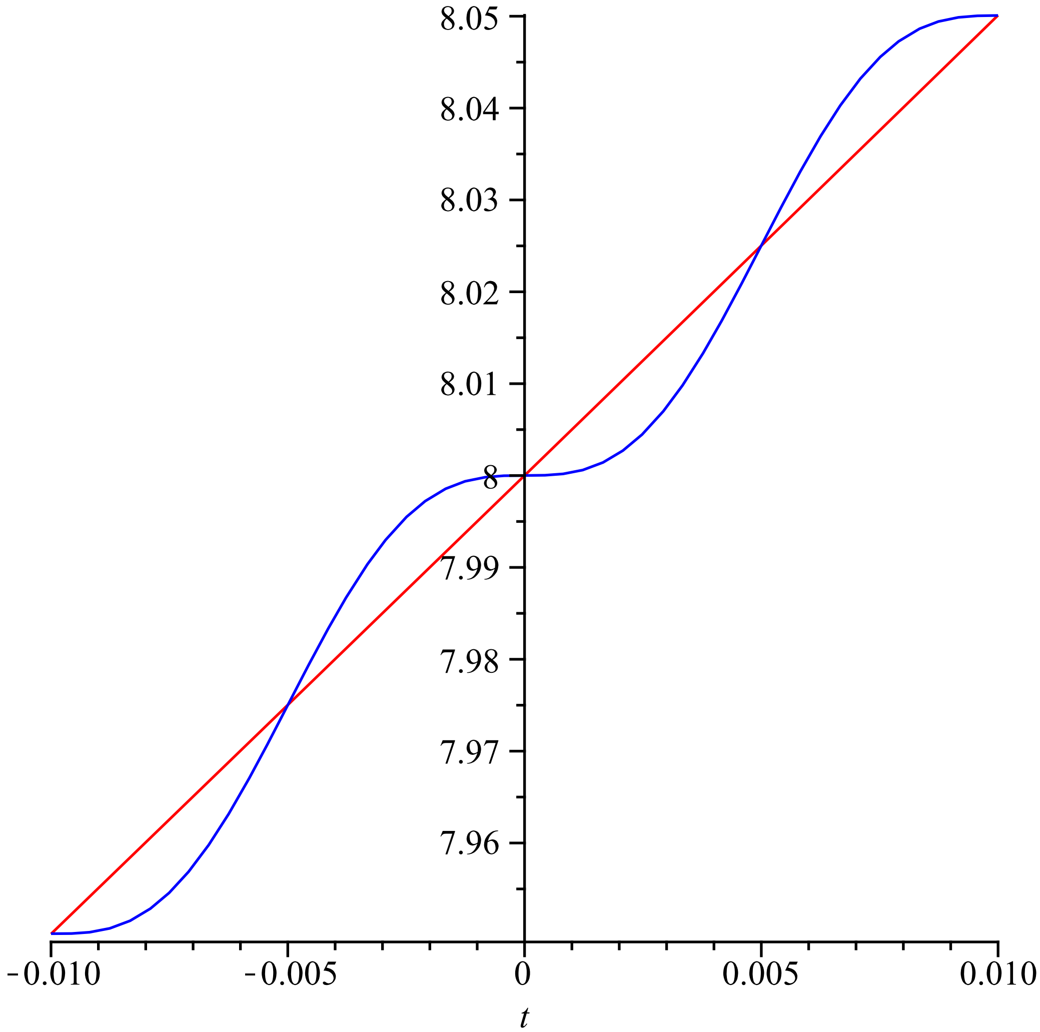
$$STF := \frac{49}{3} + \sum_{n=1}^{\infty} \left(\frac{100 \, (-1)^n \cos\left(\frac{1}{5} \, n \, \pi \, t\right)}{n^2 \, \pi^2} - \frac{50 \, (-1)^n \sin\left(\frac{1}{5} \, n \, \pi \, t\right)}{n \, \pi} \right) \tag{44}$$

$$> STF_{1000} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \mathrm{Pi} \cdot t}{L}\right), n = 1..1000\right):$$

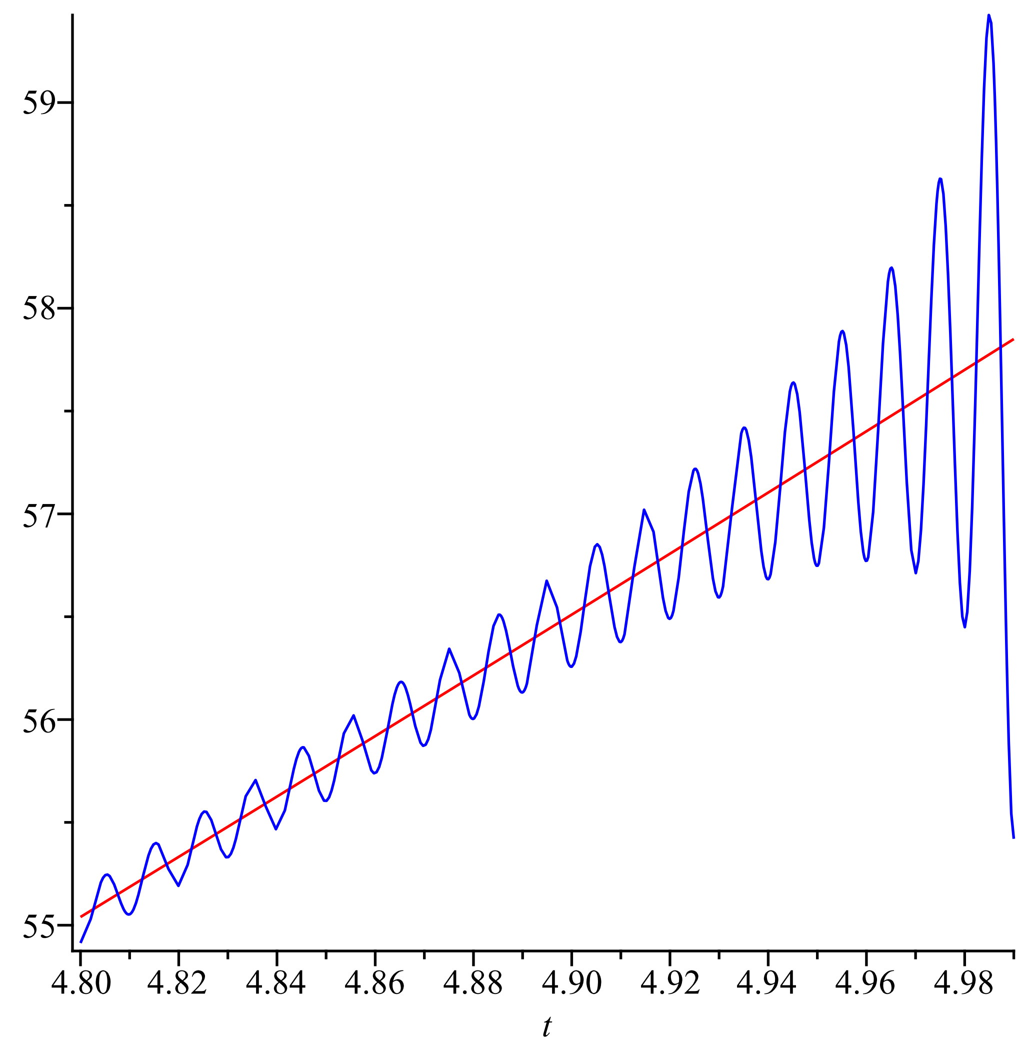
$$> plot(STF_{1000}, t = -L..L)$$



> $plot([f, STF_{1000}], t=-0.01..0.01, color=[red, blue])$



> $plot([f, STF_{1000}], t = 4.8 ..4.99, color = [red, blue])$



59
58
57
56
55
4.80
4.82
4.84
4.86
4.88
4.90
4.92
4.94
4.96
4.98
t