

$$\begin{aligned} \frac{dx_1}{dt} &= 2x_1 + 3x_2 \\ \frac{dx_2}{dt} &= x_1 + 4x_2 \end{aligned} \Rightarrow \frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\lambda_1 = \quad \lambda_2 =$$

$$e^{At} = B_0(t)I + B_1(t)A \Rightarrow e^{\lambda_1 t} = B_0(t)(1) + B_1(t)\lambda_1$$

$$\text{Ex. Carac. } \det(A - \lambda I) = 0 \quad e^{\lambda_2 t} = B_0(t)(1) + B_1(t)\lambda_2$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 1 & 4-\lambda \end{vmatrix} = 0 \quad (2-\lambda)(4-\lambda) - (3)(1) = 0$$

$$\lambda^2 - 6\lambda + 5 = 0 \quad \lambda_1 = 1$$

$$(\lambda - 1)(\lambda - 5) = 0 \quad \lambda_2 = 5$$

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$$\begin{aligned} e^t &= B_0 + B_1 \\ e^{5t} &= B_0 + 5B_1 \\ -e^t &= -B_0 - B_1 \\ \hline e^{5t} - e^t &= 4B_1 \end{aligned} \quad \left| \begin{aligned} B_1 &= \frac{1}{4}e^{5t} - \frac{1}{4}e^t \\ B_0 &= e^t - B_1 \\ B_0 &= e^t - \frac{1}{4}e^{5t} + \frac{1}{4}e^t \\ B_0 &= -\frac{1}{4}e^{5t} + \frac{5}{4}e^t \end{aligned} \right.$$

$$e^{At} = B_0(t)I + B_1(t)A$$

$$e^{At} = \left(-\frac{1}{4}e^{5t} + \frac{5}{4}e^t\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4}e^{5t} - \frac{1}{4}e^t\right) \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -\frac{1}{4} + \frac{2}{4} & \frac{3}{4} \\ \frac{1}{4} & -\frac{1}{4} + \frac{4}{4} \end{bmatrix} e^{5t} + \begin{bmatrix} \frac{5}{4} - \frac{2}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{5}{4} - \frac{4}{4} \end{bmatrix} e^t$$

$$e^{At} = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} e^{5t} + \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} e^t$$

$$e^{A(0)} = \begin{bmatrix} \frac{4}{4} & 0 \\ 0 & \frac{4}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{d}{dt}e^{At} = \begin{bmatrix} \frac{5}{4} & \frac{15}{4} \\ \frac{3}{4} & \frac{15}{4} \end{bmatrix} e^{5t} + \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} e^t$$

$$Ae^{At} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix} e^{5t} + \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} e^t$$

$$\begin{bmatrix} \frac{5}{4} & \frac{15}{4} \\ \frac{5}{4} & \frac{15}{4} \end{bmatrix} e^{5t} + \begin{bmatrix} \frac{3}{4} & -\frac{3}{4} \\ -\frac{1}{4} & \frac{1}{4} \end{bmatrix} e^t$$

$$MatExp := \begin{bmatrix} \frac{3}{4} e^t + \frac{1}{4} e^{5t} & \frac{3}{4} e^{5t} - \frac{3}{4} e^t \\ \frac{1}{4} e^{5t} - \frac{1}{4} e^t & \frac{1}{4} e^t + \frac{3}{4} e^{5t} \end{bmatrix}$$

$$\mathcal{Q}^{At} = \begin{bmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \mathcal{Q}^{St} \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix} \mathcal{Q}^t$$

$$\frac{dx_1}{dt} = x_1 + x_2 + x_3$$

$$x_1(0) = 1$$

$$\frac{dx_2}{dt} = x_1 - x_2 + x_3$$

$$x_2(0) = 2$$

$$\frac{dx_3}{dt} = -x_1 + x_2 - x_3$$

$$x_3(0) = 3$$

$$\frac{d}{dt} \bar{x} = A \bar{x} \longrightarrow \bar{x} = e^{At} \bar{x}(0)$$

$$\frac{d}{dt} \bar{x} = A \bar{x} + \bar{b}(t)$$

$$\bar{x} = e^{At} \bar{x}(0) + \int_0^t e^{A(t-\tau)} \bar{b}(\tau) d\tau$$

$$\bar{x} = e^{A(t-a)} \bar{x}(a) + \int_a^t e^{A(t-\tau)} \bar{b}(\tau) d\tau$$

$$P_{T&S} = \frac{\sum_{n=1}^3 T_n + \sum_{j=1}^5 S_j}{n+j}$$

$$P_{EP} = \frac{\sum_{i=1}^3 E_i}{3}$$

$$P_{sem} = \frac{P_{res} + P_{ep}}{2}$$

$$C_f = \frac{P_{sem} + E_f}{2}$$