> *restart*

> *Ecuacion* := *diff*$(y(x), x\$3) - 3 \cdot diff(y(x), x\$2) + 4 \cdot diff(y(x), x) - 12 \cdot y(x) = 0$

$$Ecuacion := \frac{d^3}{dx^3} y(x) - 3 \left(\frac{d^2}{dx^2} y(x) \right) + 4 \left(\frac{d}{dx} y(x) \right) - 12 \, y(x) = 0 \tag{1}$$

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> *EcuaCaract* := $m \cdot\cdot 3 - 3 \cdot m \cdot\cdot 2 + 4 \cdot m - 12 = 0$

$$EcuaCaract := m^3 - 3 \, m^2 + 4 \, m - 12 = 0 \tag{2}$$

> *Raiz* := *solve*(*EcuaCaract*)

$$Raiz := 3, 2 \, I, -2 \, I \tag{3}$$

> $Sol_1 := y(x) = \exp(Raiz_1 \cdot x)$

$$Sol_1 := y(x) = e^{3 \, x} \tag{4}$$

> $Sol_2 := y(x) = \exp\left(\mathrm{Re}(Raiz_2) \cdot x \right) \cdot \cos\left(\mathrm{Im}(Raiz_2) \cdot x \right)$

$$Sol_2 := y(x) = \cos(2 \, x) \tag{5}$$

> $Sol_3 := y(x) = \exp\left(\mathrm{Re}(Raiz_2) \cdot x \right) \cdot \sin\left(\mathrm{Im}(Raiz_2) \cdot x \right)$

$$Sol_3 := y(x) = \sin(2 \, x) \tag{6}$$

> $SolGral := y(x) = C_1 \cdot rhs(Sol_1) + C_2 \cdot rhs(Sol_2) + C_3 \cdot rhs(Sol_3)$

$$SolGral := y(x) = C_1 \, e^{3 \, x} + C_2 \cos(2 \, x) + C_3 \sin(2 \, x) \tag{7}$$

> *Sistema* := *diff*$(SolGral, x), diff(SolGral, x\$2), diff(SolGral, x\$3) : Sistema_1; Sistema_2; Sistema_3;$

$$\frac{d}{dx} y(x) = 3 \, C_1 \, e^{3 \, x} - 2 \, C_2 \sin(2 \, x) + 2 \, C_3 \cos(2 \, x)$$

$$\frac{d^2}{dx^2} y(x) = 9 \, C_1 \, e^{3 \, x} - 4 \, C_2 \cos(2 \, x) - 4 \, C_3 \sin(2 \, x)$$

$$\frac{d^3}{dx^3} y(x) = 27 \, C_1 \, e^{3 \, x} + 8 \, C_2 \sin(2 \, x) - 8 \, C_3 \cos(2 \, x) \tag{8}$$

> *Parametro* := *solve*$\left(\{Sistema\}, \{C_1, C_2, C_3\} \right) : Parametro_1; Parametro_2; Parametro_3$

$$C_1 = \frac{1}{39} \, \frac{\frac{d^3}{dx^3} y(x) + 4 \left(\frac{d}{dx} y(x) \right)}{e^{3 \, x}}$$

$$C_2 = \frac{1}{52} \, \frac{1}{\cos(2 \, x)^2 + \sin(2 \, x)^2} \left(3 \cos(2 \, x) \left(\frac{d^3}{dx^3} y(x) \right) + 12 \cos(2 \, x) \left(\frac{d}{dx} y(x) \right) \right.$$

$$\left. - 13 \cos(2 \, x) \left(\frac{d^2}{dx^2} y(x) \right) + 2 \sin(2 \, x) \left(\frac{d^3}{dx^3} y(x) \right) - 18 \left(\frac{d}{dx} y(x) \right) \sin(2 \, x) \right)$$

$$C_3 = \frac{1}{52} \, \frac{1}{\cos(2 \, x)^2 + \sin(2 \, x)^2} \left(3 \sin(2 \, x) \left(\frac{d^3}{dx^3} y(x) \right) + 12 \left(\frac{d}{dx} y(x) \right) \sin(2 \, x) \right. \tag{9}$$

$$\left. - 2 \cos(2 \, x) \left(\frac{d^3}{dx^3} y(x) \right) + 18 \cos(2 \, x) \left(\frac{d}{dx} y(x) \right) - 13 \left(\frac{d^2}{dx^2} y(x) \right) \sin(2 \, x) \right)$$

> $EcuacionInicial := simplify\big(subs\big(C_1 = rhs(Parametro_1), C_2 = rhs(Parametro_2), C_3$
$= rhs(Parametro_3), SolGral\big)\big)$

$$EcuacionInicial := y(x) = \frac{1}{12}\frac{d^3}{dx^3}y(x) + \frac{1}{3}\frac{d}{dx}y(x) - \frac{1}{4}\frac{d^2}{dx^2}y(x) \tag{10}$$

> $EcuacionFinal := rhs(EcuacionInicial)\cdot 12 - lhs(EcuacionInicial)\cdot 12 = 0$

$$EcuacionFinal := \frac{d^3}{dx^3}y(x) - 3\left(\frac{d^2}{dx^2}y(x)\right) + 4\left(\frac{d}{dx}y(x)\right) - 12\,y(x) = 0 \tag{11}$$

> $Ecuacion$

$$\frac{d^3}{dx^3}y(x) - 3\left(\frac{d^2}{dx^2}y(x)\right) + 4\left(\frac{d}{dx}y(x)\right) - 12\,y(x) = 0 \tag{12}$$

> $SolucionGral := dsolve(Ecuacion)$

$$SolucionGral := y(x) = \_C1\,e^{3x} + \_C2\,\sin(2x) + \_C3\,\cos(2x) \tag{13}$$

> $Q := 3\cdot x\cdot\cdot 2 + 4\cdot x + 6$

$$Q := 3\,x^2 + 4\,x + 6 \tag{14}$$

> $EcuacionNoHom := lhs(Ecuacion) = Q$

$$EcuacionNoHom := \frac{d^3}{dx^3}y(x) - 3\left(\frac{d^2}{dx^2}y(x)\right) + 4\left(\frac{d}{dx}y(x)\right) - 12\,y(x) = 3\,x^2 + 4\,x + 6 \tag{15}$$

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> $with(linalg):$

> $WW := wronskian\big(\,[\,rhs(Sol_1), rhs(Sol_2), rhs(Sol_3)\,]\,,x\big)$

$$WW := \begin{bmatrix} e^{3x} & \cos(2x) & \sin(2x) \\ 3\,e^{3x} & -2\sin(2x) & 2\cos(2x) \\ 9\,e^{3x} & -4\cos(2x) & -4\sin(2x) \end{bmatrix} \tag{16}$$

> $Comprobacion := simplify(det(WW)) \neq 0$

$$Comprobacion := 26\,e^{3x} \neq 0 \tag{17}$$

> $UU := array([0,0,Q])$

$$UU := \begin{bmatrix} 0 & 0 & 3\,x^2 + 4\,x + 6 \end{bmatrix} \tag{18}$$

> $PARA := linsolve(WW, UU): AAprima := PARA_1; BBprima := simplify(PARA_2); DDprima$
$:= simplify(PARA_3)$

$$AAprima := \frac{1}{13}\frac{3\,x^2 + 4\,x + 6}{e^{3x}}$$

$$BBprima := -\frac{1}{26}\,(3\,x^2 + 4\,x + 6)\,(-3\sin(2x) + 2\cos(2x))$$

$$DDprima := -\frac{1}{26}\,(3\cos(2x) + 2\sin(2x))\,(3\,x^2 + 4\,x + 6) \tag{19}$$

> $AA := int(AAprima, x) + C_1; BB := int(BBprima, x) + C_2; DD := int(DDprima, x) + C_3$

$$AA := -\frac{1}{39}\frac{8 + 6\,x + 3\,x^2}{e^{3x}} + C_1$$

$$BB := -\frac{9}{52} x^2 \cos(2x) - \frac{35}{104} \cos(2x) + \frac{1}{52} x \sin(2x) - \frac{3}{26} x^2 \sin(2x) - \frac{3}{52} \sin(2x)$$

$$- \frac{9}{26} x \cos(2x) + C_2$$

$$DD := -\frac{9}{52} x^2 \sin(2x) - \frac{35}{104} \sin(2x) - \frac{1}{52} x \cos(2x) + \frac{3}{52} \cos(2x) - \frac{9}{26} x \sin(2x)$$

$$+ \frac{3}{26} x^2 \cos(2x) + C_3$$

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> $SolucionNoHom := y(x) = simplify(AA \cdot rhs(Sol_1) + BB \cdot rhs(Sol_2) + DD \cdot rhs(Sol_3))$

$$SolucionNoHom := y(x) = -\frac{13}{24} - \frac{1}{2} x - \frac{1}{4} x^2 + C_1 e^{3x} + C_2 \cos(2x) + C_3 \sin(2x)$$

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>