S(1) EDO (1) L cc NH.

$$\bar{x}(t) = e^{A(t-a)}\bar{x}(a) + \int_a^t e^{A(t-z)} b(z)\,dz$$

$a \neq 0$

$$\left.\begin{aligned} \frac{dx_1(t)}{dt} &= 3x_1(t) + 4x_2(t) + 5e^t \\ \frac{dx_2(t)}{dt} &= 2x_1(t) + 5x_2(t) + 3e^{2t} \end{aligned}\right\} \quad \begin{aligned} \bar{x}_1(2) &= 4 \\ x_2(2) &= -6 \end{aligned}$$

$$\frac{d}{dt}\bar{x}(t) = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}\bar{x} + \begin{bmatrix} 5e^t \\ 3e^{2t} \end{bmatrix} \qquad \bar{x}(2) = \begin{bmatrix} 4 \\ -6 \end{bmatrix}$$

$$\frac{dx_1(t)}{dt} = x_1(t) + x_2(t) + 1 \qquad x_1(0) = 2$$

$$\frac{dx_2(t)}{dt} = 2x_1(t) + 2x_2(t) + t \qquad x_2(0) = 1$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad \det(A - \lambda I) = 0 \qquad \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - 2 = 0 \qquad \lambda^2 - 3\lambda = 0 \qquad \lambda(\lambda-3) = 0$$

$$\lambda_1 = 3 \quad \lambda_2 = 0$$

$$e^{At} = B_0 I + B_1 A \qquad \begin{cases} e^{3t} = B_0 + 3B_1 \\ 1 = B_0 + (0)B_1 \quad B_0 = 1 \end{cases}$$

$$e^{At} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} e^{3t} - \qquad B_1 = \frac{e^{3t} - B_0}{3} \Rightarrow B_1 = \frac{1}{3}(e^{3t} - 1)$$

$$- \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$e^{At} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} e^{3t} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$e^{At} \times \bar{x}(0) = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{3t} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3 \\ 6 \end{bmatrix} e^{3t} + \frac{1}{3}\begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$e^{A(t-z)} = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} e^{3(t-z)} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} e^{-3z} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$e^{A(t-z)} \cdot \vec{b}(z) = \frac{e^{3t}}{3}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ z \end{bmatrix} e^{-3z} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ z \end{bmatrix}$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} (1+z) \\ 2(1+z) \end{bmatrix} e^{-3z} + \frac{1}{3}\begin{bmatrix} 2-z \\ -2+z \end{bmatrix}$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}(1+z)e^{-3z} + \frac{1}{3}\begin{bmatrix} 1 \\ -1 \end{bmatrix}(2-z)$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(\int_0^t e^{-3z}\,dz + \int_0^t z e^{-3z}\,dz\right) + \frac{1}{3}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\left(2\int_0^t dz - \int_0^t z\,dz\right)$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(\left(\frac{e^{-3z}}{-3}\right)_0^t + \left(\frac{z e^{-3z}}{-3} - \frac{e^{-3z}}{9}\right)_0^t\right) + \frac{1}{3}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\left((2z)_0^t - \left(\frac{z^2}{2}\right)_0^t\right)$$

$$= \frac{e^{3t}}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(\left(\frac{e^{-3t}}{-3} + \frac{1}{3}\right) + \left(\frac{t e^{-3t}}{-3} \cancel{- \frac{e^{-3t}}{9}} \cancel{+ \frac{e^{-3t}}{9}}\right)\right) + \frac{1}{3}\begin{bmatrix} +1 \\ -1 \end{bmatrix}\left(2t - \frac{t^2}{2}\right)$$

$$= \frac{1}{3}\begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(-\frac{1}{3} + \frac{e^{3t}}{3} + \frac{t}{-3}\right) + \frac{1}{3}\begin{bmatrix} 1 \\ -1 \end{bmatrix}\left(2t - \frac{t^2}{2}\right)$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(-\frac{1}{9} + \frac{e^{3t}}{9} - \frac{t}{9}\right) + \begin{bmatrix} 1 \\ -1 \end{bmatrix}\left(\frac{2}{3}t - \frac{1}{6}t^2\right)$$