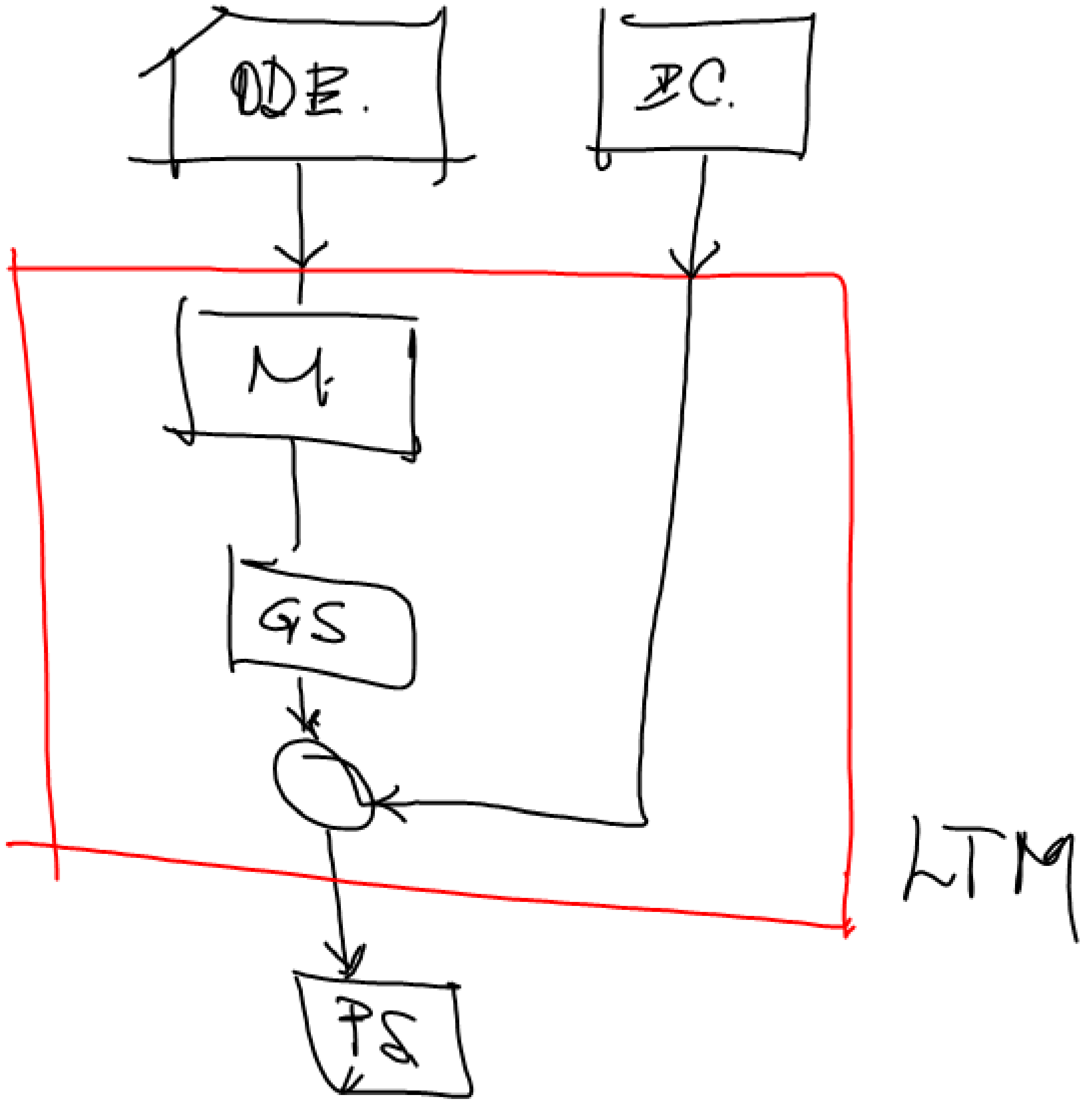
Chapter 4.- Laplace Transform.

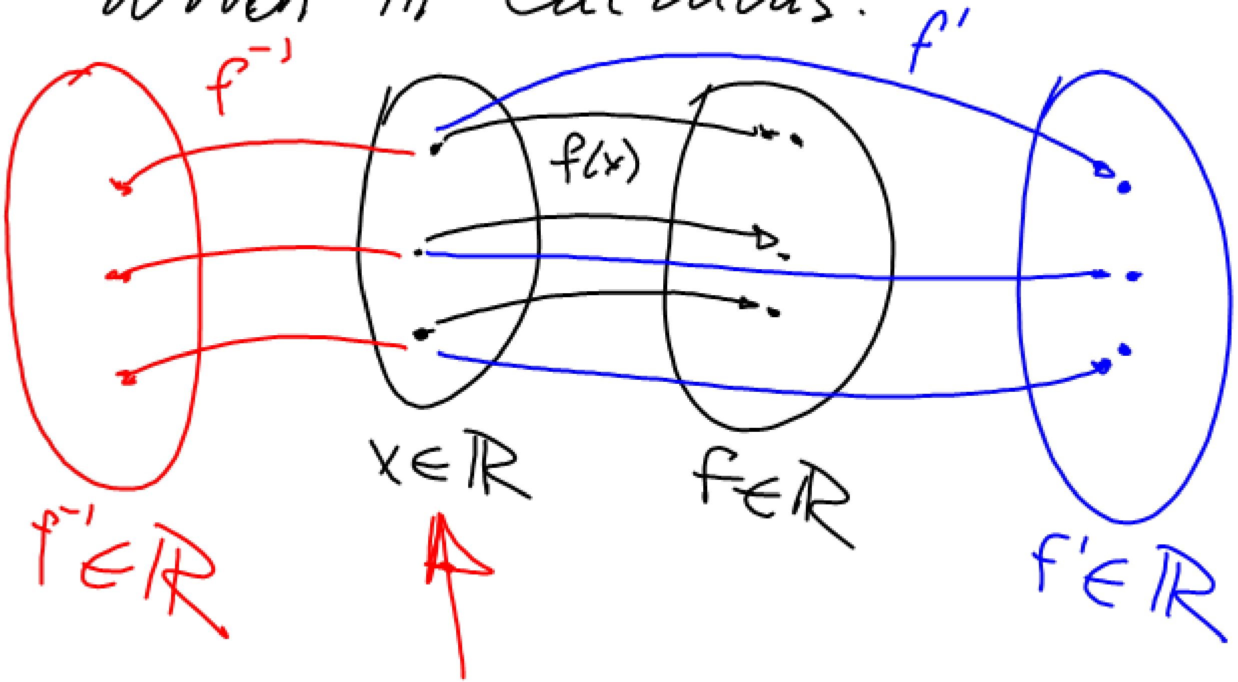
is a method for solving Initial Condition Problems of Ordinary Differential Equations

Result always in Particular Solutions


ODE.
DC.
Mi
GS
PS
LTM

What is Transform in Mathematics?

When in Calculus.



$f(t)$ $\quad F = T\{f\}$ $\quad F(s)$

$t \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\quad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

Linear

$$\boxed{af+bg \quad \left.\begin{matrix} f(t)\\ g(t)\end{matrix}\right\} \in \mathbb{R} \quad a,b\in\mathbb{R}} \longrightarrow aF+bG$$

$F(s) = T\{f(t)\}$

$G(s) = T\{g(t)\}$.

$$f' \longrightarrow L\{f'\} = sF(s) - f(0)$$

$$\int f\,dt \longleftarrow L^{-1}\left\{\frac{F(s)}{s}\right\} = \int f\,dt$$

Partial

Fractions

Transform

operator

$$T\{f(t)\} = \int_{-\infty}^{+\infty} N(t,s)\, f(t)\, dt$$

argument

Nucleous

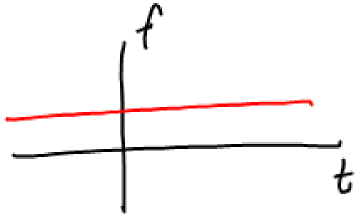
$t, f \in \mathbb{R}$

LaPlace Transform

$$N(t,s) = \begin{cases} e^{-st} & ; \ t > 0 \\ 0 & ; \ t < 0 \end{cases}$$

$$\left.\begin{array}{l} \mathcal{L}\{f(t)\} \\ L\{f(t)\} \end{array}\right\} = \int_0^{\infty} e^{-st} f(t)\, dt$$

$f(t) = 1$



$$\mathcal{L}\{1\} = \int_0^{\infty} e^{-st} \cdot (1) \cdot dt$$

$$= \left[\int e^{-st} dt \right]_0^{\infty}$$

$$= \left[\frac{e^{-st}}{-s} \right]_0^{\infty}$$

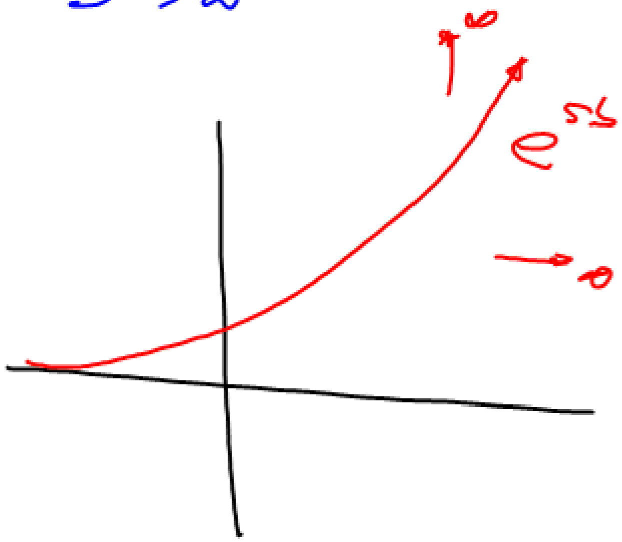
$$= -\frac{1}{s} \left(\cancelto{0}{\lim_{b \to \infty} e^{-sb}} - 1 \right)$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

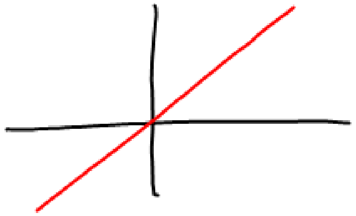
$$\lim_{b\to\infty} e^{-sb} = \lim_{b\to\infty} \frac{1}{e^{sb}}$$

$$\lim_{b\to\infty} e^{sb} \to \infty$$

$$\lim_{a\to\infty} \frac{1}{a} = 0$$



$$f(t) = t$$



$$\mathcal{L}\{t\} = \int_0^\infty e^{-st}\, t \, dt$$

$$= \left[\int t e^{-st} dt \right]_0^\infty$$

$$\int t e^{-st} dt = -\frac{t e^{-st}}{s} - \int \frac{e^{-st}}{-s} dt$$

$u = t \qquad du = dt$

$dV = e^{-st} dt \qquad V = \dfrac{e^{-st}}{-s}$

$$= -\frac{t e^{-st}}{s} + \frac{1}{s}\left(\frac{e^{-st}}{-s}\right)$$

$$= -\frac{t e^{-st}}{s} - \frac{1}{s^2} e^{-st}$$

$$\mathcal{L}\{t\} = \left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_0^\infty$$

$$= -\frac{1}{s}\left[te^{-st}\right]_0^\infty - \frac{1}{s^2}\left[e^{-st}\right]_0^\infty$$

$$= -\frac{1}{s}\left[\overset{0}{\lim_{b\to\infty}(b)e^{-sb}} - 0\right] - \frac{1}{s^2}\left[\overset{=0}{\lim_{b\to\infty} e^{-sb}} - 1\right]$$

$$\lim_{b\to\infty} b\cdot e^{-st} = \overset{\infty}{\lim_{b\to\infty} b} \cdot \overset{0}{\lim_{b\to\infty} e^{-sb}}$$

$$= 0$$

$$\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}} \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{t^4\} = \frac{4!}{s^5} \qquad n \in \mathbb{Z}^{+}$$

$$f(t) = e^{at} \quad a \in \mathbb{R}$$

$$\mathcal{L}\{e^{at}\} = \int_0^{\infty} e^{-st} e^{at}\, dt$$

$$= \left[\int e^{-(s-a)t}\, dt \right]_0^{\infty}$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^{\infty} \Rightarrow -\frac{1}{s-a} \left[\lim_{b \to \infty} e^{-(s-a)b} - 1 \right]$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\cos(bt)\} \quad b \in \mathbb{R}^{+}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}\{\cos(bt)\} = \int_0^{\infty} e^{-st} \cos(bt)\, dt$$

$$\mathcal{L}\{\sin(bt)\} = \int_0^{\infty} e^{-st} \sin(bt)\, dt$$

Theorem.

For a Laplace Transform
exists and is unique
when the argument
$f(t)$ is a Class "A" function.

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a Class "A" function is when:

a) $f(t)$ is exponential order function
b) $f(t)$ is Sectional Continous.
function.

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a) $$|f(t)| \leq Me^{At} \quad M, A \in \mathbb{R}$$