Laplace Transform propierties

1. Linear

if $\mathcal{L}\{f(t)\} = F(s)$

$\mathcal{L}\{g(t)\} = G(s)$

then

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s).$$

$a, b \in \mathbb{R}$

(2) $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

$a \in \mathbb{R}$

(3) $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

(s^0)

$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{(n-1)}(0)$

(4) if $\mathcal{L}^{-1}\{F(s)\} = f(t)$

$\mathcal{L}^{-1}\{F'(s)\} = -t f(t)$

$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$

(5) $$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{F(s)}{s}$$

(6) $$\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \frac{f(t)}{t}$$

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(8) $$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{e^{at}\cos(bt)\} = \frac{(s-a)}{(s-a)^2+b^2}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3} \rightarrow \mathcal{L}\{t^2e^{at}\} = \frac{2!}{(s-a)^3}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s)+2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+2-1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+(1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+(1)^2} - \frac{1}{(s+1)^2+(1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = e^{-t}\cos(t) - e^{-t}\sin(t).$$

Theorem

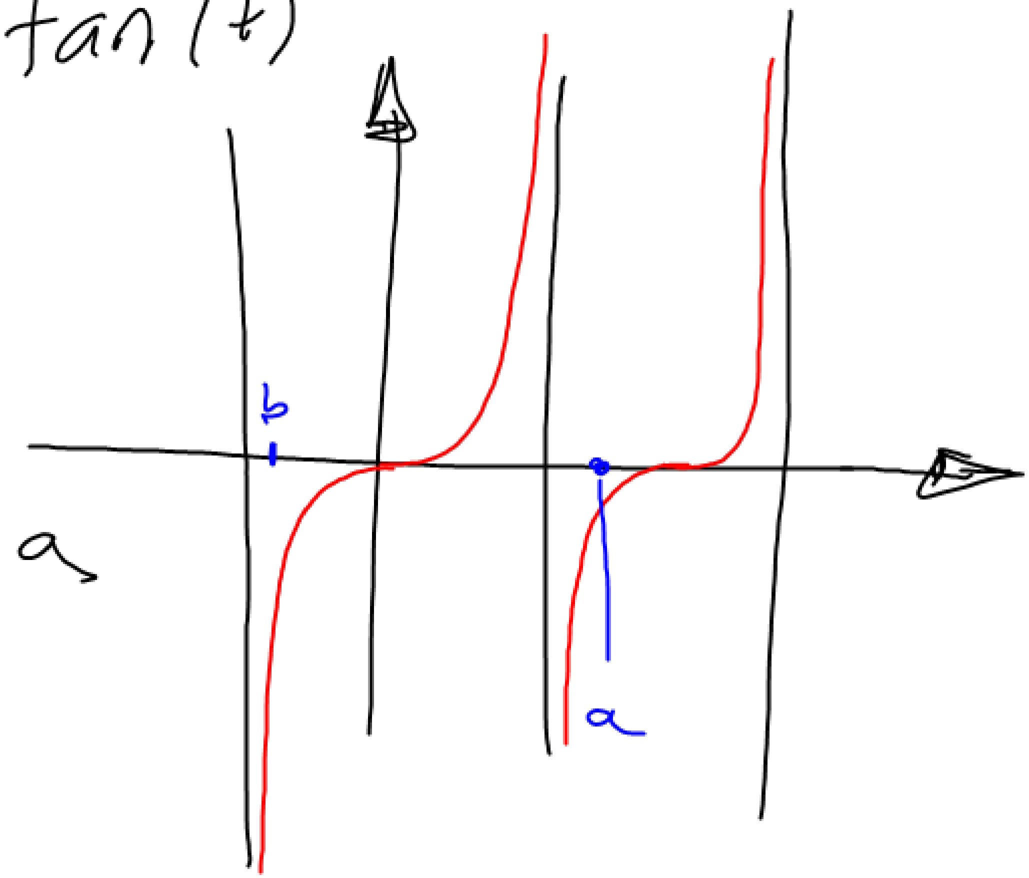
Laplace Transform of $f(t)$
exists and is unique when

a) $|f(t)| \leq Me^{At}$ $M, A \in \mathbb{R}$

b) $f(t)$ is sectional continous function
it can a finit number of discontinuities

sectional
continous function

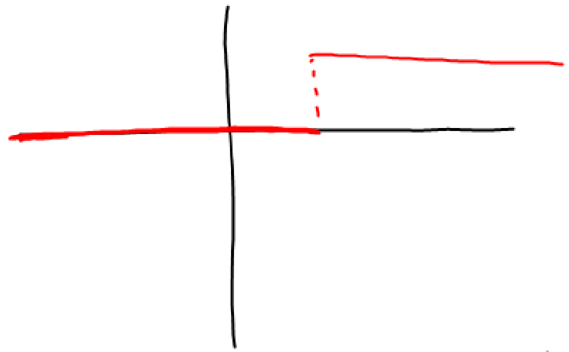
tan(t)



b<t<a

unit step function

$$u(t-a) = \begin{cases} 0 \; ; \; t \le a \\ 1 \; ; \; t > a \end{cases}$$



$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

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$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\cdot u(t-a)$$

$$f(t-a)\cdot u(t-a) = \begin{cases} 0 \; ; \; t \le a \\ f(t-a) \; ; \; t > a \end{cases}$$

10 convolution operator

$$\mathcal{L}^{-1}\left\{F(s)\cdot G(s)\right\} = f(t) * g(t)$$

$$f(t) * g(t) = \int_0^t f(z)\cdot g(t-z)\,dz$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \mathcal{L}^{-1}\left\{\left(\frac{s}{s^2+1}\right)\cdot\left(\frac{1}{s^2+1}\right)\right\}$$

$$\cos(t) * \sin(t) = \int_0^t \cos(z)\cdot\sin(t-z)\,dz$$

$$= \left[\int \cos(z)\left[\sin(t)\cdot\cos(z) - \cos(t)\cdot\sin(z)\right]dz\right]_0^t$$

$$= \left[\sin(t)\int \cos^2(z)\,dz - \cos(t)\int \cos(z)\sin(z)\,dz\right]_0^t$$

$$= \left[\sin(t)\int\left(\frac{1}{2}+\frac{1}{2}\cos(2z)\right)dz - \cos(t)\left(\frac{\sin^2(z)}{2}\right)\right]_0^t$$

$$= \left[\frac{\sin(t)}{2}(z) + \frac{\sin(t)}{4}\left(\sin(2z)\right) - \cos(t)\frac{\sin^2(z)}{2}\right]_0^t$$

$$= \frac{t\sin(t)}{2} - (0) + \frac{\sin(t)}{4}\left(\sin(2t) - (0)\right) - \frac{\cos(t)}{2}\left(\sin^2(t) - 0\right)$$

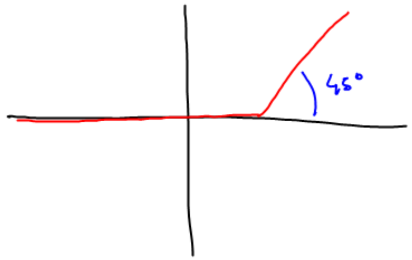
$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\} = \frac{t\sin(t)}{2} + \frac{\sin(t)\sin(2t)}{4} - \frac{\cos(t)\sin^2(t)}{2}$$

step

$$\mu(t-a) = \begin{cases} 0; & t \le a \\ 1; & t > a \end{cases}$$

slope

$$\delta(t-a) = \begin{cases} 0; & t \le a \\ t-a; & t > a \end{cases}$$



$$\mathcal{L}\{\delta(t-a)\} = \frac{e^{-as}}{s^2}$$

$$\mathcal{L}\left\{\frac{d}{dt}\delta(t-a)\right\} = s\left[\frac{e^{-as}}{s^2}\right] - \delta(t-a)\Big|_{t=0}$$

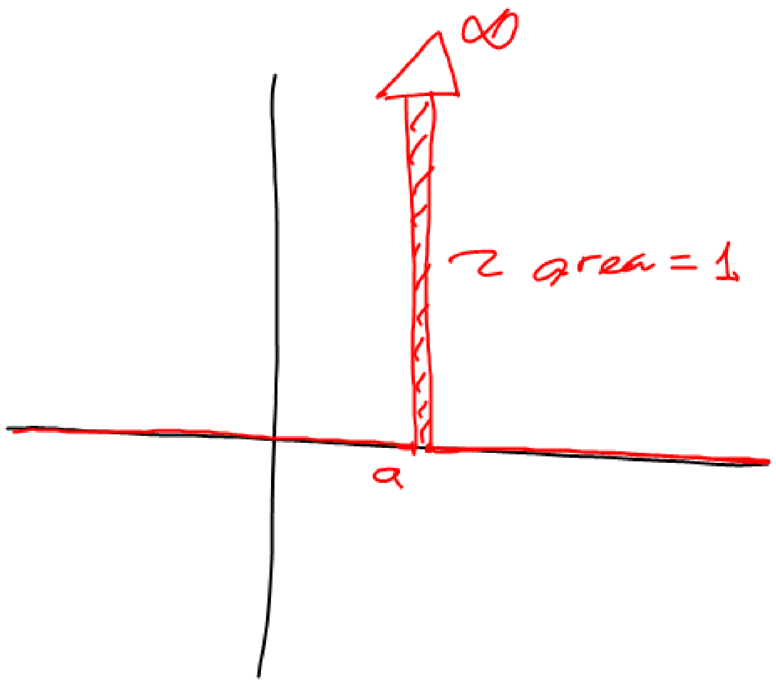
$$\mathcal{L}\left\{\frac{d}{dt}\delta(t-a)\right\} = \frac{e^{-as}}{s}$$

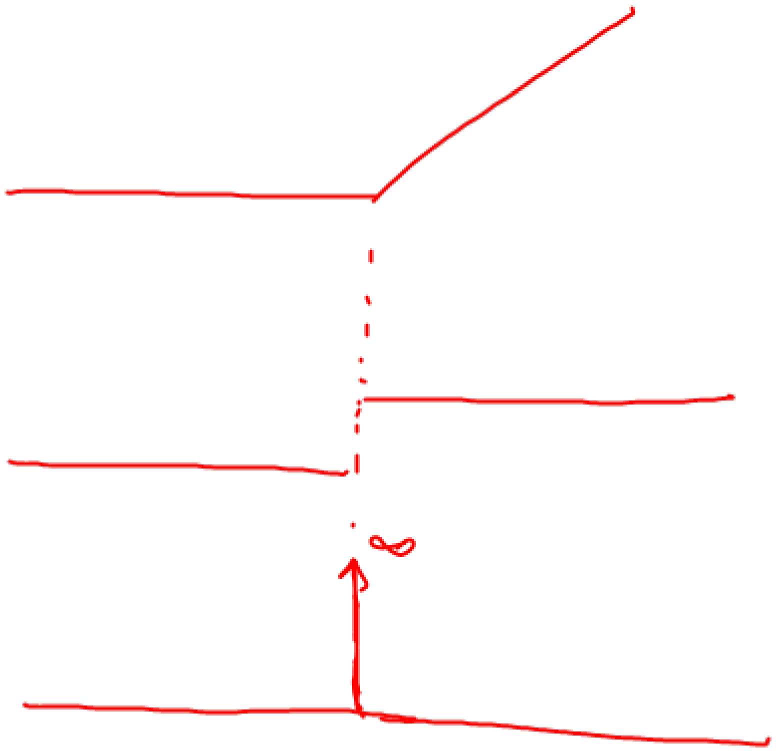
$$\mathcal{L}\left\{\frac{d}{dt}\delta(t-a)\right\} = \mathcal{L}\{\mu(t-a)\}$$

$$\frac{d}{dt}\delta(t-a) = \mu(t-a)$$

$$\frac{d}{dt}u(t-a) = D(t-a)$$

$$D(t-a) = \begin{cases} 0 & t \neq a \\ \displaystyle\int_{-\infty}^{\infty} D(t-a)\,dt = 1. \end{cases}$$

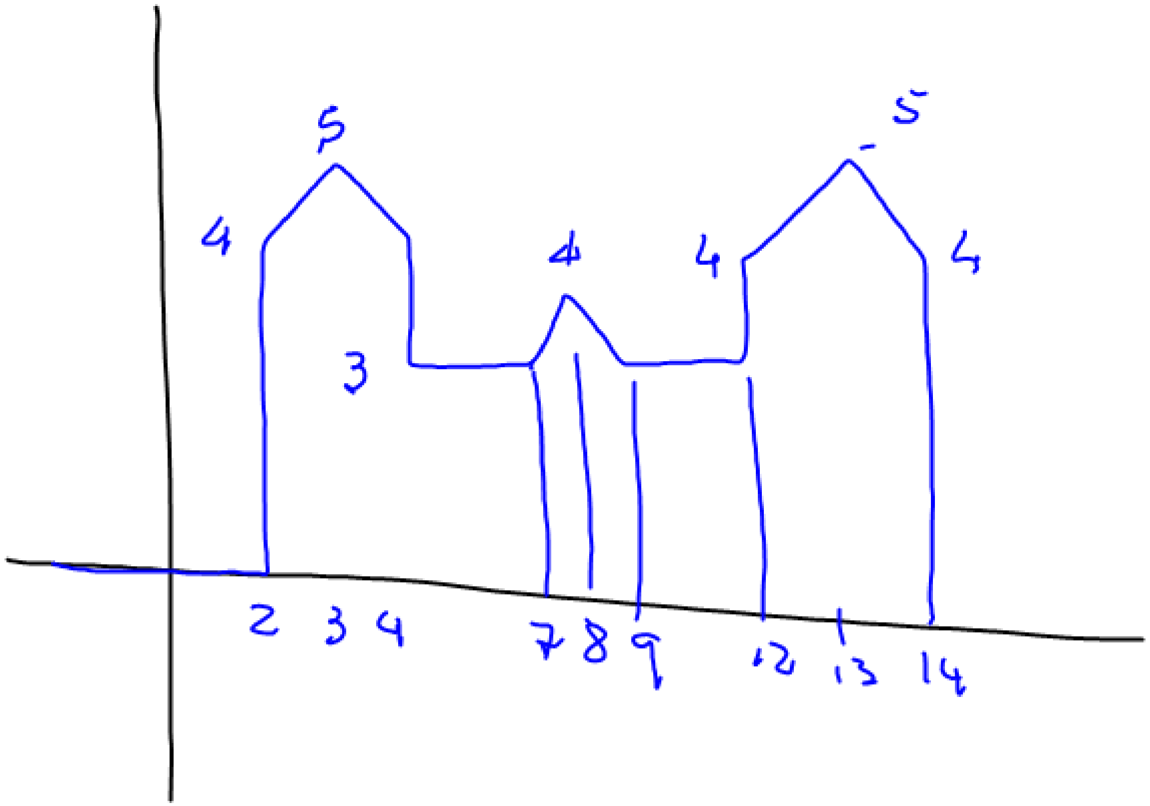




$$r(t-a)$$

$$\frac{dr(t-a)}{dt} = u(t-a)$$

$$\frac{du(t-a)}{dt} = \delta(t-a).$$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