

> restart

> Ecuacion := diff(F(x, y), x\$2) + 3·diff(F(x, y), y) = 5·F(x, y)

$$Ecuacion := \frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \quad (1)$$

> with(PDEtools)

[CanonicalCoordinates, ChangeSymmetry, CharacteristicQ, CharacteristicQInvariants, ConservedCurrentTest, ConservedCurrents, ConsistencyTest, D_Dx, DeterminingPDE, Eta_k, Euler, FromJet, InfinitesimalGenerator, Infinitesimals, IntegratingFactorTest, IntegratingFactors, InvariantSolutions, InvariantTransformation, Invariants, Laplace, Library, PDEplot, PolynomialSolutions, ReducedForm, SimilaritySolutions, SimilarityTransformation, SymmetrySolutions, SymmetryTest, SymmetryTransformation, TWSolutions, ToJet, build, casesplit, charstrip, dchange, dcoeffs, declare, diff_table, difforder, dpolyform, dsubs, mapde, separability, splitstrip, splitsys, undeclare] (2)

> Solucion := pdsolve(Ecuacion)

$$Solucion := (F(x, y) = _F1(x) _F2(y)) \&where \left[\left\{ \frac{d^2}{dx^2} _F1(x) = _c1 _F1(x), \frac{d}{dy} _F2(y) = -\frac{1}{3} _c1 _F2(y) + \frac{5}{3} _F2(y) \right\} \right] \quad (3)$$

> SolucionGeneral := build(Solucion)

$$SolucionGeneral := F(x, y) = e^{\sqrt{-c_1} x} _C3 e^{-\frac{1}{3} y - c_1 \frac{5}{3} y} _C1 + \frac{C3 e^{-\frac{1}{3} y - c_1 \frac{5}{3} y} _C2}{e^{\sqrt{-c_1} x}} \quad (4)$$

> Ecuacion;

$$\frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \quad (5)$$

> EcuacionSeparable := simplify(eval(subs(F(x, y) = f(x)·g(y), Ecuacion)))

$$EcuacionSeparable := \left(\frac{d^2}{dx^2} f(x) \right) g(y) + 3 f(x) \left(\frac{d}{dy} g(y) \right) = 5 f(x) g(y) \quad (6)$$

> EcuacionSeparada := $\frac{lhs(EcuacionSeparable) - 3 f(x) \left(\frac{d}{dy} g(y) \right)}{f(x) \cdot g(y)}$

$$= simplify \left(\frac{rhs(EcuacionSeparable) - 3 f(x) \left(\frac{d}{dy} g(y) \right)}{f(x) \cdot g(y)} \right)$$

$$EcuacionSeparada := \frac{\frac{d^2}{dx^2} f(x)}{f(x)} = \frac{5 g(y) - 3 \left(\frac{d}{dy} g(y) \right)}{g(y)} \quad (7)$$

> EcuacionX := lhs(EcuacionSeparada) = alpha

$$EcuacionX := \frac{\frac{d^2}{dx^2} f(x)}{f(x)} = \alpha \quad (8)$$

$$\begin{aligned} &> \text{EcuacionY} := \text{rhs}(\text{EcuacionSeparada}) = \alpha \\ &\quad \text{EcuacionY} := \frac{5g(y) - 3\left(\frac{d}{dy}g(y)\right)}{g(y)} = \alpha \end{aligned} \quad (9)$$

$$\begin{aligned} &> \text{SolucionXcero} := \text{dsolve}(\text{subs}(\alpha = 0, \text{EcuacionX})) \\ &\quad \text{SolucionXcero} := f(x) = _C1x + _C2 \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{SolucionYcero} := \text{dsolve}(\text{subs}(\alpha = 0, \text{EcuacionY})) \\ &\quad \text{SolucionYcero} := g(y) = _C1e^{\frac{5}{3}y} \end{aligned} \quad (11)$$

$$\begin{aligned} &> \text{SolucionGralCero} := F(x, y) = \text{rhs}(\text{SolucionXcero}) \cdot \text{subs}(_C1 = 1, \text{rhs}(\text{SolucionYcero})) \\ &\quad \text{SolucionGralCero} := F(x, y) = (_C1x + _C2)e^{\frac{5}{3}y} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{Ecuacion} \\ &\quad \frac{\partial^2}{\partial x^2} F(x, y) + 3\left(\frac{\partial}{\partial y} F(x, y)\right) = 5F(x, y) \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{Comprobacion}_0 := \text{simplify}(\text{eval}(\text{subs}(F(x, y) = \text{rhs}(\text{SolucionGralCero}), \text{lhs}(\text{Ecuacion}) \\ &\quad - \text{rhs}(\text{Ecuacion}) = 0))) \\ &\quad \text{Comprobacion}_0 := 0 = 0 \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{SolucionXneg} := \text{dsolve}(\text{subs}(\alpha = -\beta \cdot 2, \text{EcuacionX})) \\ &\quad \text{SolucionXneg} := f(x) = _C1\sin(\beta x) + _C2\cos(\beta x) \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{SolucionYneg} := \text{dsolve}(\text{subs}(\alpha = -\beta \cdot 2, \text{EcuacionY})) \\ &\quad \text{SolucionYneg} := g(y) = _C1e^{\frac{1}{3}(5 + \beta^2)y} \end{aligned} \quad (16)$$

$$\begin{aligned} &> \text{SolucionGralNeg} := F(x, y) = \text{rhs}(\text{SolucionXneg}) \cdot \text{subs}(_C1 = 1, \text{rhs}(\text{SolucionYneg})) \\ &\quad \text{SolucionGralNeg} := F(x, y) = (_C1\sin(\beta x) + _C2\cos(\beta x))e^{\frac{1}{3}(5 + \beta^2)y} \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{Ecuacion} \\ &\quad \frac{\partial^2}{\partial x^2} F(x, y) + 3\left(\frac{\partial}{\partial y} F(x, y)\right) = 5F(x, y) \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{Comprobacion}_1 := \text{simplify}(\text{eval}(\text{subs}(F(x, y) = \text{rhs}(\text{SolucionGralNeg}), \text{lhs}(\text{Ecuacion}) \\ &\quad - \text{rhs}(\text{Ecuacion}) = 0))) \\ &\quad \text{Comprobacion}_1 := 0 = 0 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{SolucionXpos} := \text{dsolve}(\text{subs}(\alpha = \beta \cdot 2, \text{EcuacionX})) \\ &\quad \text{SolucionXpos} := f(x) = _C1e^{-\beta x} + _C2e^{\beta x} \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{SolucionYpos} := \text{dsolve}(\text{subs}(\alpha = \beta \cdot 2, \text{EcuacionY})) \\ &\quad \text{SolucionYpos} := g(y) = _C1e^{-\frac{1}{3}(-5 + \beta^2)y} \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{SolucionGralPos} := F(x, y) = \text{rhs}(\text{SolucionXpos}) \cdot \text{subs}(_C1 = 1, \text{rhs}(\text{SolucionYpos})) \\ &\quad \text{SolucionGralPos} := F(x, y) = (_C1e^{-\beta x} + _C2e^{\beta x})e^{-\frac{1}{3}(-5 + \beta^2)y} \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{Ecuacion} \\ &\quad \frac{\partial^2}{\partial x^2} F(x, y) + 3\left(\frac{\partial}{\partial y} F(x, y)\right) = 5F(x, y) \end{aligned} \quad (23)$$

$$\frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \quad (23)$$

> $Comprobacion_2 := simplify(eval(subs(F(x, y) = rhs(SolucionGralPos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad (24)$$

> $EcuacionSeparadaDos$

$$:= simplify \left(\frac{lhs(EcuacionSeparable) - 3 f(x) \left(\frac{d}{dy} g(y) \right) - 5 f(x) g(y)}{f(x) \cdot g(y)} \right)$$

$$= simplify \left(\frac{rhs(EcuacionSeparable) - 3 f(x) \left(\frac{d}{dy} g(y) \right) - 5 f(x) g(y)}{f(x) \cdot g(y)} \right)$$

$$EcuacionSeparadaDos := \frac{\frac{d^2}{dx^2} f(x) - 5 f(x)}{f(x)} = - \frac{3 \left(\frac{d}{dy} g(y) \right)}{g(y)} \quad (25)$$

> $EcuacionXdos := lhs(EcuacionSeparadaDos) = \alpha$

$$EcuacionXdos := \frac{\frac{d^2}{dx^2} f(x) - 5 f(x)}{f(x)} = \alpha \quad (26)$$

> $EcuacionYdos := rhs(EcuacionSeparadaDos) = \alpha$

$$EcuacionYdos := - \frac{3 \left(\frac{d}{dy} g(y) \right)}{g(y)} = \alpha \quad (27)$$

> $SolucionXceroDos := dsolve(subs(alpha=0, EcuacionXdos))$

$$SolucionXceroDos := f(x) = _C1 e^{\sqrt{5} x} + _C2 e^{-\sqrt{5} x} \quad (28)$$

> $SolucionYceroDos := dsolve(subs(alpha=0, EcuacionYdos))$

$$SolucionYceroDos := g(y) = _C1 \quad (29)$$

> $SolucionGralCeroDos := F(x, y) = rhs(SolucionXceroDos) \cdot subs(_C1 = 1, rhs(SolucionYceroDos))$

$$SolucionGralCeroDos := F(x, y) = _C1 e^{\sqrt{5} x} + _C2 e^{-\sqrt{5} x} \quad (30)$$

> $Ecuacion$

$$\frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \quad (31)$$

> $Comprobacion_3 := simplify(eval(subs(F(x, y) = rhs(SolucionGralCeroDos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_3 := 0 = 0 \quad (32)$$

> $SolucionXnegDos := dsolve(subs(alpha=-beta \cdot 2, EcuacionXdos))$

$$SolucionXnegDos := f(x) = _C1 \sin(\sqrt{-5 + \beta^2} x) + _C2 \cos(\sqrt{-5 + \beta^2} x) \quad (33)$$

> $SolucionYnegDos := dsolve(subs(alpha=-beta \cdot 2, EcuacionYdos))$

$$SolucionYnegDos := g(y) = _C1 e^{\frac{1}{3} \beta^2 y} \quad (34)$$

$$\begin{aligned} &> \text{SolucionGralNegDos} := F(x, y) = \text{rhs}(\text{SolucionXnegDos}) \cdot \text{subs}(_C1 = 1, \\ &\quad \text{rhs}(\text{SolucionYnegDos})) \\ &\text{SolucionGralNegDos} := F(x, y) = \left(_C1 \sin\left(\sqrt{-5 + \beta^2} x\right) + _C2 \cos\left(\sqrt{-5 + \beta^2} x\right) \right) e^{\frac{1}{3} \beta^2 y} \end{aligned} \quad (35)$$

$$\begin{aligned} &> \text{Ecuacion} \\ &\quad \frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \end{aligned} \quad (36)$$

$$\begin{aligned} &> \text{Comprobacion}_4 := \text{simplify}(\text{eval}(\text{subs}(F(x, y) = \text{rhs}(\text{SolucionGralNegDos}), \text{lhs}(\text{Ecuacion}) \\ &\quad - \text{rhs}(\text{Ecuacion}) = 0))) \\ &\quad \text{Comprobacion}_4 := 0 = 0 \end{aligned} \quad (37)$$

$$\begin{aligned} &> \text{SolucionXposDos} := \text{dsolve}(\text{subs}(\text{alpha} = \text{beta} \cdot 2, \text{EcuacionXdos})) \\ &\quad \text{SolucionXposDos} := f(x) = _C1 \sin\left(\sqrt{-5 - \beta^2} x\right) + _C2 \cos\left(\sqrt{-5 - \beta^2} x\right) \end{aligned} \quad (38)$$

$$\begin{aligned} &> \text{SolucionYposDos} := \text{dsolve}(\text{subs}(\text{alpha} = \text{beta} \cdot 2, \text{EcuacionYdos})) \\ &\quad \text{SolucionYposDos} := g(y) = _C1 e^{-\frac{1}{3} \beta^2 y} \end{aligned} \quad (39)$$

$$\begin{aligned} &> \text{SolucionGralPosDos} := F(x, y) = \text{rhs}(\text{SolucionXposDos}) \cdot \text{subs}(_C1 = 1, \\ &\quad \text{rhs}(\text{SolucionYposDos})) \\ &\text{SolucionGralPosDos} := F(x, y) = \left(_C1 \sin\left(\sqrt{-5 - \beta^2} x\right) \right. \\ &\quad \left. + _C2 \cos\left(\sqrt{-5 - \beta^2} x\right) \right) e^{-\frac{1}{3} \beta^2 y} \end{aligned} \quad (40)$$

$$\begin{aligned} &> \text{Ecuacion} \\ &\quad \frac{\partial^2}{\partial x^2} F(x, y) + 3 \left(\frac{\partial}{\partial y} F(x, y) \right) = 5 F(x, y) \end{aligned} \quad (41)$$

$$\begin{aligned} &> \text{Comprobacion}_5 := \text{simplify}(\text{eval}(\text{subs}(F(x, y) = \text{rhs}(\text{SolucionGralPosDos}), \text{lhs}(\text{Ecuacion}) \\ &\quad - \text{rhs}(\text{Ecuacion}) = 0))) \\ &\quad \text{Comprobacion}_5 := 0 = 0 \end{aligned} \quad (42)$$

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