

$$\begin{aligned} &> \text{restart} \\ &> \text{fun} := f(x) = x \cdot 2 - 4 \end{aligned} \qquad \text{fun} := f(x) = x^2 - 4 \quad (1)$$

$$\begin{aligned} &> L := 2 \end{aligned} \qquad L := 2 \quad (2)$$

$$\begin{aligned} &> a_0 := \left(\frac{1}{L} \right) \cdot \text{int}(\text{rhs}(\text{fun}), x = -L..L) \end{aligned} \qquad a_0 := -\frac{16}{3} \quad (3)$$

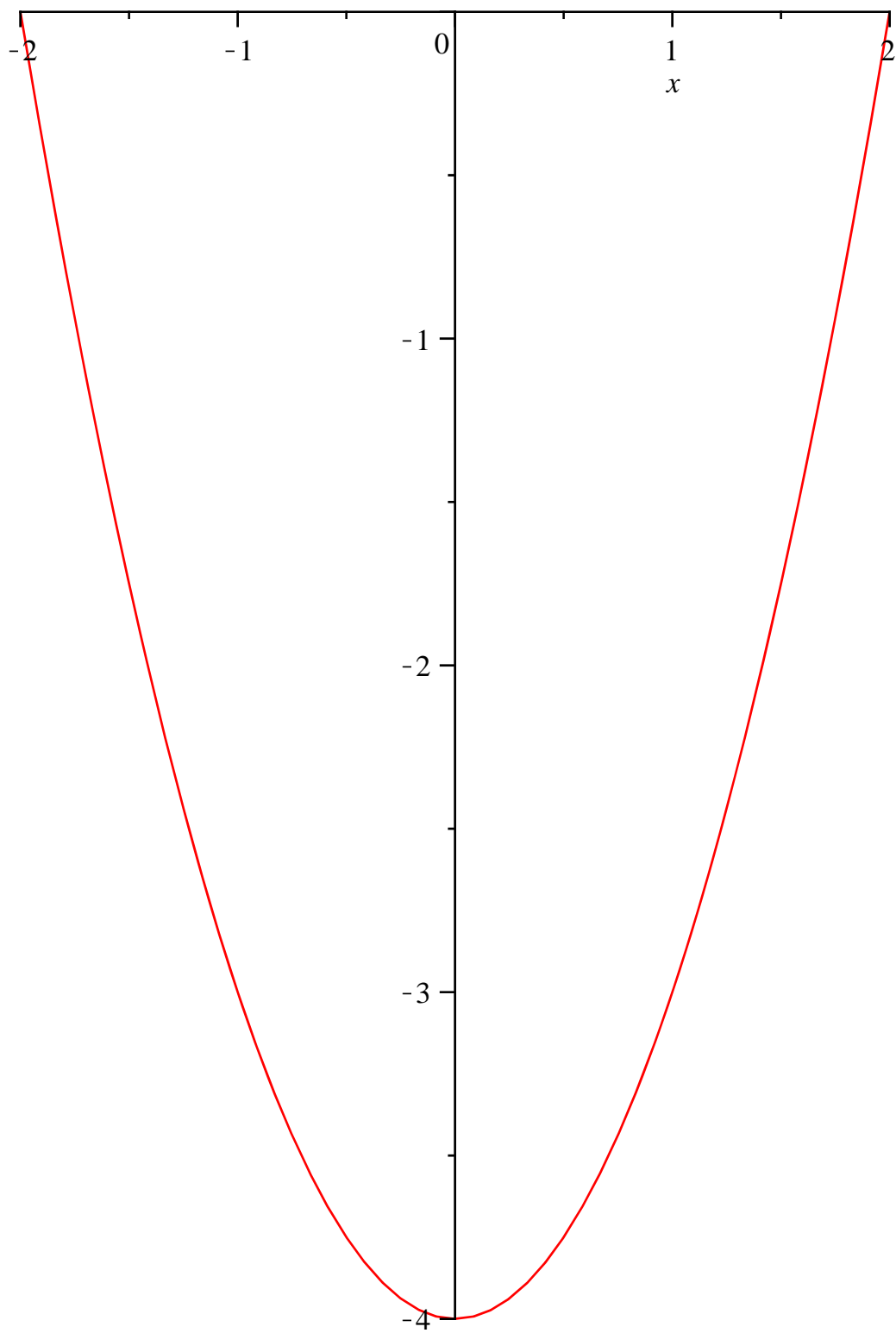
$$\begin{aligned} &> C := \frac{a_0}{2} \end{aligned} \qquad C := -\frac{8}{3} \quad (4)$$

$$\begin{aligned} &> a_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, \left(\frac{1}{L} \right) \cdot \text{int}\left(\text{rhs}(\text{fun}) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L \right. \right. \\ &\quad \left. \left. ..L \right)\right) \end{aligned} \qquad a_n := \frac{16 (-1)^n}{n^2 \pi^2} \quad (5)$$

$$\begin{aligned} &> b_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, \left(\frac{1}{L} \right) \cdot \text{int}\left(\text{rhs}(\text{fun}) \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L \right. \right. \\ &\quad \left. \left. ..L \right)\right) \end{aligned} \qquad b_n := 0 \quad (6)$$

$$\begin{aligned} &> \text{STF} := C + \text{Sum}\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1 .. \text{infinity}\right) \end{aligned} \qquad \text{STF} := -\frac{8}{3} + \sum_{n=1}^{\infty} \frac{16 (-1)^n \cos\left(\frac{1}{2} n \pi x\right)}{n^2 \pi^2} \quad (7)$$

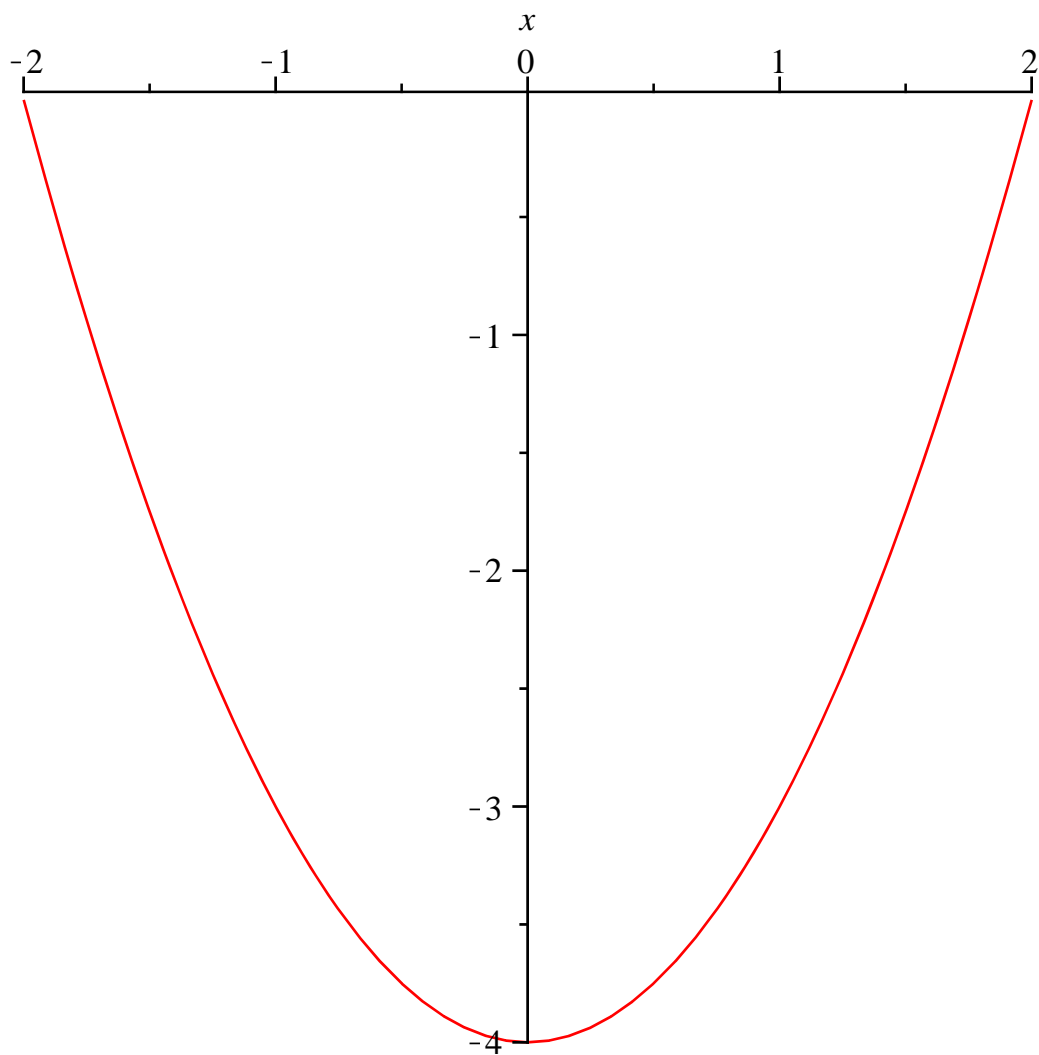
$$\begin{aligned} &> \text{with}(\text{plots}) \\ &[\text{animate}, \text{animate3d}, \text{animatecurve}, \text{arrow}, \text{changecoords}, \text{complexplot}, \text{complexplot3d}, \\ &\quad \text{conformal}, \text{conformal3d}, \text{contourplot}, \text{contourplot3d}, \text{coordplot}, \text{coordplot3d}, \text{densityplot}, \\ &\quad \text{display}, \text{dualaxisplot}, \text{fieldplot}, \text{fieldplot3d}, \text{gradplot}, \text{gradplot3d}, \text{graphplot3d}, \text{implicitplot}, \\ &\quad \text{implicitplot3d}, \text{inequal}, \text{interactive}, \text{interactiveparams}, \text{intersectplot}, \text{listcontplot}, \\ &\quad \text{listcontplot3d}, \text{listdensityplot}, \text{listplot}, \text{listplot3d}, \text{loglogplot}, \text{logplot}, \text{matrixplot}, \text{multiple}, \\ &\quad \text{odeplot}, \text{pareto}, \text{plotcompare}, \text{pointplot}, \text{pointplot3d}, \text{polarplot}, \text{polygonplot}, \text{polygonplot3d}, \\ &\quad \text{polyhedra_supported}, \text{polyhedraplot}, \text{rootlocus}, \text{semilogplot}, \text{setcolors}, \text{setoptions}, \\ &\quad \text{setoptions3d}, \text{spacecurve}, \text{sparsematrixplot}, \text{surfdata}, \text{textplot}, \text{textplot3d}, \text{tubeplot}] \\ &> \text{plot}(\text{rhs}(\text{fun}), x = -2..2) \end{aligned} \quad (8)$$



```

> STF500 := C + sum( $a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right)$ ,  $n = 1 \dots 500$ ) :
> plot(STF500, x = -2 .. 2)

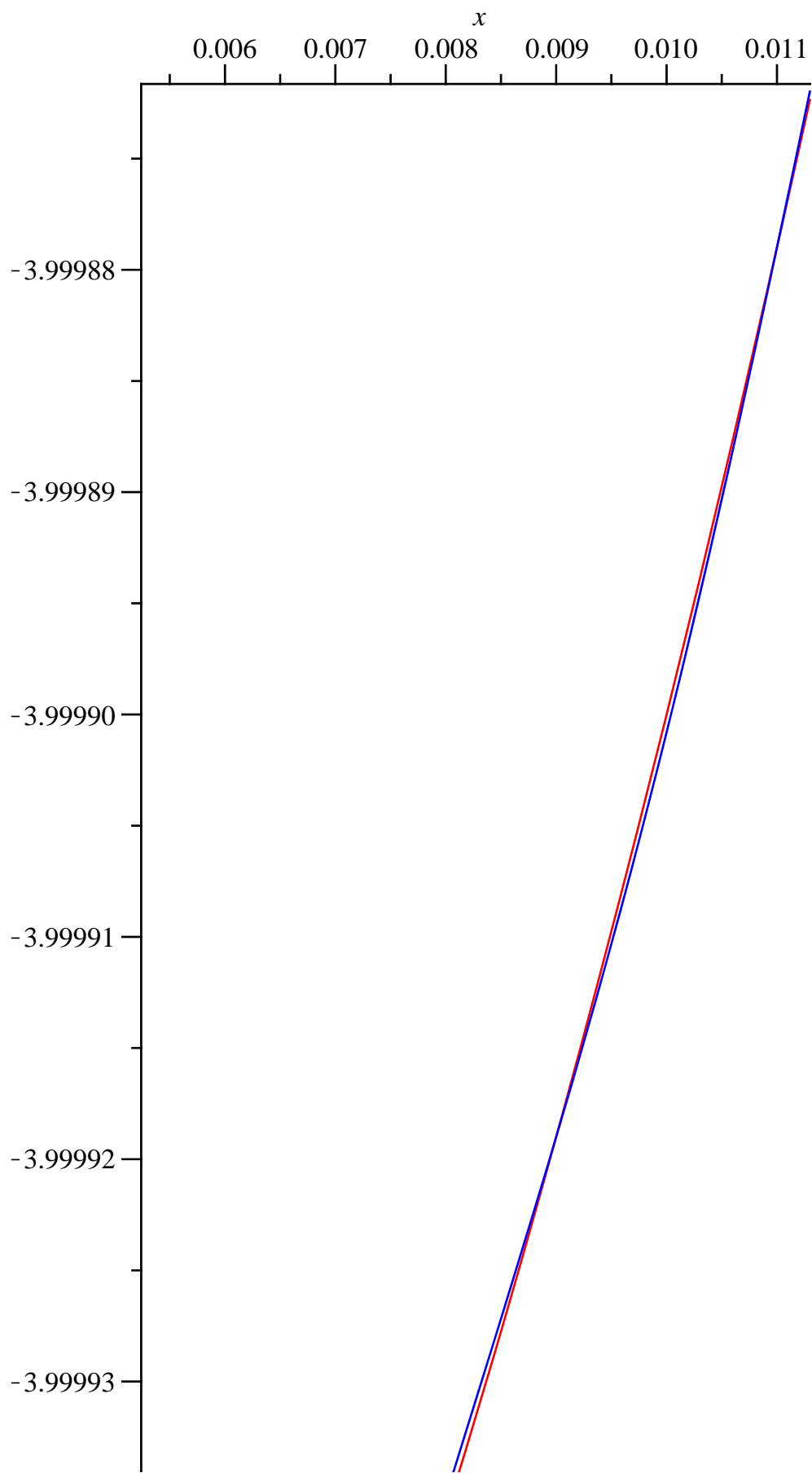
```



```

> STF1000 := C + sum( $a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right)$ , n = 1 .. 1000) :
> plot([rhs(fun), STF1000], x = 0.0053 .. 0.0113, color = [red, blue])

```

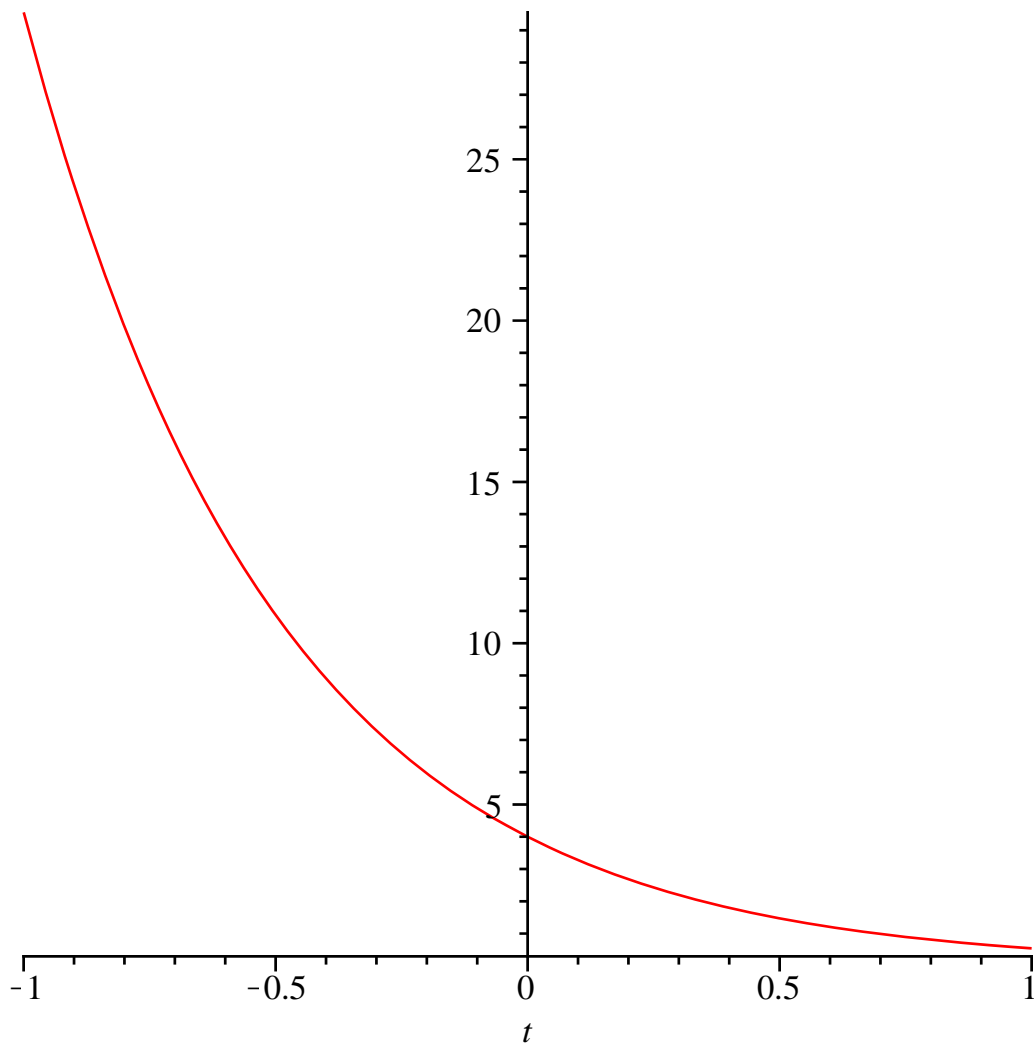


```
> restart
> f := 4·exp( -2 t)
```

$$f := 4 e^{-2t}$$

(9)

```
> plot(f, t=-1..1)
```



```
> L := 1
```

$$L := 1$$

(10)

```
> a_0 := (1/L)·int(f, t=-L..L); evalf(%)
```

$$a_0 := 2 e^2 - 2 e^{-2}$$

$$14.50744163$$

(11)

```
> C := a_0/2; evalf(%)
```

$$C := e^2 - e^{-2}$$

$$7.253720816$$

(12)

```
> a_n := subs( sin(n·Pi) = 0, cos(n·Pi) = (-1)·n, (1/L)·int( f·cos( n·Pi·t/L ), t=-L..L ) )
```

(13)

$$a_n := \frac{4 (2 e^2 (-1)^n - 2 e^{-2} (-1)^n)}{4 + n^2 \pi^2} \quad (13)$$

$$> b_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, \left(\frac{1}{L}\right) \cdot \text{int}\left(f \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t = -L..L\right)\right)$$

$$b_n := \frac{4 (e^2 n \pi (-1)^n - e^{-2} n \pi (-1)^n)}{4 + n^2 \pi^2} \quad (14)$$

$$> STF := C + \text{Sum}\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..infinity\right)$$

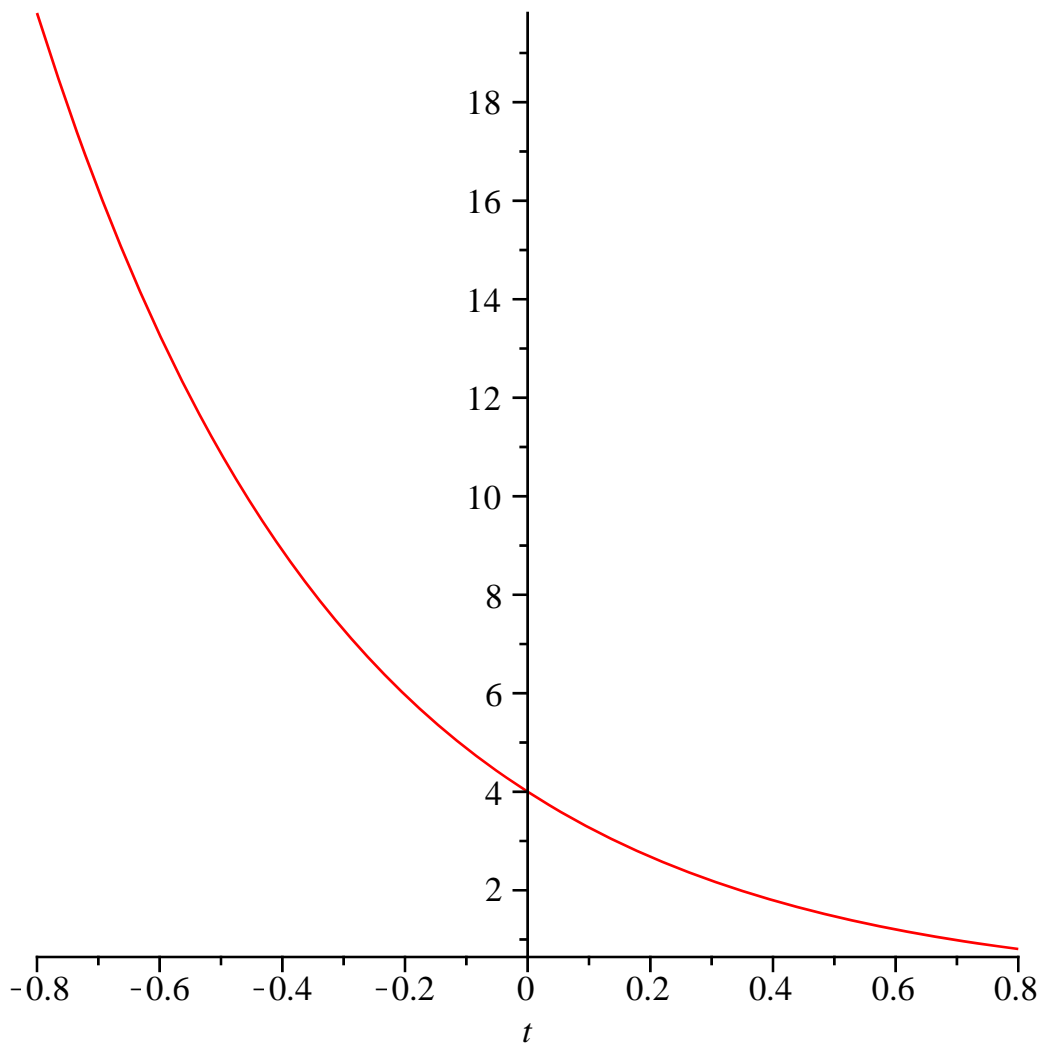
$$STF := e^2 - e^{-2} + \sum_{n=1}^{\infty} \left(\frac{4 (2 e^2 (-1)^n - 2 e^{-2} (-1)^n) \cos(n \pi t)}{4 + n^2 \pi^2} + \frac{4 (e^2 n \pi (-1)^n - e^{-2} n \pi (-1)^n) \sin(n \pi t)}{4 + n^2 \pi^2} \right) \quad (15)$$

$$> f$$

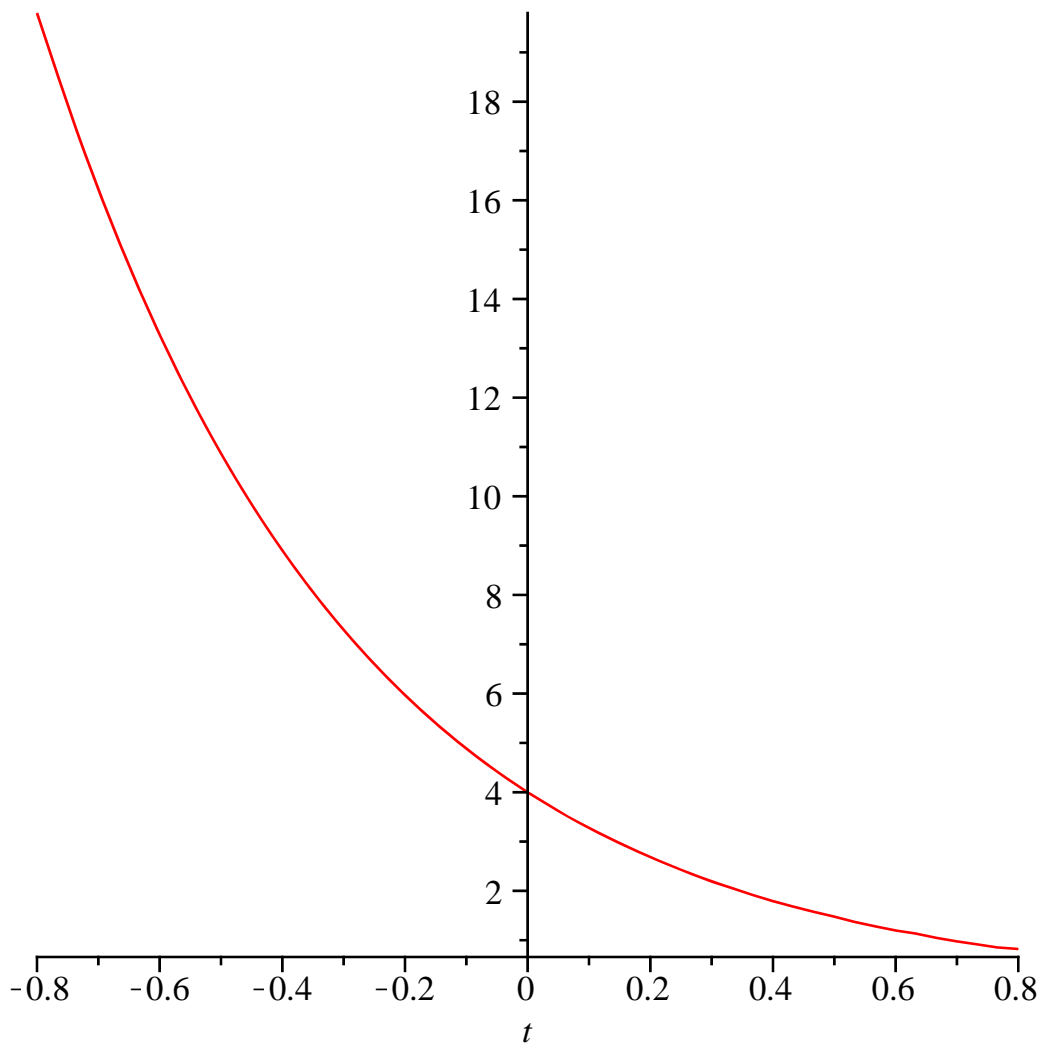
$$4 e^{-2t} \quad (16)$$

$$> STF_{1000} := C + \text{sum}\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..1000\right) :$$

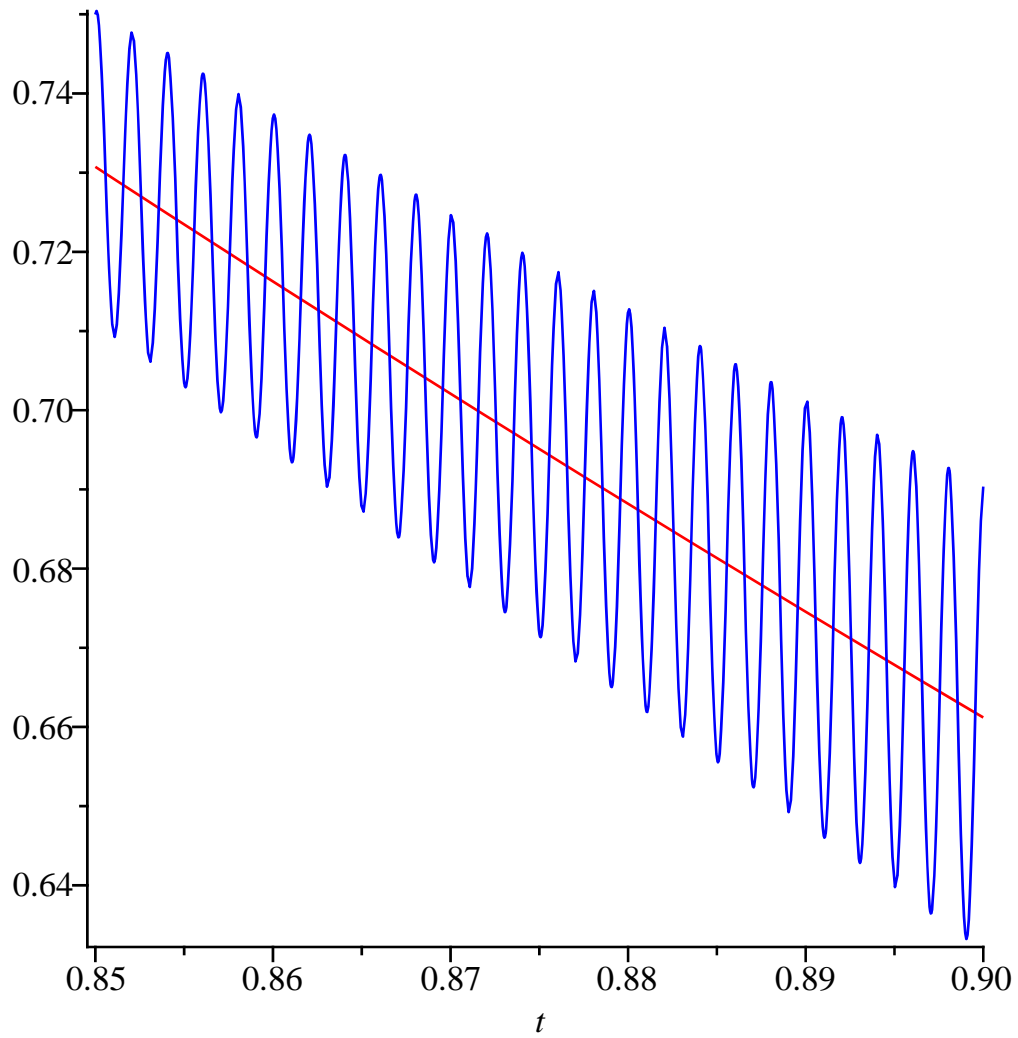
$$> \text{plot}(f, t = -0.8..0.8)$$



```
=  
> plot(STF1000, t=-0.8..0.8)
```



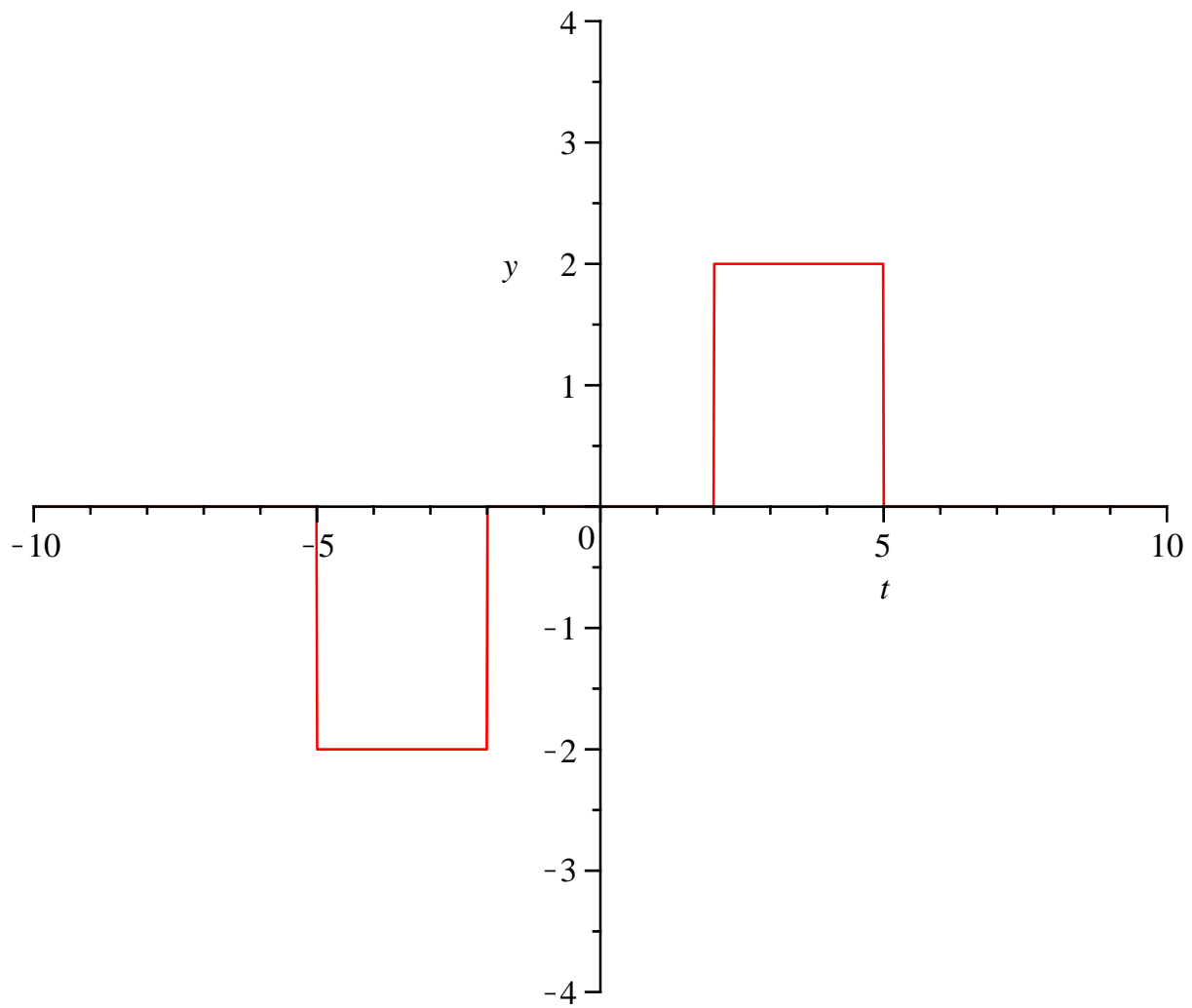
`> plot([f, STF1000], t=0.85..0.9, color=[red, blue])`



```

> restart
> G := -2·Heaviside(t + 5) + 2·Heaviside(t + 2) + 2·Heaviside(t - 2) - 2·Heaviside(t
- 5) : plot(G, t = -10 .. 10, y = -4 .. 4)

```



$$\text{> } L := 7 \quad L := 7 \quad (17)$$

$$\text{> } a_0 := \left(\frac{1}{L} \right) \cdot \text{int}(G, t = -L..L) \quad a_0 := 0 \quad (18)$$

$$\text{> } a_n := \left(\frac{1}{L} \right) \cdot \text{int} \left(G \cdot \cos \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), t = -L..L \right) \quad a_n := 0 \quad (19)$$

$$\text{> } b_n := \left(\frac{1}{L} \right) \cdot \text{int} \left(G \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), t = -L..L \right) \quad b_n := -\frac{4 \cos \left(\frac{5}{7} n \pi \right)}{n \pi} + \frac{4 \cos \left(\frac{2}{7} n \pi \right)}{n \pi} \quad (20)$$

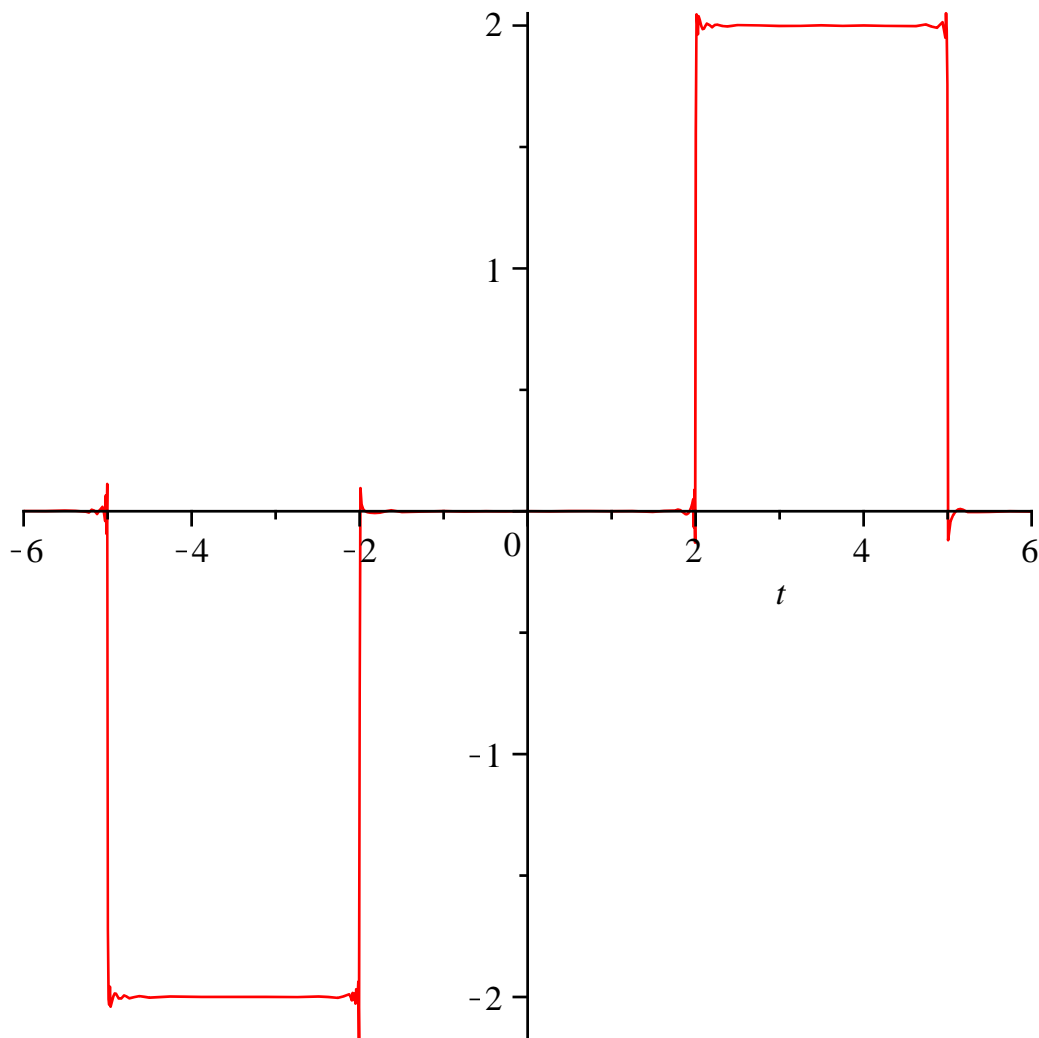
$$\text{> } STF := \text{Sum} \left(b_n \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), n = 1..infinity \right) \quad (21)$$

$$STF := \sum_{n=1}^{\infty} \left(-\frac{4 \cos\left(\frac{5}{7} n \pi\right)}{n \pi} + \frac{4 \cos\left(\frac{2}{7} n \pi\right)}{n \pi} \right) \sin\left(\frac{1}{7} n \pi t\right) \quad (21)$$

```
> G
-2 Heaviside(t + 5) + 2 Heaviside(t + 2) + 2 Heaviside(t - 2) - 2 Heaviside(t - 5) (22)
```

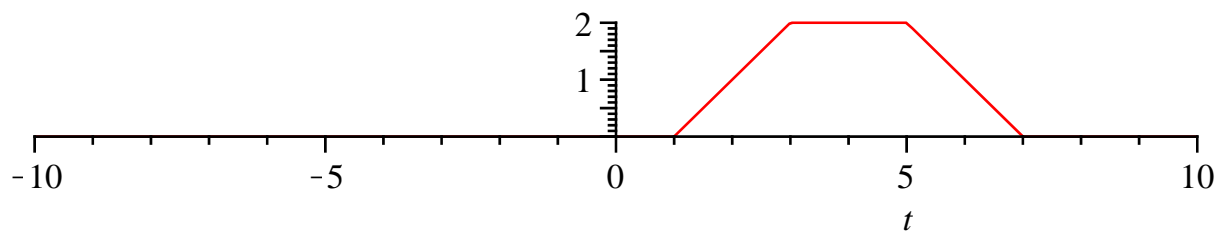
```
> STF1000 := sum(bn·sin(n·Pi·t/L), n = 1..1000) :
```

```
> plot(STF1000, t = -6..6)
```



```
> restart
```

```
> H := (t - 1)·Heaviside(t - 1) - (t - 3)·Heaviside(t - 3) - (t - 5)·Heaviside(t - 5) + (t - 7)·Heaviside(t - 7) : plot(H, t = -10..10, scaling = CONSTRAINED)
```



$$> L := 4$$

$$L := 4 \quad (23)$$

$$> a_0 := \left(\frac{1}{L} \right) \cdot \text{int}(H, t=0..8); C := \frac{a_0}{2}$$

$$a_0 := 2$$

$$C := 1$$

$$(24)$$

$$> a_n := \left(\frac{1}{L} \right) \cdot \text{int} \left(H \cdot \cos \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), t=0..8 \right)$$

$$a_n := - \frac{4 \left(\cos \left(\frac{1}{4} n \pi \right) + \frac{1}{4} n \pi \sin \left(\frac{1}{4} n \pi \right) \right)}{n^2 \pi^2} + \frac{\sin \left(\frac{1}{4} n \pi \right)}{n \pi} \\ + \frac{4 \left(\cos \left(\frac{3}{4} n \pi \right) + \frac{3}{4} n \pi \sin \left(\frac{3}{4} n \pi \right) \right)}{n^2 \pi^2} - \frac{3 \sin \left(\frac{3}{4} n \pi \right)}{n \pi}$$

$$(25)$$

$$+ \frac{4 \left(\cos\left(\frac{5}{4} n \pi\right) + \frac{5}{4} n \pi \sin\left(\frac{5}{4} n \pi\right) \right)}{n^2 \pi^2} - \frac{5 \sin\left(\frac{5}{4} n \pi\right)}{n \pi}$$

$$- \frac{4 \left(\cos\left(\frac{7}{4} n \pi\right) + \frac{7}{4} n \pi \sin\left(\frac{7}{4} n \pi\right) \right)}{n^2 \pi^2} + \frac{7 \sin\left(\frac{7}{4} n \pi\right)}{n \pi}$$

$$> b_n := \left(\frac{1}{L} \right) \cdot \text{int} \left(H \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), t = 0 .. 8 \right)$$

$$b_n := - \frac{4 \left(\sin\left(\frac{1}{4} n \pi\right) - \frac{1}{4} \cos\left(\frac{1}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} - \frac{\cos\left(\frac{1}{4} n \pi\right)}{n \pi}$$

$$+ \frac{4 \left(\sin\left(\frac{3}{4} n \pi\right) - \frac{3}{4} \cos\left(\frac{3}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} + \frac{3 \cos\left(\frac{3}{4} n \pi\right)}{n \pi}$$

$$+ \frac{4 \left(\sin\left(\frac{5}{4} n \pi\right) - \frac{5}{4} \cos\left(\frac{5}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} + \frac{5 \cos\left(\frac{5}{4} n \pi\right)}{n \pi}$$

$$- \frac{4 \left(\sin\left(\frac{7}{4} n \pi\right) - \frac{7}{4} \cos\left(\frac{7}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} - \frac{7 \cos\left(\frac{7}{4} n \pi\right)}{n \pi}$$

(26)

$$> STF := C + \text{Sum} \left(a_n \cdot \cos \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right) + b_n \cdot \sin \left(\frac{n \cdot \text{Pi} \cdot t}{L} \right), n = 1 .. \text{infinity} \right)$$

$$STF := 1 + \sum_{n=1}^{\infty} \left(\left(- \frac{4 \left(\cos\left(\frac{1}{4} n \pi\right) + \frac{1}{4} n \pi \sin\left(\frac{1}{4} n \pi\right) \right)}{n^2 \pi^2} + \frac{\sin\left(\frac{1}{4} n \pi\right)}{n \pi} \right. \right.$$

$$+ \frac{4 \left(\cos\left(\frac{3}{4} n \pi\right) + \frac{3}{4} n \pi \sin\left(\frac{3}{4} n \pi\right) \right)}{n^2 \pi^2} - \frac{3 \sin\left(\frac{3}{4} n \pi\right)}{n \pi}$$

$$+ \frac{4 \left(\cos\left(\frac{5}{4} n \pi\right) + \frac{5}{4} n \pi \sin\left(\frac{5}{4} n \pi\right) \right)}{n^2 \pi^2} - \frac{5 \sin\left(\frac{5}{4} n \pi\right)}{n \pi}$$

$$\left. - \frac{4 \left(\cos\left(\frac{7}{4} n \pi\right) + \frac{7}{4} n \pi \sin\left(\frac{7}{4} n \pi\right) \right)}{n^2 \pi^2} + \frac{7 \sin\left(\frac{7}{4} n \pi\right)}{n \pi} \right) \cos\left(\frac{1}{4} n \pi t\right) + \left(\right.$$

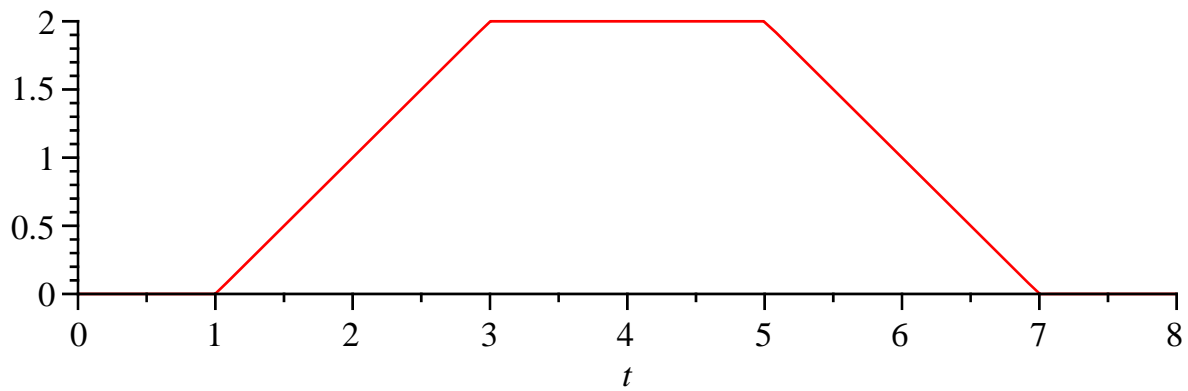
$$- \frac{4 \left(\sin\left(\frac{1}{4} n \pi\right) - \frac{1}{4} \cos\left(\frac{1}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} - \frac{\cos\left(\frac{1}{4} n \pi\right)}{n \pi}$$

$$\left. + \frac{4 \left(\sin\left(\frac{3}{4} n \pi\right) - \frac{3}{4} \cos\left(\frac{3}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} + \frac{3 \cos\left(\frac{3}{4} n \pi\right)}{n \pi} \right)$$

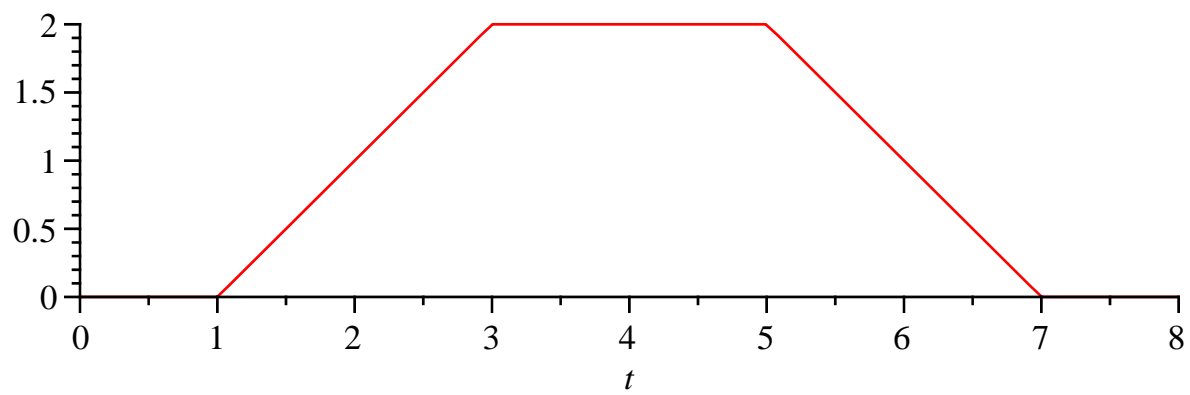
(27)

$$+ \frac{4 \left(\sin\left(\frac{5}{4} n \pi\right) - \frac{5}{4} \cos\left(\frac{5}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} + \frac{5 \cos\left(\frac{5}{4} n \pi\right)}{n \pi} \\ - \frac{4 \left(\sin\left(\frac{7}{4} n \pi\right) - \frac{7}{4} \cos\left(\frac{7}{4} n \pi\right) n \pi \right)}{n^2 \pi^2} - \frac{7 \cos\left(\frac{7}{4} n \pi\right)}{n \pi} \right) \sin\left(\frac{1}{4} n \pi t\right) \Bigg)$$

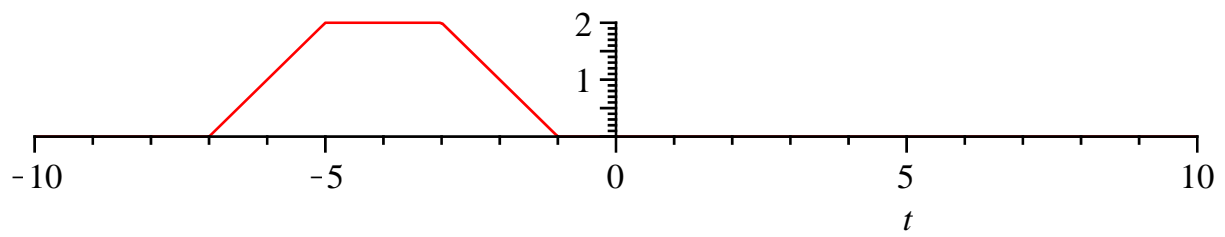
```
> STF500 := C + Sum(an·cos(n·Pi·t/L) + bn·sin(n·Pi·t/L), n = 1..500) :
> plot(STF500, t = 0..8, scaling = CONSTRAINED)
```



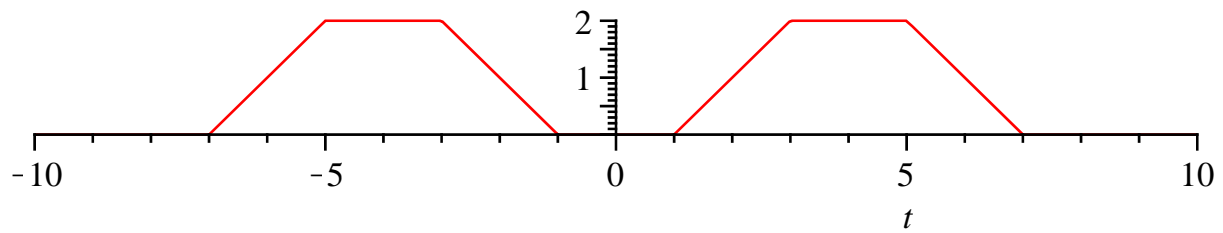
```
> plot(H, t = 0..8, scaling = CONSTRAINED)
```



$$F := (t + 7) \cdot \text{Heaviside}(t + 7) - (t + 5) \cdot \text{Heaviside}(t + 5) - (t + 3) \cdot \text{Heaviside}(t + 3) + (t + 1) \cdot \text{Heaviside}(t + 1) : \text{plot}(F, t = -10..10, \text{scaling} = \text{CONSTRAINED})$$



=
> $R := F + H$; plot($R, t = -10..10, scaling = CONSTRAINED$)



> $L := 9$

$L := 9$

(28)

> $a_{00} := \left(\frac{1}{L}\right) \cdot \text{int}(R, t=-L..L); CC := \frac{a_{00}}{2}$

$a_0 := \frac{16}{9}$

$CC := \frac{8}{9}$

(29)

> $a_{nn} := \text{simplify}\left(\left(\frac{1}{L}\right) \cdot \text{int}\left(R \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

$a_{nn} := -\frac{18 \left(\cos\left(\frac{7}{9} n \pi\right) - \cos\left(\frac{5}{9} n \pi\right) - \cos\left(\frac{1}{3} n \pi\right) + \cos\left(\frac{1}{9} n \pi\right)\right)}{n^2 \pi^2}$

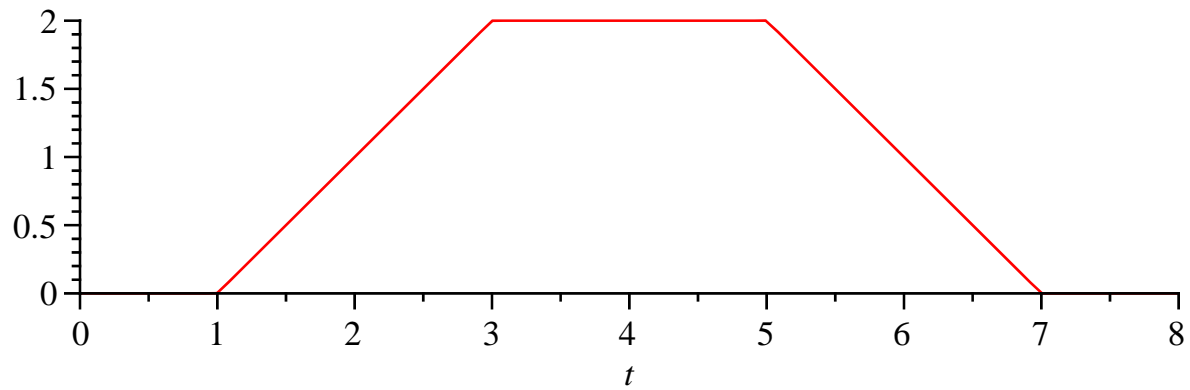
(30)

> $b_{nn} := \text{simplify}\left(\left(\frac{1}{L}\right) \cdot \text{int}\left(R \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), t=-L..L\right)\right)$

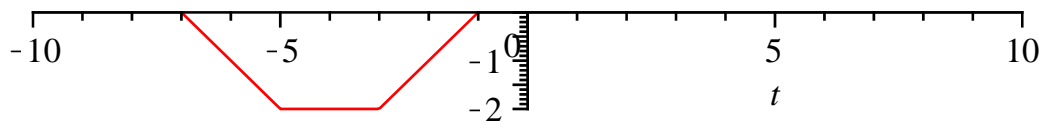
$b_{nn} := 0$

(31)

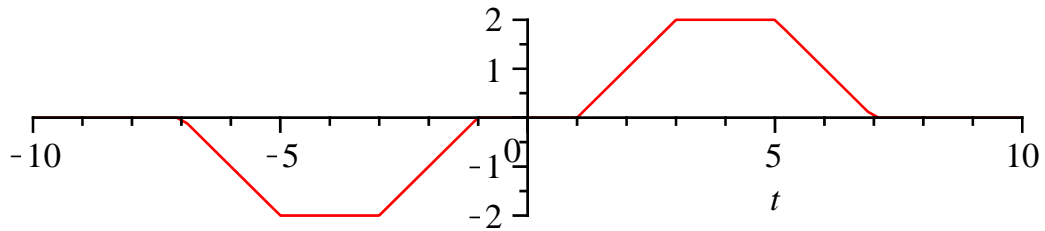
```
> STFF500 := CC + sum( $a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right)$ , n = 1 .. 500) :
> plot(STFF500, t = 0 .. 8, scaling = CONSTRAINED)
```



```
> FF := - (t + 7) · Heaviside(t + 7) + (t + 5) · Heaviside(t + 5) + (t + 3) · Heaviside(t + 3) - (t + 1) · Heaviside(t + 1) : plot(FF, t = -10 .. 10, scaling = CONSTRAINED)
```



> $S := FF + H$; $plot(S, t = -10 .. 10, scaling = CONSTRAINED)$



```
> L := 10
```

```
L := 10
```

(32)

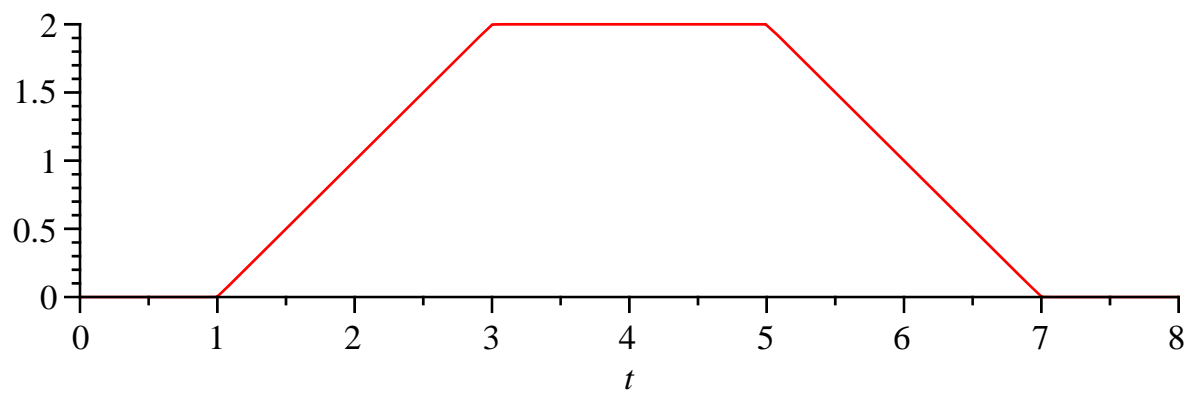
```
> b_mnn := simplify( ( ( 1/L ) · int( S · sin( (n·Pi·t)/L ), t = -L..L ) )
```

$$b_{mn} := \frac{20 \left(\sin\left(\frac{3}{10} n \pi\right) - \sin\left(\frac{1}{10} n \pi\right) - \sin\left(\frac{7}{10} n \pi\right) + \sin\left(\frac{1}{2} n \pi\right) \right)}{n^2 \pi^2}$$

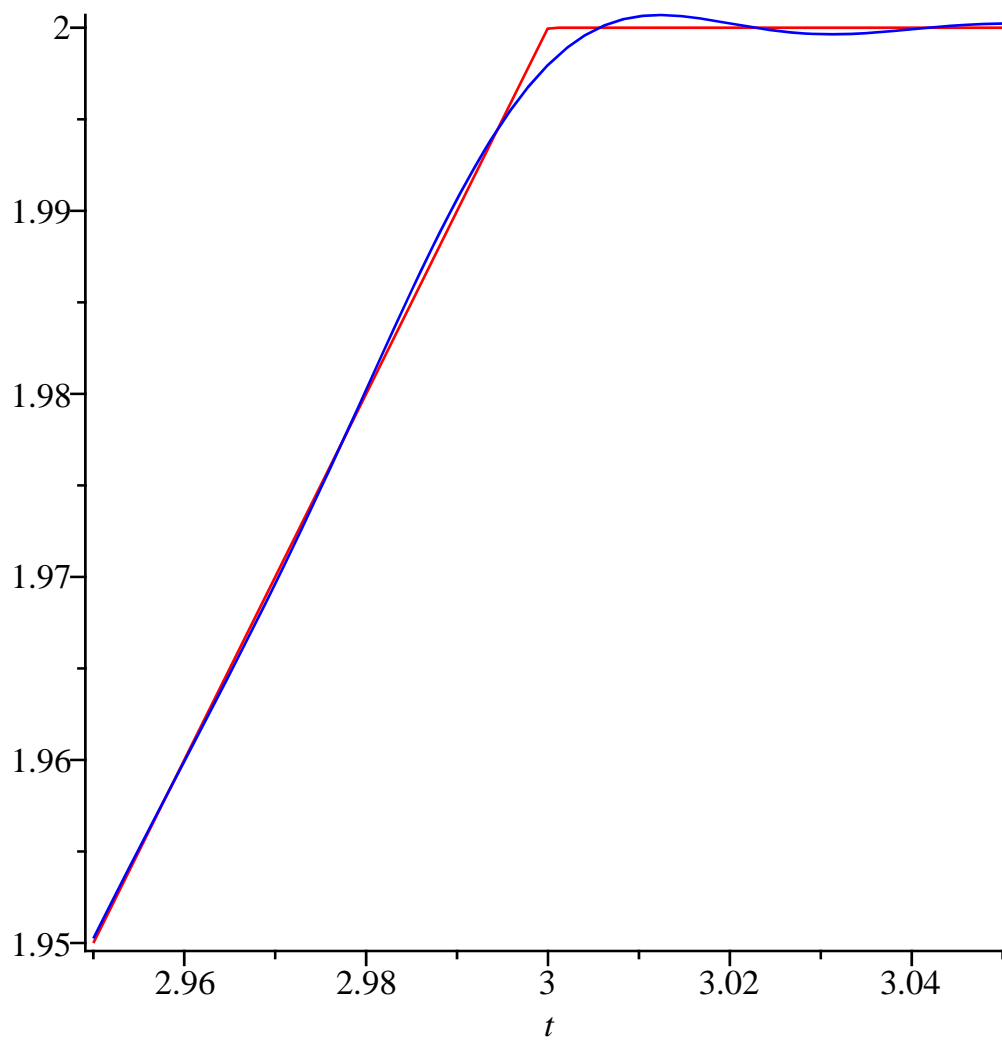
(33)

```
> STFFF_500 := sum( b_mnn · sin( (n·Pi·t)/L ), n = 1..500 ) :
```

```
> plot( STFFF_500, t = 0..8, scaling = CONSTRAINED )
```



```
> plot([S, STFFF500], t = 2.95 .. 3.05, color = [red, blue])
```



[>
[>
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