

> restart

> Ecuacion := $\frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 0$

$$Ecuacion := \frac{d^2}{dx^2} y(x) + 2 \left(\frac{d}{dx} y(x) \right) + 2 y(x) = 0 \quad (1)$$

> Condiciones := $y(0) = 1, D(y)(0) = 1$

$$Condiciones := y(0) = 1, D(y)(0) = 1 \quad (2)$$

> EcuacionCaract := $m \cdot 2 + 2 \cdot m + 2 = 0$

$$EcuacionCaract := m^2 + 2m + 2 = 0 \quad (3)$$

> Raiz := solve(EcuacionCaract)

$$Raiz := -1 + I, -1 - I \quad (4)$$

> SolUno := $y(x) = \exp(\operatorname{Re}(Raiz_1) \cdot x) \cdot \cos(\operatorname{Im}(Raiz_1) \cdot x)$; SolDos := $y(x) = \exp(\operatorname{Re}(Raiz_1) \cdot x) \cdot \sin(\operatorname{Im}(Raiz_1) \cdot x)$

$$SolUno := y(x) = e^{-x} \cos(x)$$

$$SolDos := y(x) = e^{-x} \sin(x) \quad (5)$$

> SolGral := $y(x) = C_1 \cdot rhs(SolUno) + C_2 \cdot rhs(SolDos)$

$$SolGral := y(x) = C_1 e^{-x} \cos(x) + C_2 e^{-x} \sin(x) \quad (6)$$

> Sistemita := $subs(x=0, rhs(SolGral) = rhs(Condiciones_1))$, $subs(x=0, rhs(diff(SolGral, x))) = rhs(Condiciones_2)$: Sistemita₁; Sistemita₂

$$C_1 = 1$$

$$-C_1 + C_2 = 1 \quad (7)$$

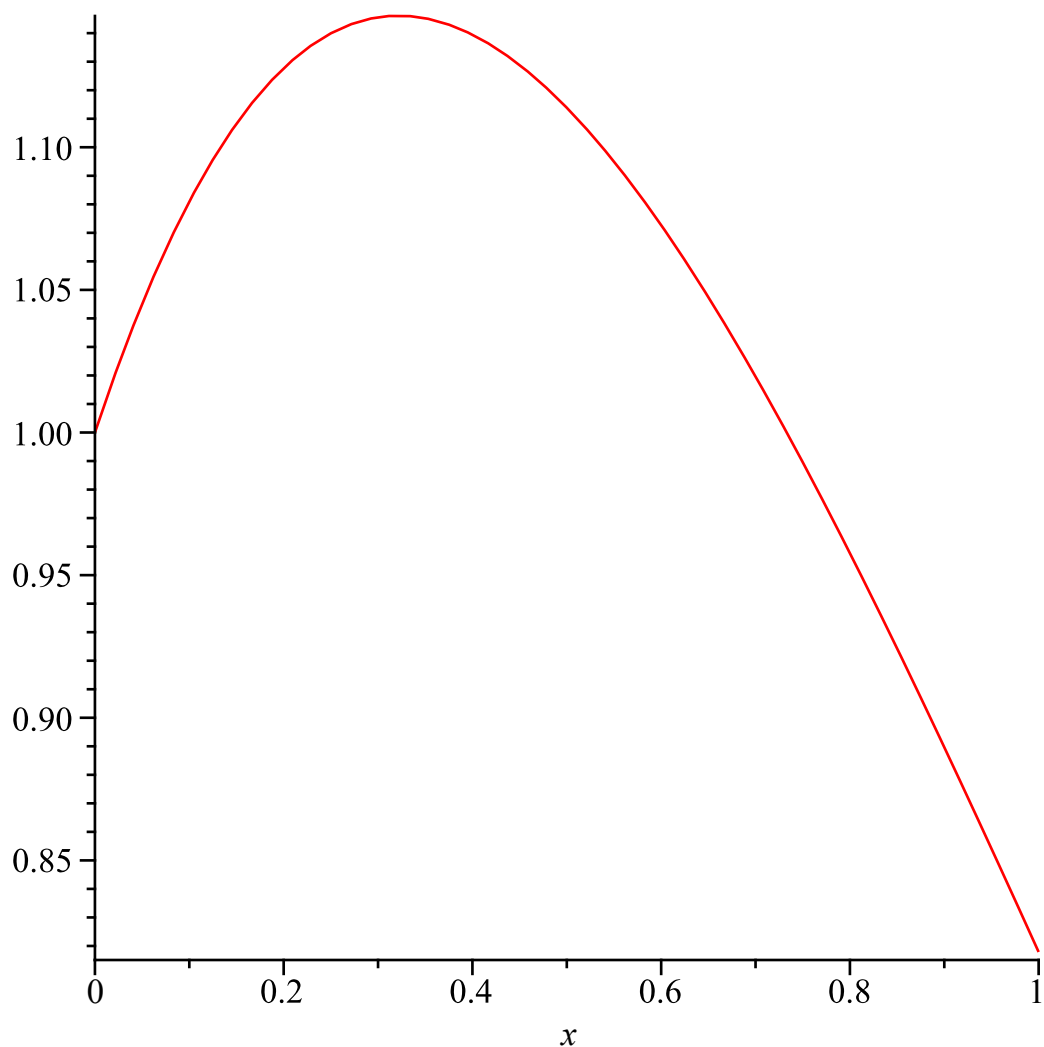
> Parametro := solve({Sistemita}, {C₁, C₂})

$$Parametro := \{C_1 = 1, C_2 = 2\} \quad (8)$$

> SolucionParticular := $subs(C_1 = rhs(Parametro_1), C_2 = rhs(Parametro_2), SolGral)$

$$SolucionParticular := y(x) = e^{-x} \cos(x) + 2 e^{-x} \sin(x) \quad (9)$$

> plot(rhs(SolucionParticular), x = 0 .. 1)



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> Sistema := diff(x(t), t) = 3·x(t) + 4·y(t) + 5·t, diff(y(t), t) = 4·x(t) + 3·y(t) + 1 :
Sistema1; Sistema2
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$$\frac{d}{dt} x(t) = 3x(t) + 4y(t) + 5t$$

$$\frac{d}{dt} y(t) = 4x(t) + 3y(t) + 1$$

(10)

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> Condiciones := x(0) = -2, y(0) = -4
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Condiciones := x(0) = -2, y(0) = -4

(11)

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> AA := array([ [3, 4], [4, 3] ])
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$$AA := \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

(12)

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> Xcero := array([ -2, -4 ])
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$$Xcero := \begin{bmatrix} -2 & -4 \end{bmatrix}$$

(13)

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> BB := array([ 5 t, 1 ])
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$$BB := \begin{bmatrix} 5t & 1 \end{bmatrix}$$

(14)

> with(linalg) :

> MatExp := exponential(AA, t)

$$\text{MatExp} := \begin{bmatrix} \frac{1}{2} e^{-t} + \frac{1}{2} e^{7t} & \frac{1}{2} e^{7t} - \frac{1}{2} e^{-t} \\ \frac{1}{2} e^{7t} - \frac{1}{2} e^{-t} & \frac{1}{2} e^{-t} + \frac{1}{2} e^{7t} \end{bmatrix} \quad (15)$$

> SolHom := evalm(MatExp &* Xcero)

$$\text{SolHom} := \begin{bmatrix} e^{-t} - 3 e^{7t} & -3 e^{7t} - e^{-t} \end{bmatrix} \quad (16)$$

> MatExpTau := map(rcurry(eval, t='t - tau'), MatExp)

$$\text{MatExpTau} := \begin{bmatrix} \frac{1}{2} e^{-t+\tau} + \frac{1}{2} e^{7t-7\tau} & \frac{1}{2} e^{7t-7\tau} - \frac{1}{2} e^{-t+\tau} \\ \frac{1}{2} e^{7t-7\tau} - \frac{1}{2} e^{-t+\tau} & \frac{1}{2} e^{-t+\tau} + \frac{1}{2} e^{7t-7\tau} \end{bmatrix} \quad (17)$$

> BBtau := map(rcurry(eval, t='tau'), BB)

$$\text{BBtau} := \begin{bmatrix} 5 \tau & 1 \end{bmatrix} \quad (18)$$

> ProdMatExpTauBBtau := evalm(MatExpTau &* BBtau)

$$\text{ProdMatExpTauBBtau} := \begin{bmatrix} 5 \left(\frac{1}{2} e^{-t+\tau} + \frac{1}{2} e^{7t-7\tau} \right) \tau + \frac{1}{2} e^{7t-7\tau} - \frac{1}{2} e^{-t+\tau}, \\ 5 \left(\frac{1}{2} e^{7t-7\tau} - \frac{1}{2} e^{-t+\tau} \right) \tau + \frac{1}{2} e^{-t+\tau} + \frac{1}{2} e^{7t-7\tau} \end{bmatrix} \quad (19)$$

> SolNoHom := map(int, ProdMatExpTauBBtau, tau = 0 ..t)

$$\text{SolNoHom} := \begin{bmatrix} \frac{3}{49} (49 + 2 e^{8t} + 35 t e^t - 51 e^t) e^{-t} - \frac{1}{49} (-6 e^{8t} + 147 + 140 t e^t - 141 e^t) e^{-t} \end{bmatrix} \quad (20)$$

> Comprobacion₁ := map(rcurry(eval, t='0'), SolNoHom)

$$\text{Comprobacion}_1 := \begin{bmatrix} 0 & 0 \end{bmatrix} \quad (21)$$

> Comprobacion₂ := map(rcurry(eval, t='0'), SolHom)

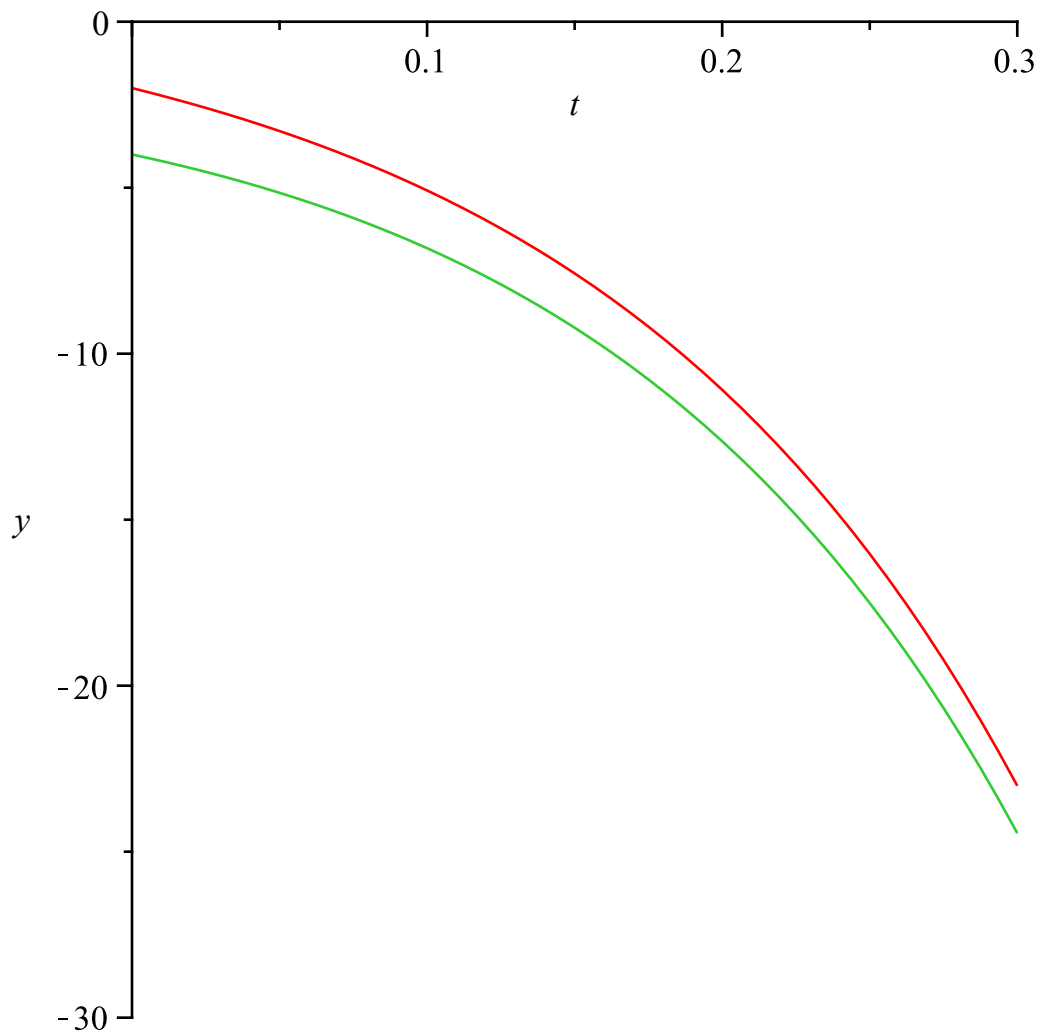
$$\text{Comprobacion}_2 := \begin{bmatrix} -2 & -4 \end{bmatrix} \quad (22)$$

> SolucionFinal := evalm(SolHom + SolNoHom) : SolucionUno := x(t)
= simplify(SolucionFinal₁); SolucionDos := y(t) = simplify(SolucionFinal₂)

$$\text{SolucionUno} := x(t) = 4 e^{-t} - \frac{141}{49} e^{7t} + \frac{15}{7} t - \frac{153}{49}$$

$$\text{SolucionDos} := y(t) = -\frac{141}{49} e^{7t} - 4 e^{-t} - \frac{20}{7} t + \frac{141}{49} \quad (23)$$

> plot([rhs(SolucionUno), rhs(SolucionDos)], t=0 ..0.3, y=-30 ..0)



$$\begin{aligned} &> \text{Comprobacion}_3 := \text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolucionUno}), y(t) = \text{rhs}(\text{SolucionDos}), \\ &\quad \text{lhs}(\text{Sistema}_1) - \text{rhs}(\text{Sistema}_1) = 0)) \\ &\quad \text{Comprobacion}_3 := 0 = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{Comprobacion}_4 := \text{eval}(\text{subs}(x(t) = \text{rhs}(\text{SolucionUno}), y(t) = \text{rhs}(\text{SolucionDos}), \\ &\quad \text{lhs}(\text{Sistema}_2) - \text{rhs}(\text{Sistema}_2) = 0)) \\ &\quad \text{Comprobacion}_4 := 0 = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \text{SolucionPart} := \text{dsolve}(\{\text{Sistema}, \text{Condiciones}\}) : \text{SolucionPart}_1; \text{SolucionPart}_2 \\ &\quad x(t) = 4 e^{-t} - \frac{141}{49} e^{7t} + \frac{15}{7} t - \frac{153}{49} \\ &\quad y(t) = -\frac{141}{49} e^{7t} - 4 e^{-t} - \frac{20}{7} t + \frac{141}{49} \end{aligned} \quad (26)$$

$$\begin{aligned} &> \text{SolucionUno}; \text{SolucionDos} \\ &\quad x(t) = 4 e^{-t} - \frac{141}{49} e^{7t} + \frac{15}{7} t - \frac{153}{49} \\ &\quad y(t) = -\frac{141}{49} e^{7t} - 4 e^{-t} - \frac{20}{7} t + \frac{141}{49} \end{aligned} \quad (27)$$

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