CAP I 1ª Parte. Definiciones

ED
- EDO CAP. IV TL.
  - LINEALES CAP III SISTEMAS.
    - 1er orden CAP II
    - orden n≥1 CAP II
    - CV / CC
    - H / N-H.
  - NO LINEALES CAP. III XX
    - 1er orden 2ª parte CAP 1.
    - orden n≥1 XX
- EDenDP cap V

CAP I. 1ª parte

CAP II
CAP III } LINEALES

EP1

CAP I 2ª parte NO-LINEALES 1er orden

EP2

CAP IV TL.
CAP V: EDenDP

EP3

EDO(n) LINEALES

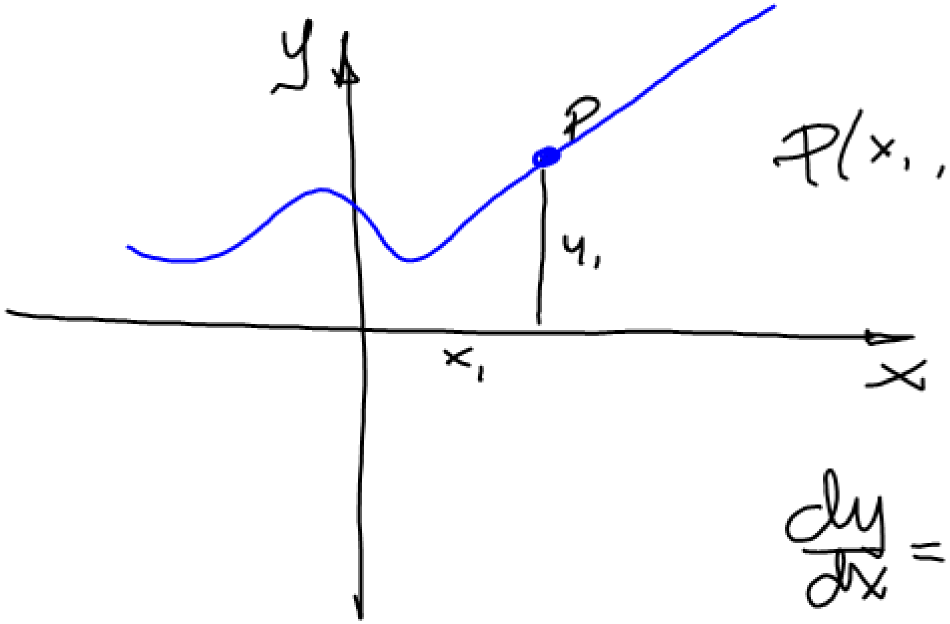
$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots\ldots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)\,y = Q(x)$$

$y(x)$ incógnita

coeficientes $\begin{cases} a_i(x) \longrightarrow i=0\ldots n \longrightarrow \text{coef. variables} \\ \forall a_i(x) = k_i \longrightarrow \text{coef. constantes} \end{cases}$

homogeneidad $\begin{cases} Q(x) \neq 0 & \text{NO HOMOGÉNEA.} \\ Q(x) = 0 & \text{HOMOGENEA.} \end{cases}$

Teorema Existencia y unicidad Solución



$P(x_1, y_1)$

$$\frac{dy}{dx} = F(x,y)$$

$F(x,y)$ es continua (x_1, y_1)

&

$\frac{\partial F}{\partial y}$ es continua en (x_1, y_1).

$$\frac{dy}{dx} = \frac{y}{x} \longrightarrow \frac{dy}{dx} - \left(\frac{1}{x}\right) y = 0$$

EDO(1) LINEAL. CV. H.

$x = 0$

$F(x,y)$ no es continua

$\frac{\partial F}{\partial y} = \frac{1}{x}$ no es continua

$\longrightarrow x = 1,\ y = 1$

$$\frac{dy}{dx} = \frac{y}{x} \longrightarrow \frac{\left(\frac{dy}{dx}\right)}{y} = \frac{1}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

Método de las Variables Separables

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$L(y) + k_1 = L(x) + k_2$$

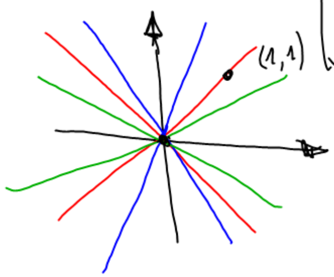
$$L(y) - L(x) = k_2 - k_1$$

$$L\left(\frac{y}{x}\right) = k_2 - k_1 \longrightarrow \frac{y}{x} = e^{k_2 - k_1}$$

$$\frac{y}{x} = C_1$$

$$\boxed{y = C_1 x}$$

$$\boxed{\frac{dy}{dx} = \frac{y}{x}}$$



$$\frac{dy}{dx} = yx \longrightarrow \frac{dy}{dx} - (x)\,y = 0$$

EDO(1) L. CV. H.

$F(x,y) \Rightarrow yx$ existe para todo punto

$\frac{\partial F}{\partial y} \Rightarrow x$ existe para todo punto.

$$\frac{dy}{y} = x\,dx \Rightarrow \int \frac{dy}{y} = \int x\,dx$$

$$\Downarrow$$

$$L(y) + k_1 = \frac{x^2}{2} + k_2$$

$$\downarrow$$

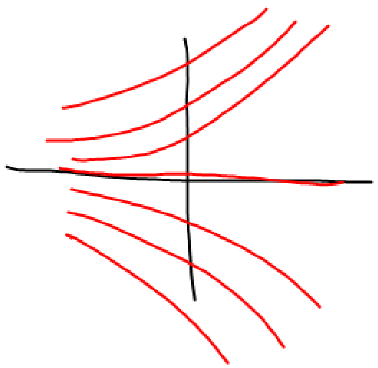
$$L(y) = \frac{x^2}{2} + (k_2 - k_1)$$

$$\Leftarrow$$

$$y = e^{\left(\frac{x^2}{2} + (k_2 - k_1)\right)}$$

$$y = e^{k_2 - k_1}\, e^{\frac{x^2}{2}}$$

$$y = C_1 e^{\frac{x^2}{2}}$$



CAPÍTULO II.- La EDO Lineal

$$a_0(x)\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_{n-1}(x)\frac{dy}{dx} + a_n(x)\, y = Q(x)$$

$y(x)$ EDO(n) L. cv. NH

$n=1$

$$a_0(x)\frac{dy}{dx} + a_1(x)\, y = Q(x) \qquad \text{EDO(1) L. cv. NH}$$

Regla de Oro

"Procura que el coeficiente de la derivada de mayor orden sea siempre uno"

normalizar
estandarizar

$$\frac{dy}{dx} + \frac{a_1(x)}{a_0(x)}\, y = \frac{Q(x)}{a_0(x)}$$

$$\frac{dy}{dx} + p(x)\, y = q(x) \qquad \text{EDO(1) L. cv. NH}$$

$q(x)=0$

$$\boxed{\frac{dy}{dx} + p(x)\, y = 0} \qquad \text{EDO(1) L cv H.}$$

$$\frac{dy}{dx} + p(x)y = 0$$

Método de Variables Separables

$$\frac{dy}{dx} = -p(x)y$$

$$\frac{dy}{y} = -p(x)dx$$

$$\int \frac{dy}{y} = -\int p(x)dx$$

$$L(y) + k_1 = \left[-\int p(x)dx\right] + k_2$$

$$L(y) = \left[-\int p(x)dx\right] + k_2 - k_1$$

$$y = e^{\left(\left[-\int p(x)dx\right] + (k_2 - k_1)\right)}$$

$$y = e^{(k_2 - k_1)} e^{-\int p(x)dx}$$

$$\boxed{y = C_1 e^{-\int p(x)dx}}$$

$$\frac{dy}{dx} - \frac{y}{x} = 0$$

$$\frac{dy}{dx} + \left(-\frac{1}{x}\right) y = 0$$

$$p(x) = -\frac{1}{x}$$

$$\int p(x)\,dx = \int \left(-\frac{1}{x}\right) dx$$

$$= -\int \frac{dx}{x}$$

$$\int p(x)\,dx = -L(x)$$

$$-\int p(x)\,dx = Lx$$

$$u = e^{Lx}$$

$$Lu = Le^{Lx}$$

$$Lu = Lx\,\cancel{Le}^{\,=1}$$

$$u = x$$

$$\cancel{y = C_1 e^{Lx}}$$

$$\boxed{y = C_1 x}$$

$$\frac{dy}{dx} = xy$$

$$\frac{dy}{dx} - xy = 0$$

$$p(x) = -x$$

$$-\int p(x)\,dx = -\int -x\,dx$$

$$= \frac{x^2}{2}$$

$$\boxed{y = C_1 e^{\frac{x^2}{2}}}$$

$$\frac{dy}{dx} + \cos(4x)\, y = 0$$

$$\frac{dy}{dx} + \cosh(5x)\, y = 0$$

$$e^{e^{\sim}}$$