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> restart
> Ecuacion := y'' - 5 y' + 6 y = 4 * exp(x)
      Ecuacion :=  $\frac{d^2}{dx^2} y(x) - 5 \left( \frac{d}{dx} y(x) \right) + 6 y(x) = 4 e^x$  (1)
> EcuaHom := lhs(Ecuacion) = 0
      EcuaHom :=  $\frac{d^2}{dx^2} y(x) - 5 \left( \frac{d}{dx} y(x) \right) + 6 y(x) = 0$  (2)
> Q := rhs(Ecuacion)
      Q :=  $4 e^x$  (3)
> EcuaCarac := m * 2 - 5 * m + 6 = 0
      EcuaCarac :=  $m^2 - 5 m + 6 = 0$  (4)
> Raiz := solve(EcuaCarac)
      Raiz := 3, 2 (5)
> SolUno := y(x) = exp(Raiz_1 * x); SolDos := y(x) = exp(Raiz_2 * x)
      SolUno :=  $y(x) = e^{3x}$ 
      SolDos :=  $y(x) = e^{2x}$  (6)
> SolHom := y(x) = C_1 * rhs(SolUno) + C_2 * rhs(SolDos)
      SolHom :=  $y(x) = C_1 e^{3x} + C_2 e^{2x}$  (7)
> SolNoHom := y(x) = A * rhs(SolUno) + B * rhs(SolDos)
      SolNoHom :=  $y(x) = A e^{3x} + B e^{2x}$  (8)
> with(linalg) :
> WW := wronskian([rhs(SolUno), rhs(SolDos)], x)
      WW :=  $\begin{bmatrix} e^{3x} & e^{2x} \\ 3 e^{3x} & 2 e^{2x} \end{bmatrix}$  (9)
> BB := array([0, Q])
      BB :=  $\begin{bmatrix} 0 & 4 e^x \end{bmatrix}$  (10)
> SOL := simplify(linsolve(WW, BB))
      SOL :=  $\begin{bmatrix} 4 e^{-2x} & -4 e^{-x} \end{bmatrix}$  (11)
> A := int(SOL_1, x) + C_1; B := int(SOL_2, x) + C_2
      A :=  $-2 e^{-2x} + C_1$ 
      B :=  $4 e^{-x} + C_2$  (12)
> Solucion := simplify(SolNoHom)
      Solucion :=  $y(x) = 2 e^x + C_1 e^{3x} + C_2 e^{2x}$  (13)
> SolGral := dsolve(Ecuacion)
      SolGral :=  $y(x) = e^{2x} _C2 + e^{3x} _C1 + 2 e^x$  (14)
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> restart
> Ecuacion := y''' + y'' + y' + y = 8·exp(x) + 5·cos(2 x)
      Ecuacion :=  $\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 8 e^x + 5 \cos(2 x)$  (15)
> EcuacionHom := lhs(Ecuacion) = 0; Q := rhs(Ecuacion);
      EcuacionHom :=  $\frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 0$ 
      Q :=  $8 e^x + 5 \cos(2 x)$  (16)
> EcuacionCarac := m·3 + m·2 + m + 1 = 0
      EcuacionCarac :=  $m^3 + m^2 + m + 1 = 0$  (17)
> Raiz := solve(EcuacionCarac)
      Raiz := -1, I, -I (18)
> SolUno := y(x) = exp(Raiz1·x); SolDos := y(x) = cos(Im(Raiz2)·x); SolTres := y(x)
      = sin(Im(Raiz2)·x)
      SolUno :=  $y(x) = e^{-x}$ 
      SolDos :=  $y(x) = \cos(x)$ 
      SolTres :=  $y(x) = \sin(x)$  (19)
> SolucionHom := y(x) = C1·rhs(SolUno) + C2·rhs(SolDos) + C3·rhs(SolTres)
      SolucionHom :=  $y(x) = C_1 e^{-x} + C_2 \cos(x) + C_3 \sin(x)$  (20)
> SolucionNoHom := y(x) = A·rhs(SolUno) + B·rhs(SolDos) + DD·rhs(SolTres)
      SolucionNoHom :=  $y(x) = A e^{-x} + B \cos(x) + DD \sin(x)$  (21)
> with(linalg) :
> WW := wronskian([rhs(SolUno), rhs(SolDos), rhs(SolTres)], x)
      WW :=  $\begin{bmatrix} e^{-x} & \cos(x) & \sin(x) \\ -e^{-x} & -\sin(x) & \cos(x) \\ e^{-x} & -\cos(x) & -\sin(x) \end{bmatrix}$  (22)
> comprobacion0 := simplify(det(WW)) ≠ 0
      comprobacion0 :=  $2 e^{-x} \neq 0$  (23)
>
> BB := array([0, 0, Q])
      BB :=  $\begin{bmatrix} 0 & 0 & 8 e^x + 5 \cos(2 x) \end{bmatrix}$  (24)
> SOL := simplify(linsolve(WW, BB)) :
> Aprima := SOL1; Bprima := SOL2; Dprima := SOL3
      Aprima :=  $\frac{1}{2} (8 e^x + 5 \cos(2 x)) e^x$ 

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$$\begin{aligned}
 Bprima &:= -4 \cos(x) e^x - \frac{5}{2} \cos(x) \cos(2x) - 4 \sin(x) e^x - \frac{5}{2} \sin(x) \cos(2x) \\
 Dprima &:= -4 \sin(x) e^x + 4 \cos(x) e^x - \frac{5}{2} \sin(x) \cos(2x) + \frac{5}{2} \cos(x) \cos(2x)
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 &> A := \text{int}(Aprima, x) + C_1; B := \text{int}(Bprima, x) + C_2; DD := \text{int}(Dprima, x) + C_3; \\
 &\quad A := 2 (e^x)^2 + (\cos(x) + 2 \sin(x)) e^x \cos(x) - \frac{1}{2} e^x + C_1 \\
 &\quad B := -4 \sin(x) e^x - \frac{5}{4} \sin(x) - \frac{5}{12} \sin(3x) + \frac{5}{12} \cos(3x) - \frac{5}{4} \cos(x) + C_2 \\
 &\quad DD := 4 \cos(x) e^x + \frac{5}{12} \cos(3x) - \frac{5}{4} \cos(x) + \frac{5}{4} \sin(x) + \frac{5}{12} \sin(3x) + C_3
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 &> \text{SolucionFinal} := \text{simplify}(\text{SolucionNoHom}) \\
 \text{SolucionFinal} &:= y(x) = 2 e^x - \frac{3}{2} \cos(x)^2 - \frac{1}{2} \sin(x) \cos(x) + \frac{3}{4} + C_1 e^{-x} \\
 &\quad - \frac{5}{12} \cos(x) \sin(3x) + \frac{5}{12} \cos(x) \cos(3x) + C_2 \cos(x) + \frac{5}{12} \sin(x) \cos(3x) \\
 &\quad + \frac{5}{12} \sin(x) \sin(3x) + C_3 \sin(x)
 \end{aligned} \tag{27}$$

$$\begin{aligned}
 &> \text{Comprobacion} := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolucionFinal}), \text{lhs}(\text{Ecuacion}) \\
 &\quad - \text{rhs}(\text{Ecuacion}) = 0))) \\
 &\quad \text{Comprobacion} := 0 = 0
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 &> \text{Ecuacion} \\
 &\quad \frac{d^3}{dx^3} y(x) + \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) + y(x) = 8 e^x + 5 \cos(2x)
 \end{aligned} \tag{29}$$

$$\begin{aligned}
 &> \text{SolucionAlterna} := \text{dsolve}(\text{Ecuacion}) \\
 \text{SolucionAlterna} &:= y(x) = -\frac{1}{3} \cos(2x) - \frac{2}{3} \sin(2x) + 2 e^x + \_C1 \cos(x) + \_C2 \sin(x) \\
 &\quad + \_C3 e^{-x}
 \end{aligned} \tag{30}$$

$$\begin{aligned}
 &> \text{SolPartUno} := y(x) = 2 e^x - \frac{3}{2} \cos(x)^2 - \frac{1}{2} \sin(x) \cos(x) + \frac{3}{4} - \frac{5}{12} \cos(x) \sin(3x) \\
 &\quad + \frac{5}{12} \cos(x) \cos(3x) + \frac{5}{12} \sin(x) \cos(3x) + \frac{5}{12} \sin(x) \sin(3x) \\
 \text{SolPartUno} &:= y(x) = 2 e^x - \frac{3}{2} \cos(x)^2 - \frac{1}{2} \sin(x) \cos(x) + \frac{3}{4} - \frac{5}{12} \cos(x) \sin(3x) \\
 &\quad + \frac{5}{12} \cos(x) \cos(3x) + \frac{5}{12} \sin(x) \cos(3x) + \frac{5}{12} \sin(x) \sin(3x)
 \end{aligned} \tag{31}$$

$$\begin{aligned}
 &> \text{SolPartDos} := y(x) = -\frac{1}{3} \cos(2x) - \frac{2}{3} \sin(2x) + 2 e^x \\
 &\quad \text{SolPartDos} := y(x) = -\frac{1}{3} \cos(2x) - \frac{2}{3} \sin(2x) + 2 e^x
 \end{aligned} \tag{32}$$

$$\begin{aligned}
 &> \\
 &> \\
 &> \text{simplify}(\text{rhs}(\text{SolPartUno}) - \text{rhs}(\text{SolPartDos}) = 0) \\
 &\quad 0 = 0
 \end{aligned} \tag{33}$$

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