Capítulo III.- Sistemas de Ecuaciones Diferenciales Lineales.

$$\boxed{\begin{array}{l} 2x+3y=4 \\ -x+4y=-6 \end{array}}$$ Ec. simultáneas.

(1) $\frac{dx}{dt}=2x+3y$; $x(t)$; $x(0)=4$

(2) $\frac{dy}{dt}=x+4y$; $y(t)$; $y(0)=-2$

De (2) despejo "$x(t)$" S(2)EDO(1)L $\downarrow$ EDO(2)L

$$x=\frac{dy}{dt}-4y$$

$$\frac{d}{dt}: \quad \frac{dx}{dt}=\frac{d^2y}{dt^2}-4\frac{dy}{dt}$$

Sust. en (1)

$$\left(\frac{d^2y}{dt^2}-4\frac{dy}{dt}\right)=2\left(\frac{dy}{dt}-4y\right)+3y$$

$$\frac{d^2y}{dt^2}-6\frac{dy}{dt}+5y=0 \quad \text{EDO(2)LccH}$$

$$m^2-6m+5=0$$

$$(m-1)(m-5)=0 \quad \begin{cases} m_1=1 \\ m_2=5 \end{cases} \quad m_1\neq m_2$$

$$y_g(t)=C_1e^t+C_2e^{5t}$$

$$\frac{dy}{dt}=C_1e^t+5C_2e^{5t}$$

$$x(t)=\left(C_1e^t+5C_2e^{5t}\right)-4\left(C_1e^t+C_2e^{5t}\right)$$

$$\boxed{\begin{array}{l} x(t)=-3C_1e^t+C_2e^{5t} \\ y(t)=C_1e^t+C_2e^{5t} \end{array}}$$

$$\boxed{\begin{array}{l} x(t)=C_{10}e^t+C_{20}e^{5t} \\ y(t)=-\frac{C_{10}}{3}e^t+C_{20}e^{5t} \end{array}}$$

$$\frac{dx(t)}{dt}=2x(t)+3y(t)$$

$$\frac{dy(t)}{dt}=x(t)+4y(t)$$

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}=\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

$$\frac{dx_1(t)}{dt} = a_{11}x_1(t) + a_{12}x_2(t) + a_{13}x_3(t)$$

$$\frac{dx_2(t)}{dt} = a_{21}x_1(t) + a_{22}x_2(t) + a_{23}x_3(t)$$

$$\frac{dx_3(t)}{dt} = a_{31}x_1(t) + a_{32}x_2(t) + a_{33}x_3(t)$$

S(3) EDO(1) LccH

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{x}(t) = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix}$$

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$e^{At}\Big|_{t=0} = I. \qquad \frac{d}{dt}\bar{x}(t) = A\,\bar{x}(t)$$

$$\frac{d}{dt}e^{At} = A \times e^{At} \longleftarrow \quad \bar{x}(t) = \left[e^{At}\right]\bar{x}(0)$$

$$e^{At} \cdot \left[e^{At}\right]^{-1} = I.$$

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$$\left[e^{At}\right]^{-1} = e^{A(-t)}$$

$$\frac{d}{dt}\bar{x}(t) = A\bar{x}(t) + 5(t) \qquad S(n)\ EDO(1)\ Lcc\ \underline{\underline{NH}}$$

$$\bar{x}(t) = e^{At} \cdot \bar{x}(0) + \int_0^t e^{A(t-z)}\, 5(z)\, dz.$$