$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mu(t-a) = \begin{cases} 0 \; ; & t<a \\ 1 \; ; & t>a \end{cases}$$

$$\mathcal{L}\{u(t-a)\} = \int_0^\infty e^{-st}\mu(t-a)\,dt$$

$$= \int_0^a \cancelto{0}{e^{-st}\cdot(0)}\cdot dt + \int_a^\infty e^{-st}\cdot(1)\cdot dt$$

$$= \left[\int e^{-st}\,dt\right]_a^\infty$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$= -\frac{1}{s}\left(\cancelto{0}{\lim_{b\to\infty} e^{-sb}} - e^{-as}\right)$$

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$$

slope

$$r(t-a) = \begin{cases} 0\ ; & t<a \\ (t-a)\ ; & t>a \end{cases}$$

$$\mathcal{L}\{r(t-a)\} = \int_0^\infty e^{-st}\cdot r(t-a)\cdot dt$$

$$= \overbrace{\int_0^a e^{-st}\cdot(0)\cdot dt}^{0} + \int_a^\infty e^{-st}(t-a)\,dt$$

$$= \int_a^\infty e^{-st}(t-a)\,dt$$

$$= \left[\int e^{-st}(t-a)\,dt\right]_a^\infty$$

$$\int e^{-st}(t-a)\,dt = \frac{(t-a)e^{-st}}{-s} - \int \frac{e^{-st}}{-s}\,dt$$

$u=(t-a)$ $\quad du=dt$

$dv=e^{-st}dt$ $\quad v=\frac{e^{-st}}{-s}$

$$= \frac{(t-a)e^{-st}}{-s} + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]$$

$$= \frac{(t-a)e^{-st}}{-s} - \frac{1}{s^2}e^{-st}$$

$$\mathcal{L}\{r(t-a)\} = -\frac{1}{s}\left[(t-a)e^{-st}\right]_a^\infty - \frac{1}{s^2}\left[e^{-st}\right]_a^\infty$$

$$= -\frac{1}{s}\Big((0)-(0)\Big) - \frac{1}{s^2}\Big((0)-e^{-sa}\Big)$$

$$\boxed{\mathcal{L}\{r(t-a)\} = \frac{e^{-as}}{s^2}} \qquad \mathcal{L}\{t\} = \frac{1}{s^2}$$

$$\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s} \qquad \mathcal{L}\{r(t-a)\} = \frac{e^{-sa}}{s^2}$$

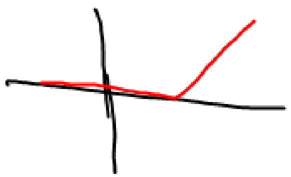
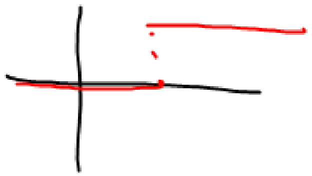
$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = s\,\mathcal{L}\{r(t-a)\} - r(t-a)\Big|_{t=0}$$

$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = s\left(\frac{e^{-as}}{s^2}\right) - (0)$$

$$= \frac{e^{-as}}{s}$$

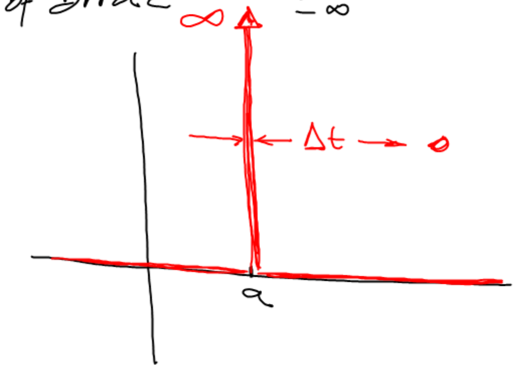
$$\mathcal{L}\left\{\frac{d}{dt} r(t-a)\right\} = \mathcal{L}\{u(t-a)\}$$

$$\frac{d}{dt} r(t-a) = u(t-a)$$

$$\delta(t-a) = \begin{cases} 0 \; ; \; t \neq a \\ \int_{-\infty}^{\infty} \delta(t-a)\,dt = 1 \end{cases}$$

delta of Dirac



$$\mathcal{L}\{\delta(t-a)\} = e^{-st}$$

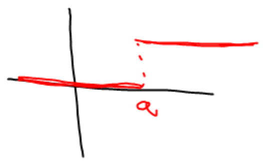
$$\mathcal{L}\left\{\frac{d}{dt}\left(u(t-a)\right)\right\} = s\mathcal{L}\{u(t-a)\} - u(t-a)\Big|_{t=0}$$

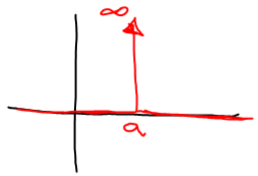
$$= s\left[\frac{e^{-st}}{s}\right] - (0)$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\} = e^{-st}$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\} = \mathcal{L}\{\delta(t-a)\}$$

$$\frac{d}{dt}u(t-a) = \delta(t-a)$$





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$$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{4t} \cdot 1\} = \frac{1}{s-4}$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} \qquad \mathcal{L}\{t^3 e^{4t}\} = \frac{3!}{(s-4)^4}$$

$$\mathcal{L}\{\cos(3t)\} = \frac{s}{s^2+3^2} \qquad \mathcal{L}\{e^{4t}\cos(3t)\} = \frac{(s-4)}{(s-4)^2+9}$$

$$\boxed{\mathcal{L}\{e^{4t}\cos(3t)\} = \frac{s-4}{s^2-8s+25}}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s^2+2s+1)+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2+(1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)-1}{(s+1)^2+(1)^2}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+(1)^2}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+(1)^2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s+2}\right\} = e^{-t}\cos(t) - e^{-t}\sin(t)$$

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$$\mathcal{L}^{-1}\left\{ e^{-as} F(s) \right\} = f(t-a) \cdot \mu(t-a)$$

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$$\mathcal{L}\left\{ e^{at} f(t) \right\} = F(s-a)$$

$$\mathcal{L}^{-1}\left\{ \frac{6e^{-4s}}{(s-8)^4} \right\} = (t-4)^3 e^{8(t-4)} \cdot \mu(t-4)$$

$$\mathcal{L}^{-1}\left\{ \frac{3!}{(s-8)^4} \right\} = e^{8t} t^3$$

$$\mathcal{L}^{-1}\left\{ \frac{3!}{s^4} \right\} = t^3$$

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convolution

$$\mathcal{L}^{-1}\left\{F(s)\cdot G(s)\right\} = f(t) \underset{\uparrow}{*} g(t)$$

$$f(t)*g(t)=\int_0^t f(\tau)\,g(t-\tau)\,d\tau$$

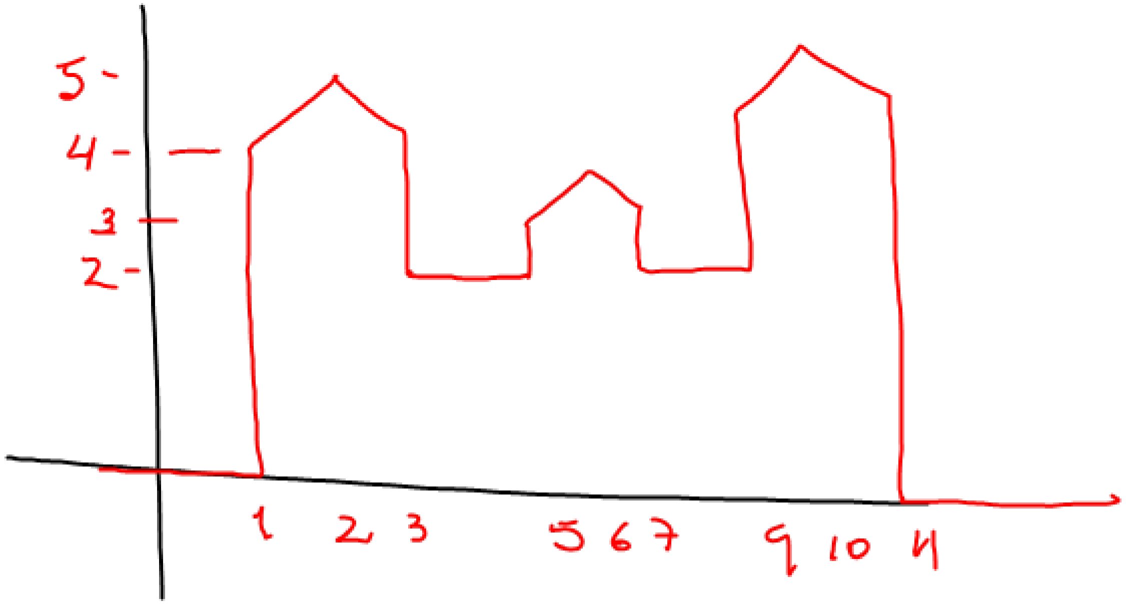
$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+(2)^2}\cdot\frac{1}{s^2+(2)^2}\right\}$$

$$=\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+(2)^2}\cdot\frac{2}{s^2+(2)^2}\right\}$$

$$\left.\begin{array}{l}\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4}\right\}=\cos(2t)\\ \mathcal{L}^{-1}\left\{\dfrac{2}{s^2+4}\right\}=\sin(2t)\end{array}\right\}=\frac{1}{2}\cos(2t)*\sin(2t)$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}=\frac{1}{2}\int_0^t\cos(2\tau)\cdot\operatorname{sen}(2(t-\tau))\,d\tau$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}=\frac{1}{4}t\cdot\sin(2t)$$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