

> restart

>

$$\frac{\partial^2 z(x,y)}{\partial y^2} + x^2 \frac{\partial^2 z(x,y)}{\partial x^2} = z(x,y)$$

> Ecuacion := diff(z(x,y), y\$2) + x^2 * diff(z(x,y), x) = z(x,y)

$$Ecuacion := \frac{\partial^2}{\partial y^2} z(x,y) + x^2 \left(\frac{\partial}{\partial x} z(x,y) \right) = z(x,y) \quad (1)$$

> EcuaSeparable := simplify(eval(subs(z(x,y) = F(x) * G(y), Ecuacion)))

$$EcuaSeparable := F(x) \left(\frac{d^2}{dy^2} G(y) \right) + x^2 \left(\frac{d}{dx} F(x) \right) G(y) = F(x) G(y) \quad (2)$$

> EcuaSeparadaUno := $\frac{\left(lhs(EcuaSeparable) - x^2 \left(\frac{d}{dx} F(x) \right) G(y) \right)}{F(x) \cdot G(y)}$

$$= simplify \left(\frac{\left(rhs(EcuaSeparable) - x^2 \left(\frac{d}{dx} F(x) \right) G(y) \right)}{F(x) \cdot G(y)} \right)$$

$$EcuaSeparadaUno := \frac{\frac{d^2}{dy^2} G(y)}{G(y)} = \frac{F(x) - x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} \quad (3)$$

> EcuaSeparadaDos

$$:= simplify \left(\frac{\left(lhs(EcuaSeparable) - x^2 \left(\frac{d}{dx} F(x) \right) G(y) - F(x) G(y) \right)}{F(x) \cdot G(y)} \right)$$

$$= \frac{\left(rhs(EcuaSeparable) - x^2 \left(\frac{d}{dx} F(x) \right) G(y) - F(x) G(y) \right)}{F(x) \cdot G(y)}$$

$$EcuaSeparadaDos := \frac{\frac{d^2}{dy^2} G(y) - G(y)}{G(y)} = - \frac{x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} \quad (4)$$

> EcuaSepUnoX := rhs(EcuaSeparadaUno) = alpha; EcuaSepUnoY := lhs(EcuaSeparadaUno) = alpha

$$EcuaSepUnoX := \frac{F(x) - x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} = \alpha$$

$$EcuaSepUnoY := \frac{\frac{d^2}{dy^2} G(y)}{G(y)} = \alpha \quad (5)$$

> $SolUnoXcero := dsolve(subs(alpha = 0, EcuaSepUnoX)); SolUnoYcero := dsolve(subs(alpha = 0, EcuaSepUnoY))$

$$\begin{aligned} SolUnoXcero &:= F(x) = _C1 e^{-\frac{1}{x}} \\ SolUnoYcero &:= G(y) = _C1 y + _C2 \end{aligned} \quad (6)$$

> $SolUnoCero := z(x, y) = subs(_C1 = 1, rhs(SolUnoXcero)) \cdot rhs(SolUnoYcero)$

$$SolUnoCero := z(x, y) = e^{-\frac{1}{x}} (_C1 y + _C2) \quad (7)$$

> $Comprobacion_1 := simplify(eval(subs(z(x, y) = rhs(SolUnoCero), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_1 := 0 = 0 \quad (8)$$

> $SolUnoXpos := dsolve(subs(alpha = beta \cdot 2, EcuaSepUnoX)); SolUnoYpos := dsolve(subs(alpha = beta \cdot 2, EcuaSepUnoY))$

$$\begin{aligned} SolUnoXpos &:= F(x) = _C1 e^{\frac{(\beta - 1)(\beta + 1)}{x}} \\ SolUnoYpos &:= G(y) = _C1 e^{\beta y} + _C2 e^{-\beta y} \end{aligned} \quad (9)$$

> $SolUnoPos := z(x, y) = subs(_C1 = 1, rhs(SolUnoXpos)) \cdot rhs(SolUnoYpos)$

$$SolUnoPos := z(x, y) = e^{\frac{(\beta - 1)(\beta + 1)}{x}} (_C1 e^{\beta y} + _C2 e^{-\beta y}) \quad (10)$$

> $Comprobacion_2 := simplify(eval(subs(z(x, y) = rhs(SolUnoPos), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_2 := 0 = 0 \quad (11)$$

> $SolUnoXneg := dsolve(subs(alpha = -beta \cdot 2, EcuaSepUnoX)); SolUnoYneg := dsolve(subs(alpha = -beta \cdot 2, EcuaSepUnoY))$

$$\begin{aligned} SolUnoXneg &:= F(x) = _C1 e^{-\frac{\beta^2 + 1}{x}} \\ SolUnoYneg &:= G(y) = _C1 \sin(\beta y) + _C2 \cos(\beta y) \end{aligned} \quad (12)$$

> $SolUnoNeg := z(x, y) = subs(_C1 = 1, rhs(SolUnoXneg)) \cdot rhs(SolUnoYneg)$

$$SolUnoNeg := z(x, y) = e^{-\frac{\beta^2 + 1}{x}} (_C1 \sin(\beta y) + _C2 \cos(\beta y)) \quad (13)$$

> $Comprobacion_3 := simplify(eval(subs(z(x, y) = rhs(SolUnoNeg), lhs(Ecuacion) - rhs(Ecuacion) = 0)))$

$$Comprobacion_3 := 0 = 0 \quad (14)$$

> $EcuaSepDosX := rhs(EcuaSeparadaDos) = alpha; EcuaSepDosY := lhs(EcuaSeparadaDos) = alpha$

$$\begin{aligned} EcuaSepDosX &:= -\frac{x^2 \left(\frac{d}{dx} F(x) \right)}{F(x)} = \alpha \\ EcuaSepDosY &:= \frac{\frac{d^2}{dy^2} G(y) - G(y)}{G(y)} = \alpha \end{aligned} \quad (15)$$

> $SolDosXcero := dsolve(subs(alpha = 0, EcuaSepDosX)); SolDosYcero := dsolve(subs(alpha$

= 0, EcuSepDosY))

$$\text{SolDosXcero} := F(x) = _C1$$

$$\text{SolDosYcero} := G(y) = _C1 e^{-y} + _C2 e^y \quad (16)$$

> $\text{SolDosCero} := z(x, y) = \text{subs}(_C1 = 1, \text{rhs}(\text{SolDosXcero})) \cdot \text{rhs}(\text{SolDosYcero})$

$$\text{SolDosCero} := z(x, y) = _C1 e^{-y} + _C2 e^y \quad (17)$$

> $\text{Comprobacion}_4 := \text{simplify}(\text{eval}(\text{subs}(z(x, y) = \text{rhs}(\text{SolDosCero}), \text{lhs}(\text{Ecuacion}) - \text{rhs}(\text{Ecuacion}) = 0)))$

$$\text{Comprobacion}_4 := 0 = 0 \quad (18)$$

> $\text{SolDosXpos} := \text{dsolve}(\text{subs}(\text{alpha} = \text{beta} \cdot 2, \text{EcuSepDosX})); \text{SolDosYpos} := \text{dsolve}(\text{subs}(\text{alpha} = \text{beta} \cdot 2, \text{EcuSepDosY}))$

$$\text{SolDosXpos} := F(x) = _C1 e^{\frac{\beta^2}{x}}$$

$$\text{SolDosYpos} := G(y) = _C1 \sin(\sqrt{-1 - \beta^2} y) + _C2 \cos(\sqrt{-1 - \beta^2} y) \quad (19)$$

> $\text{SolDosPos} := z(x, y) = \text{subs}(_C1 = 1, \text{rhs}(\text{SolDosXpos})) \cdot \text{rhs}(\text{SolDosYpos})$

$$\text{SolDosPos} := z(x, y) = e^{\frac{\beta^2}{x}} \left(_C1 \sin(\sqrt{-1 - \beta^2} y) + _C2 \cos(\sqrt{-1 - \beta^2} y) \right) \quad (20)$$

> $\text{Comprobacion}_5 := \text{simplify}(\text{eval}(\text{subs}(z(x, y) = \text{rhs}(\text{SolDosPos}), \text{lhs}(\text{Ecuacion}) - \text{rhs}(\text{Ecuacion}) = 0)))$

$$\text{Comprobacion}_5 := 0 = 0 \quad (21)$$

> $\text{SolDosXneg} := \text{dsolve}(\text{subs}(\text{alpha} = -\text{beta} \cdot 2, \text{EcuSepDosX})); \text{SolDosYneg} := \text{dsolve}(\text{subs}(\text{alpha} = -\text{beta} \cdot 2, \text{EcuSepDosY}))$

$$\text{SolDosXneg} := F(x) = _C1 e^{-\frac{\beta^2}{x}}$$

$$\text{SolDosYneg} := G(y) = _C1 \sin(\sqrt{-1 + \beta^2} y) + _C2 \cos(\sqrt{-1 + \beta^2} y) \quad (22)$$

> $\text{SolDosNeg} := z(x, y) = \text{subs}(_C1 = 1, \text{rhs}(\text{SolDosXneg})) \cdot \text{rhs}(\text{SolDosYneg})$

$$\text{SolDosNeg} := z(x, y) = e^{-\frac{\beta^2}{x}} \left(_C1 \sin(\sqrt{-1 + \beta^2} y) + _C2 \cos(\sqrt{-1 + \beta^2} y) \right) \quad (23)$$

> $\text{Comprobacion}_6 := \text{simplify}(\text{eval}(\text{subs}(z(x, y) = \text{rhs}(\text{SolDosNeg}), \text{lhs}(\text{Ecuacion}) - \text{rhs}(\text{Ecuacion}) = 0)))$

$$\text{Comprobacion}_6 := 0 = 0 \quad (24)$$

> $\text{with}(\text{PDEtools}) :$

> $\text{SolucionGeneral} := \text{build}(\text{pdsolve}(\text{Ecuacion}))$

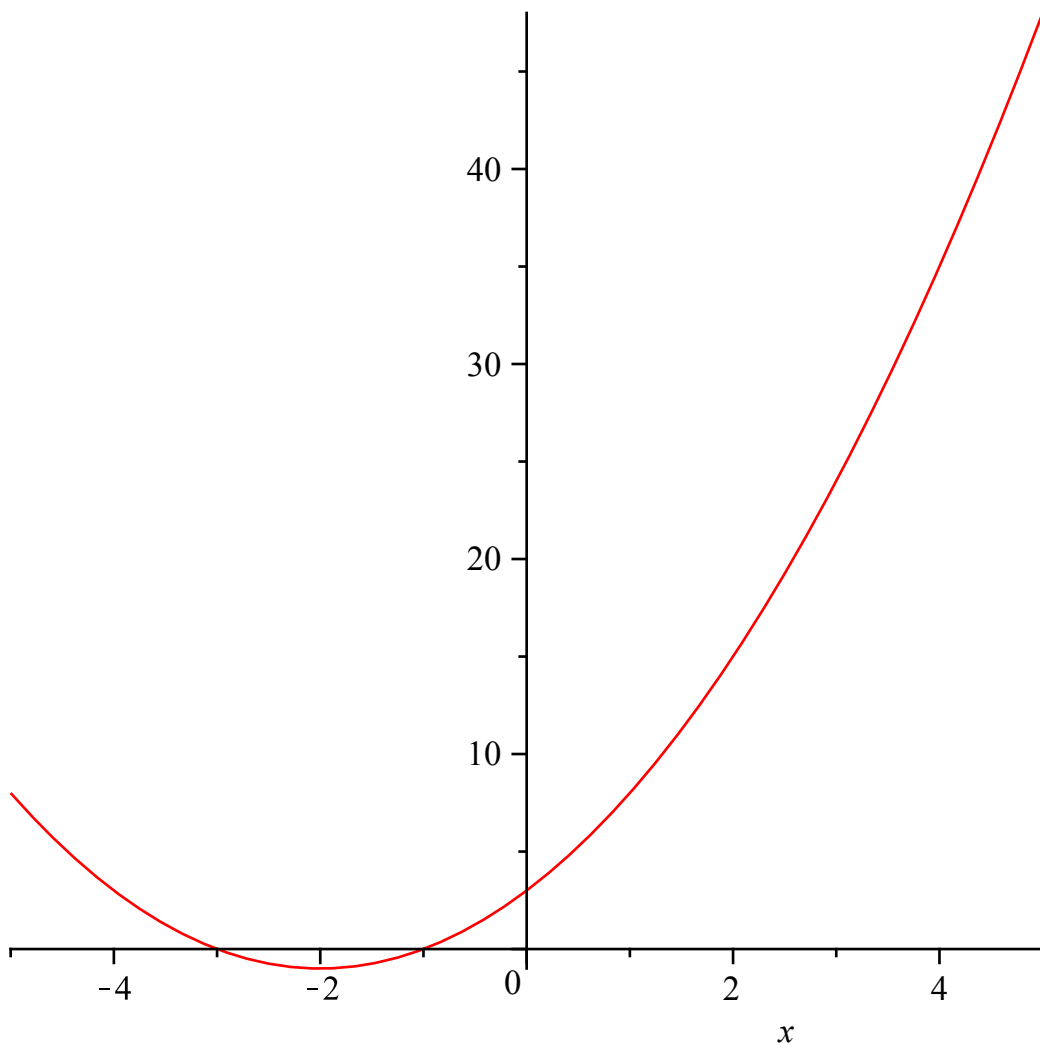
$$\text{SolucionGeneral} := z(x, y) = \frac{_C1 _C2 \sin(\sqrt{_C1 - 1} y)}{e^{\frac{_C1}{x}}} + \frac{_C1 _C3 \cos(\sqrt{_C1 - 1} y)}{e^{\frac{_C1}{x}}} \quad (25)$$

> restart

> $f := x \cdot 2 + 4 \cdot x + 3$

$$f := x^2 + 4x + 3 \quad (26)$$

> $\text{plot}(f, x = -5 \dots 5)$



> $L := 5$

$L := 5$

(27)

> $a_0 := \left(\frac{1}{L}\right) \cdot \text{int}(f, x = -L..L)$

$a_0 := \frac{68}{3}$

(28)

> $C := \frac{a_0}{2}$

$C := \frac{34}{3}$

(29)

> $a_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, \left(\frac{1}{L}\right) \cdot \text{int}\left(f \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

$a_n := \frac{100 \cdot (-1)^n}{n^2 \cdot 2}$

(30)

> $b_n := \text{subs}\left(\sin(n \cdot \text{Pi}) = 0, \cos(n \cdot \text{Pi}) = (-1) \cdot n, \left(\frac{1}{L}\right) \cdot \text{int}\left(f \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), x = -L..L\right)\right)$

(31)

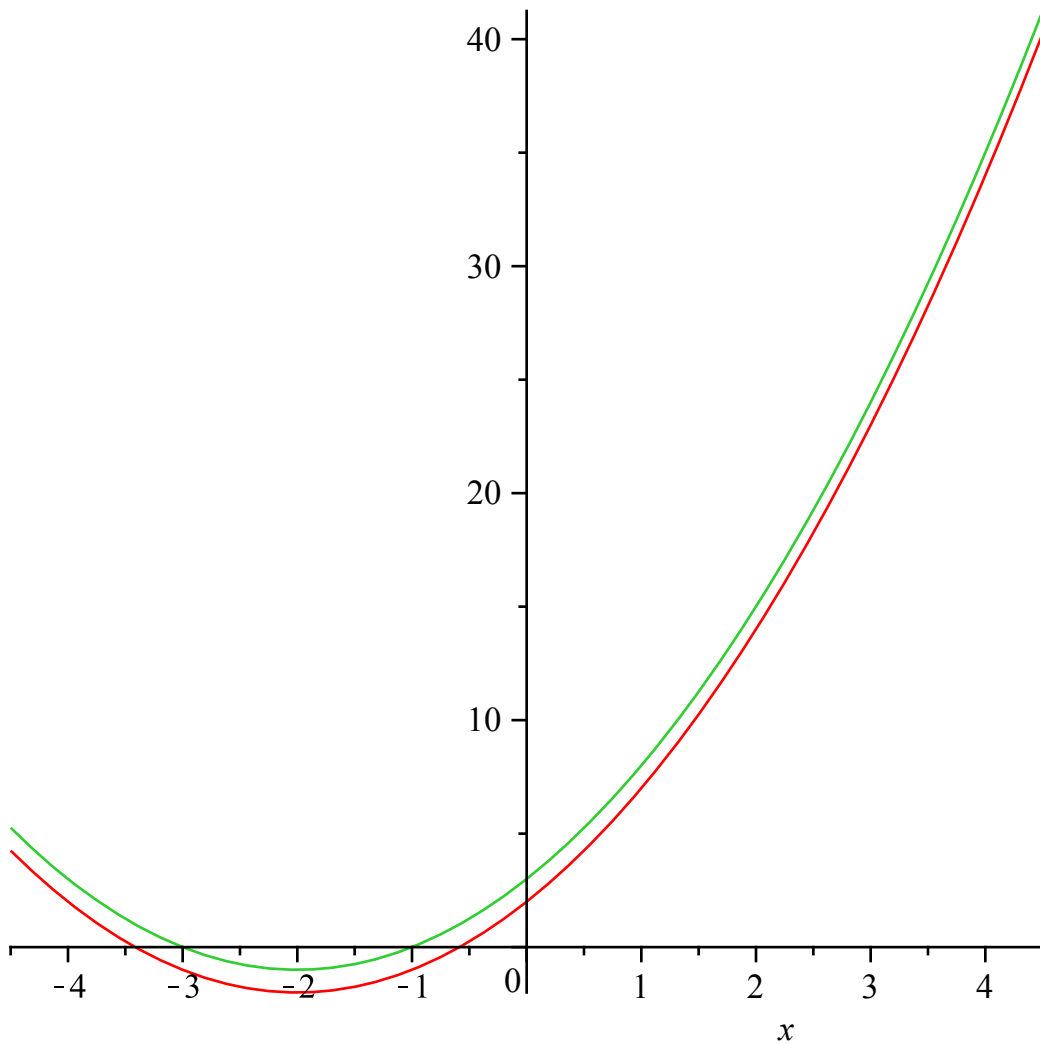
$$b_n := -\frac{40 (-1)^n}{n \pi} \quad (31)$$

$$> STF := C + Sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1 .. \text{infinity}\right)$$

$$STF := \frac{34}{3} + \sum_{n=1}^{\infty} \left(\frac{100 (-1)^n \cos\left(\frac{1}{5} n \pi x\right)}{n^2 \pi^2} - \frac{40 (-1)^n \sin\left(\frac{1}{5} n \pi x\right)}{n \pi} \right) \quad (32)$$

$$> STF_{5000} := C + sum\left(a_n \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right) + b_n \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot x}{L}\right), n = 1 .. 5000\right) :$$

$$> plot([f - 1, STF_{5000}], x = -4.5 .. 4.5)$$



$$> plot([f, STF_{5000}], x = -2.02 .. -1.98)$$

