Ecuaciones Diferenciales Ordinarias (2°)
Lineales de coef. constantes
No-homogéneas.

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = Q(x) \qquad Q(x) \neq 0.$$

$$y_g = C_1 y_1^{P} + C_2 y_2^{P} + y_3^{P}$$

$$y_g = C_1 y_1^{(p)} + C_2 y_2^{(p)} + y_3^{(p)}$$

FUNDAMENTALES

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = Q(x)$$

y_1, y_2: e^{mx}; x^n $n \in E^{(+)}$ (serie completa); $\begin{cases}\cos(bx)\\ \operatorname{sen}(bx)\end{cases}$ (por pareja) $b \in \mathbb{R}^+$

C.C.

$$y = C_1 e^{2x} + C_2 e^{3x} + \underbrace{e^{4x} + e^{5x}}_{y_3^{(p)}}$$

$y_1^{(p)}$, $y_2^{(p)}$

$$(m-2)(m-3)=0$$

$$m^2 - 5m + 6 = 0$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$y_p = e^{4x} + e^{5x}$$

$$y' = 4e^{4x} + 5e^{5x}$$

$$y'' = 16e^{4x} + 25e^{5x}$$

$$Q = \left[16e^{4x} + 25e^{5x}\right] - 5\left[4e^{4x} + 5e^{5x}\right] + 6\left[e^{4x} + e^{5x}\right]$$

$$Q = (16 - 20 + 6)e^{4x} + (25 - 25 + 6)e^{5x}$$

$$Q = 2e^{4x} + 6e^{5x}$$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 2e^{4x} + 6e^{5x}$$

$$y_g = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x} + 8x^3e^{-x}$$

$$y_g = e^{-x}\left(c_1 + c_2x + c_3x^2 + 8x^3\right)$$

$$\frac{d^3y}{dx^3} + a_1\frac{d^2y}{dx^2} + a_2\frac{dy}{dx} + a_3y = Q(x)$$

EDO(3) L cc NH

CASO II $\qquad m_1 = m_2 = m_3 \Rightarrow -1$

$$(m+1)^3 = 0$$

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = Q \leftarrow$$

$$y_p = 8x^3e^{-x} \rightarrow y' = 8\left(-x^3e^{-x} + 3x^2e^{-x}\right)$$

$$y' = -8x^3e^{-x} + 24x^2e^{-x}$$

$$y'' = -8\left(-x^3e^{-x} + 3x^2e^{-x}\right) + 24\left(-x^2e^{-x} + 2xe^{-x}\right)$$

$$y'' = 8x^3e^{-x} - 48x^2e^{-x} + 48xe^{-x}$$

$$y''' = 8\left(-x^3e^{-x} + 3x^2e^{-x}\right) - 48\left(-x^2e^{-x} + 2xe^{-x}\right) + 48\left(-xe^{-x} + e^{-x}\right)$$

$$y''' = -8x^3e^{-x} + 72x^2e^{-x} - 144xe^{-x} + 48e^{-x}$$

$$\begin{array}{l|l} y''' & -8x^3e^{-x} + 72x^2e^{-x} - 144xe^{-x} + 48e^{-x} \\ + & \\ 3y'' & 24x^3e^{-x} - 144x^2e^{-x} + 144xe^{-x} \\ + & \\ 3y' & -24x^3e^{-x} + 72x^2e^{-x} \\ + & \\ y & 8x^3e^{-x} \\ \hline Q & = (0)x^3e^{-x} + (0)x^2e^{-x} + (0)xe^{-x} + 48e^{-x} \end{array}$$

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 48e^{-x}.$$

$$y = e^{-x}\left(c_1 + c_2x + c_3x^2\right) + 8x^3e^{-x}.$$

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

EDO(3)L cc H

1
1 1
1 2 1
1 3 3 1

$$m^3 - 3m^2 + 3m - 1 = 0$$

$$(m-1)^3 = 0 \qquad m_1 = m_2 = m_3 = 1 \quad \text{CASO II.}$$

$$e^{mx} \xrightarrow{m=1} e^x$$

$$\frac{d}{dm}$$

$$xe^{mx} \xrightarrow{m=1} xe^x$$

$$\frac{d}{dm}$$

$$x^2e^{mx} \xrightarrow{m=1} x^2e^x$$

$$y = C_1e^x + C_2xe^x + C_3x^2e^x$$

CC

$$y = C_1xe^x + C_2x^2e^x + C_3x^3e^x \cdot \times$$

CV

$$\frac{dy}{dx} + a_1 y = q(x) \qquad y_g = C_1 e^{-a_1 x} + e^{-a_1 x} \int e^{a_1 x} q \, dx.$$

$$\frac{dy}{dx} + p(x)\, y = q(x) \qquad y_g = C_1 e^{-\int p\,dx} + e^{-\int p\,dx} \int e^{\int p\,dx} q \, dx.$$

$$y_g = \left(C_1 + \int e^{a_1 x} q \, dx\right) e^{-a_1 x} \qquad \begin{cases} y_H = C_1 y_1 \\ y_{NH} = (C_1 + y_p)\, y_1 \end{cases}$$

$$y_g = \left(C_1 + \int e^{+\int p\,dx} q(x)\, dx\right) e^{-\int p\,dx} \qquad \begin{cases} y_H = C_1 y_1 \\ y_{NH} = (C_1 + y_p)\, y_1 \end{cases}$$

Si tenemos EDO(n) L $\begin{Bmatrix} CC \\ CV \end{Bmatrix}$ NH.

$\downarrow$

Conocemos EDO(n) L $\begin{Bmatrix} CC \\ CV \end{Bmatrix}$ H asociada

podemos resolverla por el.

MÉTODO DE COEFICIENTES (PARÁMETROS) VARIABLES

MPV

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 5e^{-3x}$$

EDO(2) L cc NH.

Encontrar la sol. homogénea asociada

① $$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$$ Homogénea asociada

$m^2 - 6m + 8 = 0$ $\quad m_1 = 2$

$(m-2)(m-4) = 0$ $\quad m_2 = 4$ $\Big\}$ CASO I. $m_1 \neq m_2$

$$y_g = c_1 e^{2x} + c_2 e^{4x}$$ solución general de la homogénea asociada

$$\longrightarrow y_{n-h} = A(x)\, e^{2x} + B(x) e^{4x}$$

$$y_{n-h} = u_1(x) e^{2x} + u_2(x) e^{4x}$$

MÉTODO DE LOS PARÁMETROS VARIABLES

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 5e^{-3x}$$

$$\boxed{y_H = C_1 e^{2x} + C_2 e^{4x}}$$

$$y_{NH} = A(x)e^{2x} + B(x)e^{4x}$$

$$\frac{d}{dx}$$

$$\frac{dy}{dx} = 2A(x)e^{2x} + 4B(x)e^{4x} + \underbrace{\boxed{A'(x)e^{2x} + B'(x)e^{4x}}}_{=0}$$

$$\frac{dy}{dx} = 2A(x)e^{2x} + 4B(x)e^{4x} + (0)$$

$$\frac{d}{dx}$$

$$\frac{d^2y}{dx^2} = 4A(x)e^{2x} + 16B(x)e^{4x} + \underbrace{\boxed{2A'(x)e^{2x} + 4B'(x)e^{4x}}}_{=Q(x)}$$

$$\frac{d^2y}{dx^2} = 4A(x)e^{2x} + 16B(x)e^{4x} + Q(x)$$

$$\left[4A(x)e^{2x} + 16B(x)e^{4x} + Q(x)\right] - 6\left[2A(x)e^{2x} + 4B(x)e^{4x}\right] + 8\left[A(x)e^{2x} + B(x)e^{4x}\right] = 5e^{-x}$$

$$(4-12+8)A(x)e^{2x} + (16-24+8)B(x)e^{4x} + Q(x) = 5e^{-3x}$$

$$(0)A(x)e^{2x} + (0)B(x)e^{4x} + Q(x) = 5e^{-3x}$$

$$Q(x) = 5e^{-3x}$$

$$A'(x)e^{2x} + B'(x)e^{4x} = 0$$
$$2A'(x)e^{2x} + 4B'(x)e^{4x} = 5e^{-3x}$$

$$\begin{array}{ccc} WW & PRIMAS & BB \\ \begin{bmatrix} e^{2x} & e^{4x} \\ 2e^{2x} & 4e^{4x} \end{bmatrix} & \begin{bmatrix} A'(x) \\ B'(x) \end{bmatrix} = & \begin{bmatrix} 0 \\ 5e^{-3x} \end{bmatrix} \end{array}$$

$$A'(x) = \frac{\begin{vmatrix} 0 & e^{4x} \\ 5e^{-3x} & 4e^{4x} \end{vmatrix}}{\begin{vmatrix} e^{2x} & e^{4x} \\ 2e^{2x} & 4e^{4x} \end{vmatrix}} \Rightarrow \frac{-5e^{-3x}e^{4x}}{2e^{2x}e^{4x}} \Rightarrow \boxed{-\frac{5}{2}e^{-5x}}$$

$$B'(x) = \frac{\begin{vmatrix} e^{4x} & 0 \\ 2e^{2x} & 5e^{-3x} \end{vmatrix}}{2e^{2x}e^{4x}} \Rightarrow \frac{5e^{-3x}e^{2x}}{2e^{2x}e^{4x}} \Rightarrow \boxed{\frac{5}{2}e^{-7x}}$$

$$A'(x) = -\frac{5}{2}e^{-5x} \longrightarrow A(x) = -\frac{5}{2}\int e^{-5x}dx$$
$$A(x) = -\frac{5}{2}\left(\frac{e^{-5x}}{-5}\right) + C_1$$
$$\boxed{A(x) = \frac{e^{-5x}}{2} + C_1}$$

$$B'(x) = \frac{5}{2}e^{-7x} \longrightarrow B(x) = \frac{5}{2}\int e^{-7x}dx$$
$$= \frac{5}{2}\left(\frac{e^{-7x}}{-7}\right) + C_2$$
$$\boxed{B(x) = -\frac{5}{14}e^{-7x} + C_2}$$

$$y_{g/NH} = \left(\frac{e^{-5x}}{2} + C_1\right)e^{2x} + \left(-\frac{5}{14}e^{-7x} + C_2\right)e^{4x}$$
$$y_{NH} = C_1e^{2x} + C_2e^{4x} + \frac{e^{-3x}}{2} - \frac{5}{14}e^{-3x}$$
$$y_{g/NH} = C_1e^{2x} + C_2e^{4x} + \left(\frac{1}{2} - \frac{5}{14}\right)e^{-3x}$$
$$\boxed{y = C_1e^{2x} + C_2e^{4x} + \frac{e^{-3x}}{7}}$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 5e^{-3x}$$