Capítulo 2. EDO LINEAL

$$a_0(x)\frac{d^n y}{dx^n}+a_1(x)\frac{d^{n-1}y}{dx^{n-1}}+\dots\dots+a_{n-1}(x)\frac{dy}{dx}+a_n(x)y=Q(x)$$

incógnita

EDO(n) L cv NH. $y(x)$ v.i.

EDO(n)
- Lineal
  - Homogéneas $Q(x)=0$
  - No Homogeneas $Q(x)\neq 0$
    - Coef. variables
    - coef. constantes $\forall a_i(x)=k_i$, $k_i \in \mathbb{R}$
- No Lineal. ×

$$\frac{dy}{dx} = 0$$

$a_0(x) = 1$

$a_1(x) = 0$ EDO(1)L c.c. H.

$Q(x) = 0$

$$\frac{dy}{dx} + 2y = 0 \quad \text{EDO(1) L cc H}$$

$a_0(x) = 1$

$a_1(x) = 2$

$Q(x) = 0$

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \quad \text{EDO(2) L cc H.}$$

$a_0(x) = 1$

$a_1(x) = -5$

$a_2(x) = 6$

$Q(x) = 0$

$$\frac{dy}{dx} + \frac{y}{x} = 0$$

$a_0(x) = 1$

$a_1(x) = \frac{1}{x}$ EDO(1) L $\underline{\underline{cv}}$ H.

$Q(x) = 0$

$$\frac{d^2y}{dx^2} = -g$$

$a_0(x) = 1$ $a_1(x) = 0$ $a_2(x) = 0$ $Q(x) = -g$

EDO(2) L cc. $\underline{NH}$

$$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 8y = 5e^{3x} + 4x$$

$a_0(x) = x^2$

$a_1(x) = -6x$ $Q(x) = 5e^{3x} + 4x$ EDO(2) L cv NH

$a_2(x) = 8$

EDO(1)LCCH

Regla de Oro de la resolución de las Lineales:

"Procurar - siempre - que el coeficiente de la derivada de mayor orden sea igual a la unidad"

$$x^2\frac{d^2y}{dx^2}-6x\frac{dy}{dx}+8y=5e^{3x}+4x$$

$$\frac{d^2y}{dx^2}-\frac{6}{x}\frac{dy}{dx}+\frac{8}{x^2}y=\frac{5e^{3x}}{x^2}+\frac{4}{x}$$

EDO(1) L cc H.

$$2\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}+\frac{1}{2}y=0 \longrightarrow \boxed{y=c_1e^{-\frac{1}{2}x}}$$

$$\frac{dy}{dx}+a_1y=0 \qquad a_1\in\mathbb{R}$$

$$\frac{dy}{dx}=-a_1y$$

$$\frac{dy}{y}=-a_1dx$$

$$\int\frac{dy}{y}=-a_1\int dx$$

$$\ln y+k_1=-a_1(x+k_2)$$

$$\ln y=-a_1x+(-a_1k_2-k_1)$$

$$y=e^{-a_1x+(-a_1k_2-k_1)}$$

$$y=e^{-a_1x}\cdot e^{-a_1k_2-k_1}$$

$$=e^{-a_1x}\cdot c_1$$

$$\boxed{y=c_1e^{-a_1x}}$$ SOLUCIÓN GENERAL

$$\frac{dy}{dx} - \sqrt{2}\, y = 0 \quad \text{EDO(1)LCC H.}$$

$$\boxed{y = C_1 e^{\sqrt{2}x}}$$

SOLUCIÓN GENERAL

como el orden ED es uno la solución general tendrá una sola constante arbitraria asociada a una función "solución particular" conocida como "fundamental" $\rightarrow$ $\boxed{y_p = e^{\sqrt{2}x}}$

$$y_p = \sqrt{3}\, e^{\sqrt{2}x}$$

EDO(3)L CC H.

SOLUCIÓN GENERAL $y = c_1 y_1 + c_2 y_2 + c_3 y_3$

si $c_1 = 1 \quad c_2 = c_3 = 0$ $\qquad W = [y_1 \; y_2 \; y_3] \neq 0$

$y_p = y_1$

si $c_2 = 1 \quad c_1 = c_3 = 0$

$y_p = y_2$

Si $c_3 = 1 \quad c_1 = c_2 = 0$

$y_p = y_3$

élite.

$$y_p = \sqrt{5}\, y_1 + \cos(3\pi)\, y_2 + \frac{8}{4} y_3$$

$W[y_p \;\; y_1] \neq 0$ vulgares.

EDO(1)L cc H.

$$\frac{dy}{dx} + a_1 y = 0 \longrightarrow SG \quad y = C_1 e^{-a_1 x}$$

EDO(1)L cv H.

$$\frac{dy}{dx} + p(x) y = 0$$

$$\frac{dy}{dx} = -p(x) y$$

$$\frac{dy}{y} = -p(x) dx$$

$$\int \frac{dy}{y} = -\int p(x) dx$$

$$Ly + k_1 = -\int p(x) dx + k_2$$

$$Ly = -\int p(x) dx + (k_2 - k_1)$$

$$y = e^{-\int p(x) dx + (k_2 - k_1)}$$

$$y = e^{-\int p(x) dx} \cdot e^{(k_2 - k_1)}$$

$$\boxed{y = C_1 e^{-\int p(x) dx}}$$

SOLUCIÓN GENERAL

$$\frac{dy}{dx} + \frac{y}{x} = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x}\right) y = 0$$

$$p(x) = \frac{1}{x}$$

$$\boxed{a_0(x) \frac{dy}{dx} + a_1(x) y = Q(x)}$$

$$\int p(x) dx = \int \frac{dx}{x} \Rightarrow Lx$$

$$y = C_1 e^{-Lx}$$

$$y = C_1 e^{L(x)^{-1}}$$

$$y = C_1 x^{-1}$$

SG $\boxed{y = \frac{C_1}{x}}$

$$y = e^{La}$$

$$Ly = Le^{La}$$

$$Ly = La \cdot \cancel{Le}^{1}$$

$$Ly = La$$

$$y = a$$