EDO(2)L cc H.

$$H_0 \quad \boxed{\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0}$$

$$y_p = e^{mx}$$

$$\frac{dy}{dx} = m e^{mx}$$

$$\frac{d^2y}{dx^2} = m^2 e^{mx}$$

$$[m^2 e^{mx}] + a_1 [m e^{mx}] + a_2 [e^{mx}] = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0$$

$$e^{mx} = 0 \quad y = 0 \text{ trivial. } \times$$

$$\boxed{m^2 + a_1 m + a_2 = 0} \qquad \left.\begin{matrix} m_1 \\ m_2 \end{matrix}\right\} \text{raíces.}$$

ECUACIÓN CARACTERÍSTICA.

Raíces:

Tipo I	Tipo II	Tipo III
$m_1 \neq m_2$	$m_1 = m_2$	$m_1, m_2 \in \mathbb{C}$
$m_1, m_2 \in \mathbb{R}$	$m_1, m_2 \in \mathbb{R}$	$\left.\begin{matrix} m_1 = a + bi \\ m_2 = a - bi \end{matrix}\right\} m_1 \neq m_2$
		$a \in \mathbb{R}$
		$b \in \mathbb{R}^+$

$$\frac{d^2y}{dx^2}+a_1\frac{dy}{dx}+a_2y=0 \qquad y_p=e^{mx}$$

$$m^2+a_1m+a_2=0 \qquad \text{EDO}(2)\,L\,\underline{CC}\,H$$

<u>TIPO I</u> $\quad m_1 \neq m_2 \in \mathbb{R}$

$$y_1^{(p)}=e^{m_1x} \qquad y_2^{(p)}=e^{m_2x}$$

$$y_g=C_1e^{m_1x}+C_2e^{m_2x} \qquad W=\begin{vmatrix} e^{m_1x} & e^{m_2x} \\ m_1e^{m_1x} & m_2e^{m_2x}\end{vmatrix}\neq 0$$

$$\frac{dy}{dx}=y \qquad e^x$$

$$m_2e^{m_1x}e^{m_2x}-m_1e^{m_1x}e^{m_2x}\neq 0$$

$$(m_2-m_1)e^{m_1x}e^{m_2x}\neq 0 \quad \left.\begin{matrix} e^{m_1x}\neq 0 \\ e^{m_2x}\neq 0\end{matrix}\right.$$

$$m_2-m_1\neq 0$$

$$m_2\neq m_1$$

normalizada o estandarizada

EDO(2)LccH.

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

TIPO I

$m_1 = 2$, $m_2 = 3$ $\}$ $m_1 \neq m_2$

$m_1, m_2 \in \mathbb{R}$

EC. $m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$y_1 = e^{2x}$ $\quad y_2 = e^{3x}$

$$\boxed{y_g = c_1 e^{2x} + c_2 e^{3x}}$$

$$y_g = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$$

$m_1 = \sqrt{2}$

$m_2 = -\sqrt{2}$

$m_1 \neq m_2$

EC $\Rightarrow$ $(m-\sqrt{2})(m+\sqrt{2}) = 0$

$m^2 - 2 = 0$

EDO(2)LccH. $\boxed{\frac{d^2y}{dx^2} - 2y = 0}$

$$y_g = c_1 e^{\frac{x}{2}} + c_2 e^{\frac{x}{3}}$$

$m_1 = \frac{1}{2}$

$m_2 = \frac{1}{3}$ $\quad m_1 \neq m_2$

$\left(m - \frac{1}{2}\right)\left(m - \frac{1}{3}\right) = 0$

$m^2 - \frac{1}{3}m + \frac{1}{6} = 0$

$$\frac{d^2y}{dx^2} - \frac{1}{3}\frac{dy}{dy} + \frac{1}{6}y = 0$$

EDO(2)LccH.

Caso III.- $m_1, m_2 \in \mathbb{C}$

$$m_1 \neq m_2$$

$$\frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

$$m^2 + a_1 m + a_2 = 0 \qquad m_1, m_2 \in \mathbb{C} \quad \begin{matrix} m_1 = a + bi \\ m_2 = a - bi \end{matrix}$$

$$y_1 = e^{(a+bi)x} \qquad y_2 = e^{(a-bi)x}$$

$$y_g = c_1 e^{(a+bi)x} + c_2 e^{(a-bi)x} \quad \left. \right\} \begin{matrix} x \in \mathbb{R} \\ y \in \mathbb{R} \end{matrix}$$

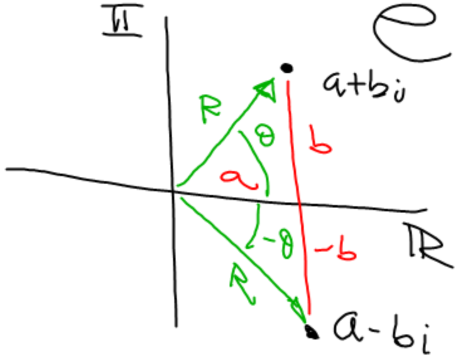
Euler

$$e^{\pi i} + 1 = 0$$

$e = 2.718 \ldots$

$\pi = 3.1415 \ldots$

$i = \sqrt{-1}$



$$R = \sqrt{a^2 + b^2}$$

$$a = R\cos(\theta)$$

$$b = R\,\mathrm{sen}(\theta)$$

$$\begin{cases} e^{\theta i} = \cos(\theta) + \mathrm{sen}(\theta)\, i \\ e^{-\theta i} = \cos(\theta) - \mathrm{sen}(\theta)\, i \end{cases}$$

Tipo III

$x \in \mathbb{R}$
$y \in \mathbb{R}$

$$y_g = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y_g = C_1 e^{ax} e^{bxi} + C_2 e^{ax} e^{-bix}$$

$$y_g = e^{ax}\left(C_1 e^{bxi} + C_2 e^{-bxi}\right)$$

$$y_g = e^{ax}\left(C_1\left[\cos(bx) + \operatorname{sen}(bx)\,i\right] + C_2\left[\cos(bx) - \operatorname{sen}(bx)\,i\right]\right)$$

$$y_g = e^{ax}\left(\left[C_1 + C_2\right]\cos(bx) + \left[C_1 i - C_2 i\right]\operatorname{sen}(bx)\right)$$

$$y_g = e^{ax}\left(C_{10}\cos(bx) + C_{20}\operatorname{sen}(bx)\right)$$

$$\boxed{y_g = C_{10} e^{ax}\cos(bx) + C_{20} e^{ax}\operatorname{sen}(bx)}$$

Caso III

$$\left.\begin{array}{l} m_1 = a + bi \\ m_2 = a - bi \end{array}\right\}$$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

$$m^2 - 2m + 2 = 0$$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$m = 1 \pm \frac{\sqrt{4}\sqrt{-1}}{2} \Rightarrow m = 1 \pm i$$

$$y = C_1 e^{x}\cos(x) + C_2 e^{x}\operatorname{sen}(x) \qquad \begin{matrix} a = 1 \\ b = 1 \end{matrix}$$

$$\frac{dy}{dx} = -C_1 e^{x}\operatorname{sen}(x) + C_1 e^{x}\cos(x) + C_2 e^{x}\cos(x) + C_2 e^{x}\operatorname{sen}(x)$$

$$\frac{dy}{dx} = (C_1 + C_2)e^{x}\cos(x) + (-C_1 + C_2)e^{x}\operatorname{sen}(x)$$

$$\frac{d^2y}{dx^2} = (C_1 + C_2)\left(-e^{x}\operatorname{sen}(x) + e^{x}\cos(x)\right) + (-C_1 + C_2)\left(e^{x}\cos(x) + e^{x}\operatorname{sen}(x)\right)$$

$$\frac{d^2y}{dx^2} = (C_1 + C_2 - C_1 + C_2)e^{x}\cos(x) + (-C_1 - C_2 - C_1 + C_2)e^{x}\operatorname{sen}(x)$$

$$\frac{d^2y}{dx^2} = 2C_2 e^{x}\cos(x) - 2C_1 e^{x}\operatorname{sen}(x)$$

$\frac{d^2y}{dx^2}$	$2C_2 e^{x}\cos(x) - 2C_1 e^{x}\operatorname{sen}(x)$
$+$	
$-2\frac{dy}{dx}$	$(-2C_1 - 2C_2)e^{x}\cos(x) + (2C_1 - 2C_2)e^{x}\operatorname{sen}(x)$
$+$	
$2y$	$2C_1 e^{x}\cos(x) + 2C_2 e^{x}\operatorname{sen}(x)$
$=$	
0	$(0)e^{x}\cos(x) + (0)e^{x}\operatorname{sen}(x)$

$$y = C_1 y_1 + C_2 y_2$$

$$\boxed{y = C_1 \cos(3x) + C_2 \operatorname{sen}(3x)}$$

$$(m - 3i)(m + 3i) = 0$$

$$m_1 = a + bi$$

$$m_2 = a - bi$$

$$a = 0 \quad a \in \mathbb{R}$$

$$b = 3 \quad b \in \mathbb{R}^+$$

$$(m^2 - (3i)^2) = 0$$

$$m^2 + 9 = 0$$

$$\boxed{\frac{d^2y}{dx^2} + 9y = 0}$$