EDO(2) L cc H.

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \qquad H. \quad y = e^{mx}$$

$$m^2 + a_1 m + a_2 = 0$$

Caso I.- $m_1 \neq m_2 \in \mathbb{R}$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x}$$

Caso III.- $m_1 \neq m_2 \in \mathbb{C}$ $\quad m_1 = a+bi \quad m_2 = a-bi \quad i = \sqrt{-1}$

$$y_g = C_1 e^{ax}\cos(bx) + C_2 e^{ax}\operatorname{sen}(bx) \qquad a \in \mathbb{R},\ b \in \mathbb{R}^+$$

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Caso II.- $m_1 = m_2 \in \mathbb{R}$ $\quad y_1 \overset{P}{=} e^{m_1 x}$

$$y_g = C_1 e^{m_1 x} + C_2 e^{m_2 x} \longrightarrow m_1 = m_2 \text{ no hay independencia lineal}$$

$$y_g = (C_1 + C_2) e^{m_1 x} \Rightarrow y_g = C_3 e^{m_1 x} + C_4 y_2 \quad ?$$

$$\left.\begin{array}{l} m^2 + a_1 m + a_2 = 0 \\ (m-m_1)^2 = 0 \end{array}\right\} \begin{array}{l} \frac{d}{dm} \rightarrow \boxed{2m + a_1 = 0} \\ \frac{d}{dm} \rightarrow 2(m-m_1) = 0 \end{array}$$

---

$$(m-m_1)(m-m_2) = 0 \qquad m_1 \neq m_2$$

$$\frac{d}{dm} \rightarrow (m-m_1) + (m-m_2) = 0$$

$$m^2 + 6m + 8 = 0$$

$$(m+2)(m+4) = 0$$

$$m_1 = -2$$

$$m_2 = -4$$

$$(m-(-2))(m-(-4)) = 0$$

---

$$m^2 - 6m + 9 = 0$$

$$(m-3)(m-3) = 0$$

$$(m-3)^2 = 0$$

$$m_1 = 3$$

$$m_2 = 3$$

$$m_1 = m_2$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0 \qquad H \Rightarrow e^{mx}$$

$$m^2 - 6m + 9 = 0 \qquad \left.\begin{matrix} m_1 = 3 \\ m_2 = 3 \end{matrix}\right\} \; m_1 = m_2$$

$$e^{mx} \xrightarrow{m_1 = 3} e^{3x}$$

$$\frac{d}{dm}\downarrow$$

$$xe^{mx} \xrightarrow{m_1 = 3} \boxed{xe^{3x}}$$

L. I.

Caso II $\quad y_g = C_1 e^{3x} + C_2 xe^{3x}$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

$y_{(P)} = e^{3x}$

$\frac{dy}{dx} = 3e^{3x}$

$\frac{d^2y}{dx^2} = 9e^{3x}$

$$\left[9e^{3x}\right] - 6\left[3e^{3x}\right] + 9\left[e^{3x}\right] = 0$$

$$(9 - 18 + 9)e^{3x} = 0$$

$$(0)e^{3x} = 0$$

$$0 \equiv 0$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

$$y_2 \overset{(P)}{=} xe^{3x}$$

$$\frac{dy}{dx} = 3xe^{3x} + e^{3x}$$

$$\frac{d^2y}{dx^2} = 3\left(3xe^{3x} + e^{3x}\right) + 3e^{3x}$$

$$\frac{d^2y}{dx^2} = 9xe^{3x} + 6e^{3x}$$

$$\left[9xe^{3x} + 6e^{3x}\right] - 6\left[3xe^{3x} + e^{3x}\right] + 9\left[xe^{3x}\right] = 0$$

$$(9 - 18 + 9)xe^{3x} + (+6 - 6)e^{3x} = 0$$

$$(0)xe^{3x} + (0)e^{3x} = 0$$

$$0 \equiv 0$$

$$\Rightarrow$$

$$\begin{vmatrix} xe^{3x} & e^{3x} \\ 3xe^{3x}+e^{3x} & 3e^{3x} \end{vmatrix} \neq 0$$

$$3xe^{3x}e^{3x} - 3xe^{3x}e^{3x} - e^{3x}e^{3x} \neq 0$$

$$-e^{3x}e^{3x} \neq 0$$

$$y_g = c_1 e^{3x} + c_2 x e^{3x}$$

$$y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x} \quad \text{CASO II.} \quad m_1 = m_2$$

$$\frac{d^2y}{dx^2}=0 \qquad \frac{d}{dx}\left(\frac{dy}{dx}\right)=0 \qquad \frac{dy}{dx}=C_1$$

EDO(2) homogénea.

$$\int dy = C_1\int dx$$

$$m^2=0 \qquad m_1=m_2=0$$

$$y+k_1=C_1(x+k_2)$$

Caso II.

$$y=C_1x+(C_1k_2-k_1)$$

$$y_g=C_1e^{m_1x}+C_2xe^{m_1x}$$

$$\boxed{y=C_1x+C_2}$$

$$y_g=C_1+C_2x$$

$y = e^{mx}$

$y_g = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + \dots + c_n e^{m_n x}$

EDO(n) L cc H.

$(m-m_1)(m-m_2)\dots\dots(m-m_n)=0$

---

II $y_1 = e^{m_1 x}$ $y_2 = x e^{m_1 x}$ $\dots y_3 = x^2 e^{m_1 x}$ $\dots\dots y_n = x^{n-1} e^{m_1 x}$

$y_g = c_1 e^{m_1 x} + c_2 x e^{m_1 x} + c_3 x^2 e^{m_1 x} + \dots + c_n x^{n-1} e^{m_1 x}$

$m_1 = m_2 = m_3 = \dots = m_n$

$(m-m_1)^n = 0$

EDO(n) L cc H.

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III $y_1 = e^{a_1 x}\cos(b_1 x)$ $y_2 = e^{a_1 x}\operatorname{sen}(b_1 x)$

$y_3 = e^{a_2 x}\cos(b_2 x)$ $y_4 = e^{a_2 x}\operatorname{sen}(b_2 x)$

$y_{n-1} = e^{a_{\frac{n}{2}} x}\cos(b_{\frac{n}{2}} x)$ $y_n = e^{a_{\frac{n}{2}} x}\operatorname{sen}(b_{\frac{n}{2}} x)$

---

$y_g = c_1 e^{a_1 x}\cos(b_1 x) + c_2 e^{a_1 x}\operatorname{sen}(b_1 x) + \dots\dots$

$\dots\dots + c_{n-1} e^{a_{\frac{n}{2}} x}\cos(b_{\frac{n}{2}} x) + c_n e^{a_{\frac{n}{2}} x}\operatorname{sen}(b_{\frac{n}{2}} x)$

$(m-(a_1+b_1 i))(m-(a_1-b_1 i))\dots\dots(m-(a_{\frac{n}{2}}+b_{\frac{n}{2}} i))(m-(a_{\frac{n}{2}}-b_{\frac{n}{2}} i))=0$

$y = e^{ax}\cos(bx)$ $y = e^{ax}\operatorname{sen}(bx)$

$a_1 = a_2$

$b_1 = b_2$

$y_g = c_1 e^{ax}\cos(bx) + c_2 e^{ax}\operatorname{sen}(bx) + c_3 x e^{ax}\cos(bx) + c_4 x e^{ax}\operatorname{sen}(bx)$

$(m-(a+bi))^2(m-(a-bi))^2=0$

EDO(4) L cc H

$$y = C_1 e^{-x}\cos(2x) + C_2 e^{-x}\operatorname{sen}(2x)$$

EDO(2) $\rightarrow$ CV H.

$\downarrow$

CC

$e^{mx}$

$x^n$

$\begin{cases} \cos(bx) \\ \operatorname{sen}(bx) \end{cases}$

$\rightarrow$ C.C H.