Capítulo 3.- Sistemas

$$3x + 4y = 8$$
$$2x + 5y = -10$$

$$\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -10 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} 8 & 4 \\ -10 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}} \Rightarrow \frac{80}{7} \qquad y = \frac{\begin{vmatrix} 3 & 8 \\ 2 & -10 \end{vmatrix}}{7} \Rightarrow \frac{-46}{7}$$

$$3\left(\frac{80}{7}\right) + 4\left(-\frac{46}{7}\right) = 8$$

$$\frac{240 - 184}{7} = 8$$

$$\frac{56}{7} = 8$$

$$8 \equiv 8$$

$$2\left(\frac{80}{7}\right) + 5\left(-\frac{46}{7}\right) = -10$$

$$\frac{160 - 230}{7} = -10$$

$$-\frac{70}{7} = -10$$

$$-10 = -10$$

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = x + 4y$$

Sistema. (2). EDO(1)L CC. Homogéneo

Método de Sustitución.

X (Método del Operador Diferencial)

Método de la Matriz Exponencial

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = x + 4y$$

$$x = \frac{dy}{dt} - 4y$$

$$\frac{d}{dt} \qquad \frac{dx}{dt} = \frac{d^2y}{dt^2} - 4\frac{dy}{dt}$$

$$\left[\frac{d^2y}{dt^2} - 4\frac{dy}{dt}\right] = 2\left(\frac{dy}{dt} - 4y\right) + 3y$$

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 2\frac{dy}{dt} + 8y - 3y = 0$$

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 5y = 0$$

$$m^2 - 6m + 5 = 0$$

$$(m-1)(m-5) = 0$$

$$y_g = c_1 e^{x} + c_2 e^{5x}$$

$$\frac{dy}{dt} = c_1 e^{x} + 5c_2 e^{5x}$$

$$x = \left(c_1 e^{x} + 5c_2 e^{5x}\right) - 4\left(c_1 e^{x} + c_2 e^{5x}\right)$$

$$S_g \quad \boxed{\begin{aligned} x &= -3c_1 e^{x} + c_2 e^{5x} \\ y &= c_1 e^{x} + c_2 e^{5x} \end{aligned}} \qquad \boxed{\begin{aligned} \frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= x + 4y \end{aligned}}$$

$$\frac{dx}{dt} = 2x + 3y$$

$$\frac{dy}{dt} = x + 4y$$

$$x = -3c_1e^{x} + c_2e^{5x} \qquad y = c_1e^{x} + c_2e^{5x}$$

$$\frac{dx}{dt} = -3c_1e^{x} + 5c_2e^{5x} \qquad \frac{dy}{dt} = c_1e^{x} + 5c_2e^{5x}$$

$$\begin{array}{c|l}
\frac{dx}{dt} & -3c_1e^{x} + 5c_2e^{5x} \\
= & \\
2x & -6c_1e^{x} + 2c_2e^{5x} \\
+ & \\
3y & 3c_1e^{x} + 3c_2e^{5x}
\end{array}
\qquad
\begin{array}{c|l}
\frac{dy}{dt} & c_1e^{x} + 5c_2e^{5x} \\
= & \\
x & -3c_1e^{x} + c_2e^{5x} \\
+ & \\
4y & 4c_1e^{x} + 4c_2e^{5x}
\end{array}$$

$$\frac{dx}{dt} = 2x + 3y \qquad x(0) = 4$$

$$\frac{dy}{dt} = x + 4y \qquad y(0) = -3$$

SG $\Rightarrow$ $x = -3c_1e^{x} + c_2e^{5x}$

$y = c_1e^{x} + c_2e^{5x}$

SP $\Rightarrow$ $x = \frac{21}{4}e^{x} - \frac{5}{4}e^{5x}$

$y = -\frac{7}{4}e^{x} - \frac{5}{4}e^{5x}$

$$-3c_1 + c_2 = 4$$
$$-c_1 - c_2 = 3$$

$-4c_1 = 7 \quad c_1 = -\frac{7}{4}$

$c_2 = -3 - c_1$

$c_2 = -3 + \frac{7}{4} \Rightarrow c_2 = -\frac{5}{4}$

$$\frac{dx}{dt} = 2x + 3y$$
$$\frac{dy}{dt} = x + 4y$$
$$\Rightarrow \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad \frac{d}{dt}\bar{X} = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} \qquad \bar{X}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$\frac{d}{dt}\bar{X} = A \times \bar{X}$$

$$\bar{X} = \left[e^{At} \right] \times \bar{X}(0)$$

$$e^{At} \Rightarrow e^{At}\Big|_{t=0} = I \qquad e^{a(0)} = 1$$

$$\frac{d}{dt} e^{At} = A e^{At} \quad \left(\frac{de^{at}}{dt} = a e^{at} \right.$$

$$\left[e^{At} \right]^{-1} = e^{A(-t)}$$

$$\text{MatExp} = \begin{bmatrix} \frac{3}{4}e^{t} + \frac{1}{4}e^{5t} & -\frac{3}{4}e^{t} + \frac{3}{4}e^{5t} \\ -\frac{1}{4}e^{t} + \frac{1}{4}e^{5t} & \frac{1}{4}e^{t} + \frac{3}{4}e^{5t} \end{bmatrix} \Rightarrow \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$\Downarrow$$

$$\begin{bmatrix} \frac{3}{4} + \frac{1}{4} & -\frac{3}{4} + \frac{3}{4} \\ -\frac{1}{4} + \frac{1}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Bigg) \quad \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} 5e^{5t}$$

$$Ae^{At} = \frac{1}{4}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$= \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 5 & 15 \\ 5 & 15 \end{bmatrix} e^{5t}$$

$$= \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{5}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$e^{At} \Rightarrow \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}$$

$$\left[e^{At}\right]^{-1} \Rightarrow \frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{-t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{-5t}$$

$$\left(\frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{5t}\right) \times \left[\frac{1}{4}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{-t} + \frac{1}{4}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{-5t}\right)$$

$$\Big(\frac{1}{16}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}(1) + \frac{1}{16}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} e^{-4t} +$$

$$+ \frac{1}{16}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -1 & 1 \end{bmatrix} e^{4t} + \frac{1}{16}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}(1)\Big)$$

$$\Rightarrow \frac{1}{16}\left(\begin{bmatrix} 12 & -12 \\ -4 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 12 \\ 4 & 12 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} e^{-4t} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} e^{4t}\right)$$

$$\Rightarrow \frac{1}{16}\begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$e^{At} \times \left[e^{At}\right]^{-1} = I$$