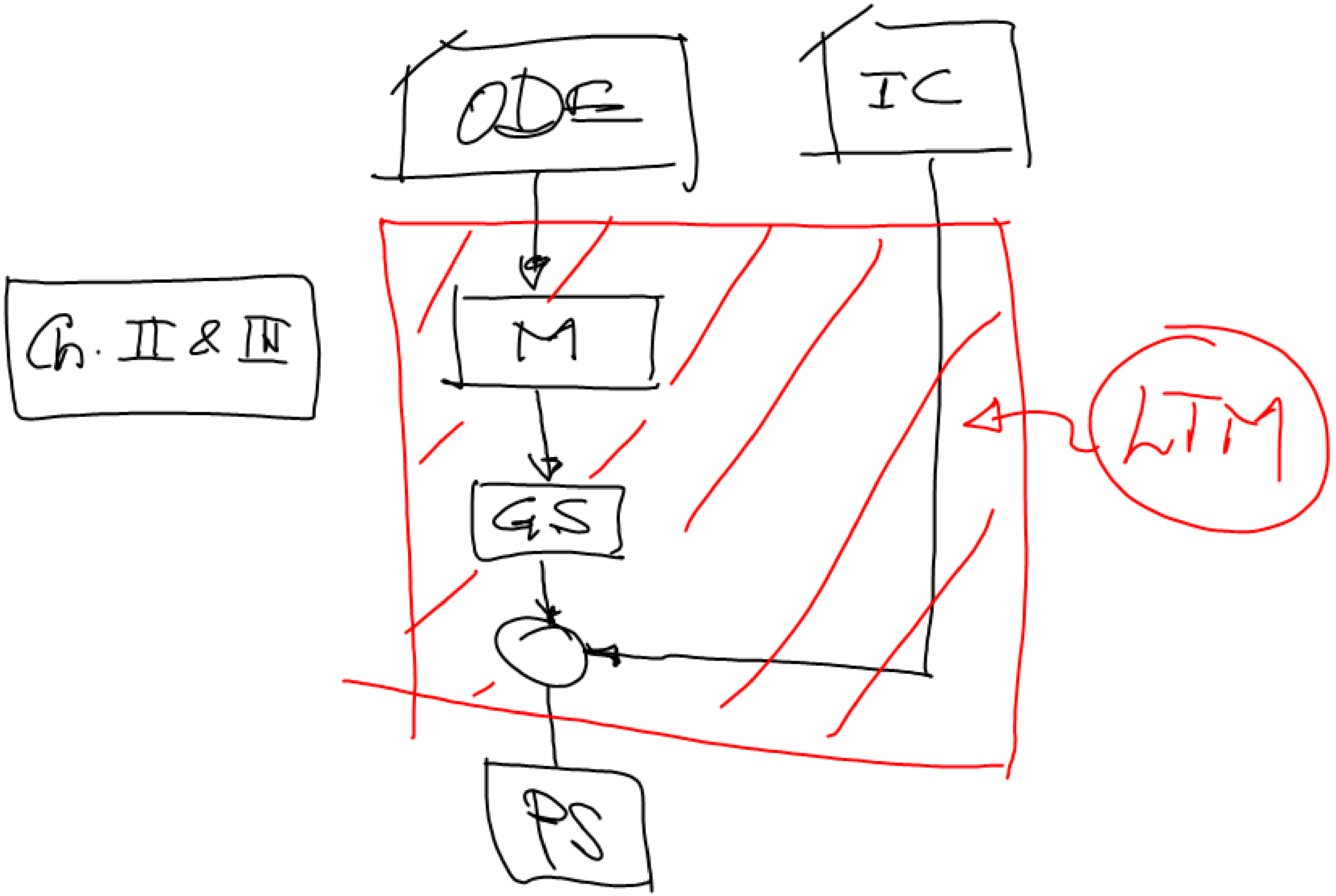
Chapter IV:

Laplace Transform as a method for solve initial condition differential Equation problems.

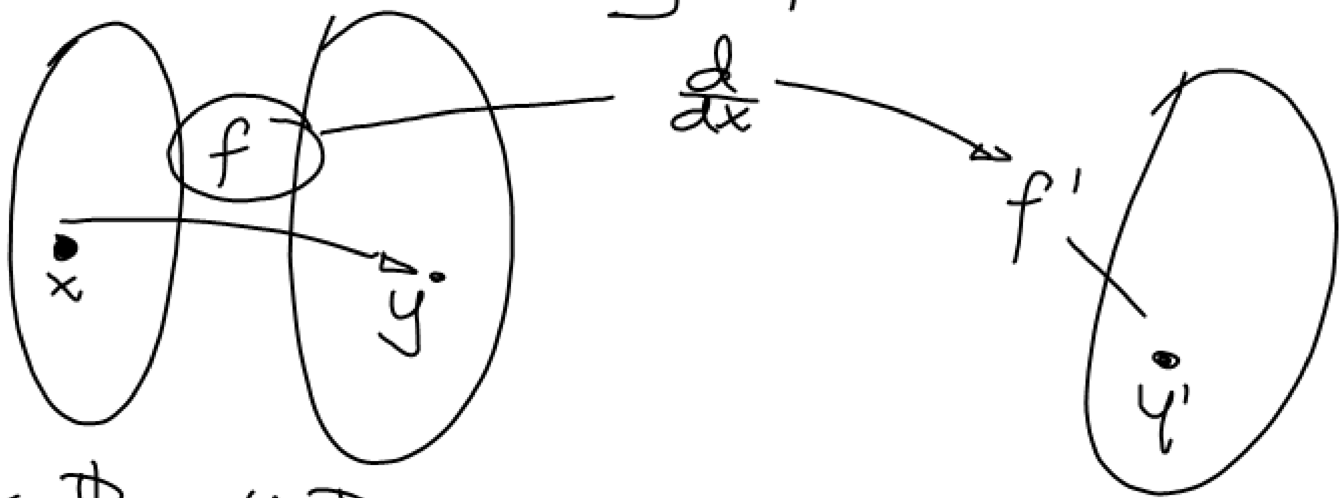
$$\boxed{\text{ODE} \to \text{I.C.}} \to \boxed{\text{LAPLACE TRANSFORM METHOD.}}$$


Ch. II & III
ODE
IC
M
GS
PS
LTM

$$x^2 \xrightarrow{\frac{d}{dx}} 2x$$

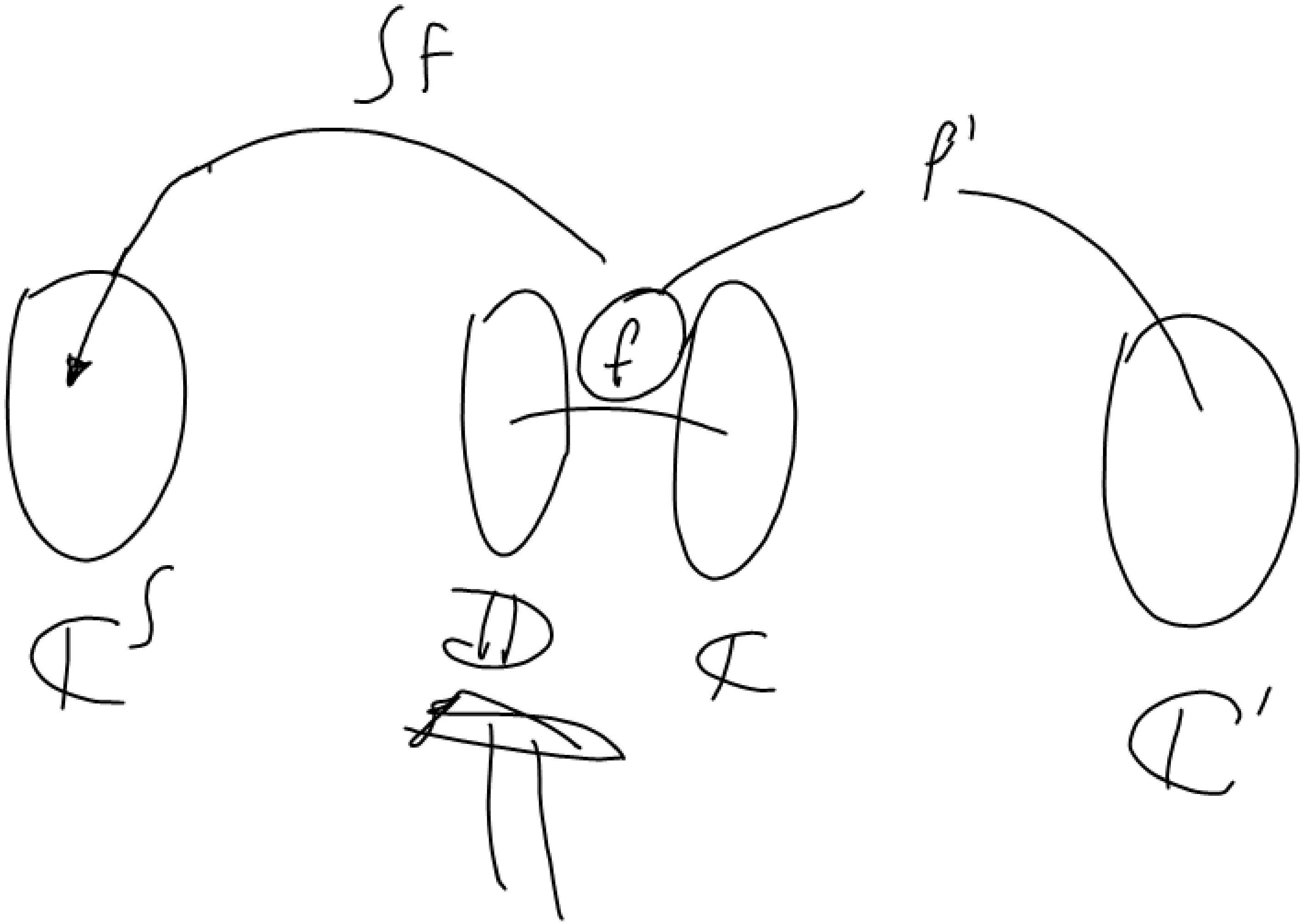
What does mean Transform?

$$y = f(x)$$



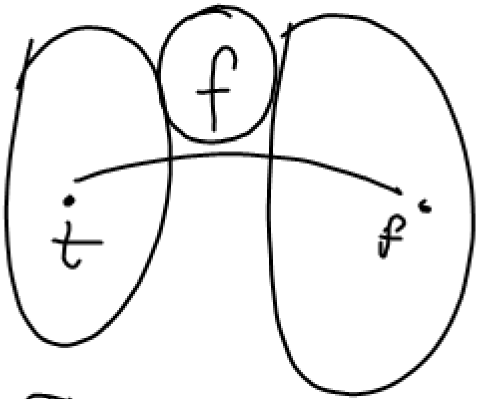
$x \in \mathbb{R}$ $\quad y \in \mathbb{R}$ $\quad y' \in \mathbb{R}$

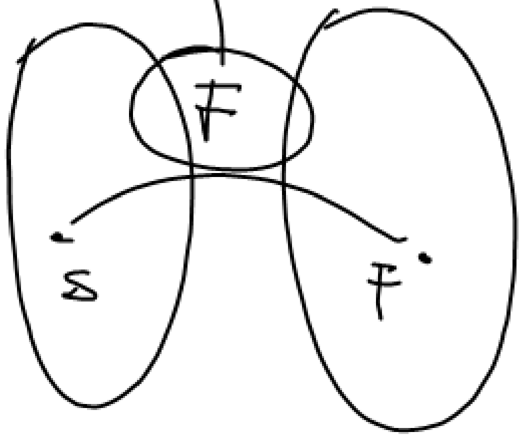
$\mathbb{D}$ $\quad \mathbb{C}$ $\quad \mathbb{C}$



$\int f$
f'
f
$\mathbb{C}^{\int}$
$\mathbb{D}$
$\mathbb{C}$
$\mathbb{C}'$

$$T\{f\} = F$$



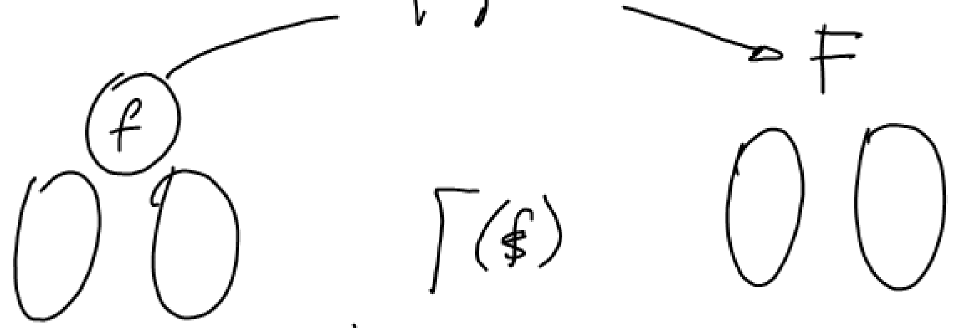


$t \in \mathbb{R}$ $\quad f \in \mathbb{R}$ $\qquad s \in \mathbb{C}$ $\quad F \in \mathbb{R}$

$\mathbb{D}$ $\quad \mathbb{C}$ $\qquad \mathbb{D}$ $\quad \mathbb{C}$

t $\qquad s = a \pm bi$

$T\{f\} = F$



$af + bg \xrightarrow{\text{Linnear Property}} aF + bG$

$a, b \in \mathbb{R}$

$f(t) \quad g(t)$

$T\{f(t)\} = F(s)$

$T\{g(t)\} = G(s)$

$F(s) \quad G(s)$

$\frac{d}{dt} f \longrightarrow sF(s) - f(0)$

$\int f\,dt \longleftarrow \frac{F(s)}{s}$

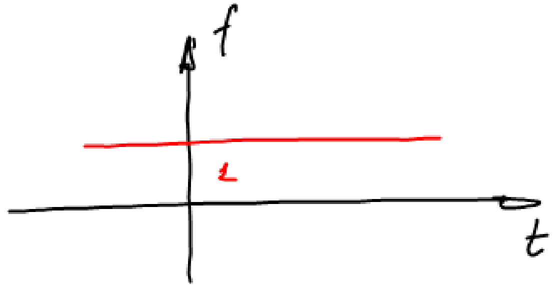
$$T\left\{f(t)\right\} = \int_{-\infty}^{\infty} N(t,s)\, f(t)\, dt \Rightarrow F(s)$$

Laplace Transform

$$N(t,s) = \begin{cases} 0 & ; t<0 \\ e^{-st} & ; t \geq 0 \end{cases}$$

$$\mathcal{L}\left\{f(t)\right\} = \int_{0}^{\infty} e^{-st} f(t)\, dt \Rightarrow F(s) \quad 5s$$

$$f(t) = 1$$



$$\mathcal{L}\{1\} = \int_0^\infty e^{-st}\cdot(1)\cdot dt$$

$$= \left[\int e^{-st}\,dt\right]_0^\infty \Rightarrow \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= -\frac{1}{s}\left(\lim_{b\to\infty} e^{-sb} - e^{-s(0)}\right)$$

$$= -\frac{1}{s}\left(0 - 1\right)$$

$$\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$$

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{-st} e^{at}\, dt$$

$$= \int_0^\infty e^{-(s-a)t}\, dt$$

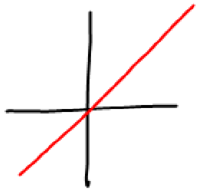
$$= \left[\int e^{-(s-a)t}\, dt \right]_0^\infty$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)} \right]_0^\infty$$

$$= -\frac{1}{s-a} \left(\lim_{b\to\infty} e^{-(s-a)b} - 1 \right) \quad \lim \to 0$$

$$\mathcal{L}\{e^{+at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{t\} = \int_0^\infty e^{-st}\, t\, dt$$



$$= \left[\int t e^{-st}\, dt\right]_0^\infty$$

$$= \left[-\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st}\right]_0^\infty$$

$$= -\frac{1}{s}\left(\lim_{t\to\infty} t e^{-st} - (0)\right) - \frac{1}{s^2}\left(\lim_{t\to\infty} e^{-st} - 1\right)$$

$$= -\frac{1}{s}\left(0 - 0\right) - \frac{1}{s^2}\left(0 - 1\right)$$

$$\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}}$$

Existent and unity Laplace Transform Theorem.

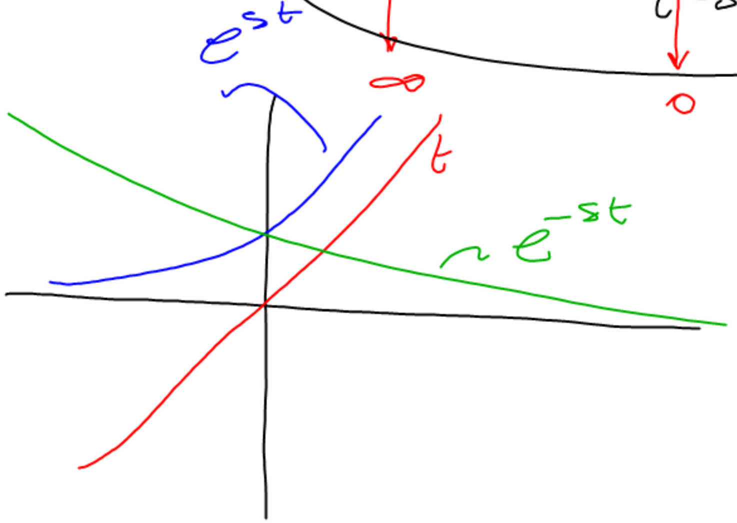
$$\int t e^{-st}\,dt = -\frac{1}{s} t e^{-st} + \frac{1}{s}\int e^{-st}\,dt$$

$$\boxed{\begin{array}{ll} u = t & dv = e^{-st}\,dt \\ du = dt & v = \dfrac{e^{-st}}{-s} \end{array}}$$

$$\int t e^{-st}\,dt = -\frac{1}{s} t e^{-st} + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]$$

$$= -\frac{1}{s} t e^{-st} - \frac{1}{s^2} e^{-st}$$

$$\lim_{t\to\infty} t e^{-st} = \underbrace{\lim_{t\to\infty} t \cdot \lim_{t\to\infty} e^{-st}}_{\infty \cdot 0} = 0$$



$$\lim_{b \to \infty} e^{-sb} = \lim_{b \to \infty} \frac{1}{e^{sb}}$$

$$= \lim_{a \to \infty} \frac{1}{a} = 0.$$