Laplace Transform

$$\mathcal{L}\{f'(t)\} = \int_0^\infty e^{-st} f'(t)\,dt$$

$$= \left[\int e^{-st} f'(t)\,dt\right]_0^\infty$$

$$\int e^{-st} f'(t)\,dt = f(t)e^{-st} + s\int e^{-st} f(t)\,dt$$

by parts method

$$u = e^{-st} \longrightarrow du = -se^{-st}\,dt$$

$$dv = f'(t)\,dt \longrightarrow v = f(t)$$

$$\int u\,dv = u\cdot v - \int v\,du$$

$$\mathcal{L}\{f'(t)\} = \left[f(t)e^{-st} + s\int e^{-st} f(t)\,dt \right]_0^\infty$$

$$\int_0^\infty e^{-st} f(t)\,dt = \mathcal{L}\{f(t)\}$$

$$= \left[f(t)\cdot e^{-st} \right]_0^\infty + s\int_0^\infty e^{-st} f(t)\,dt$$

$$\mathcal{L}\{f'(t)\} = \lim_{t\to\infty} \underset{\infty}{f(t)} \cdot \lim_{t\to\infty} \underset{0}{e^{-st}} - f(0)e^{-s(0)}$$

$$+ s\,\mathcal{L}\{f(t)\}$$

$$= (0) - f(0)(1) + s\,\mathcal{L}\{f(t)\}$$

$$\mathcal{L}\{f'(t)\} = s\,\mathcal{L}\{f(t)\} - f(0).$$

$$\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - s\cdot f(0) - f'(0)$$

$$\mathcal{L}\{f'''(t)\} = s^3\cdot\mathcal{L}\{f(t)\} - s^2\cdot f(0) - s\cdot f'(0) - f''(0)$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n\cdot\mathcal{L}\{f(t)\} - s^{n-1}\cdot f(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

$s \in \mathbb{C}$ $\qquad$ $t, f \in \mathbb{R}$

$F \in \mathbb{R}$

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{st} F(s)\,ds$$

$i = \sqrt{-1}$

$$\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y = 4e^{2t} \qquad \begin{matrix} y(0) = 4 \\ y'(0) = -3. \end{matrix}$$

Initial Condition Problem ODE(1) L cc. NH.

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 12y\right\} = \mathcal{L}\left\{4e^{2t}\right\}$$

5s

$$\mathcal{L}\left\{\frac{d^2y}{dt^2}\right\} - 7\mathcal{L}\left\{\frac{dy}{dt}\right\} + 12\mathcal{L}\{y\} = 4\mathcal{L}\left\{e^{2t}\right\}$$

$$\left[s^2\mathcal{L}\{y\} - s(4) - (-3)\right] - 7\left[s\mathcal{L}\{y\} - (4)\right] + 12\mathcal{L}\{y\} = 4\left(\frac{1}{s-2}\right)$$

$$(s^2 - 7s + 12)\mathcal{L}\{y\} - 4s + 31 = \frac{4}{s-2}$$

LT
ICP

$$(s^2 - 7s + 12)\mathcal{L}\{y\} = \frac{4}{s-2} + 4s - 31$$

$$= \frac{4 + (4s-31)(s-2)}{(s-2)}$$

$$(s^2 - 7s + 12)\mathcal{L}\{y\} = \frac{4s^2 - 39s + 66}{(s-2)}$$

LT · Ps

$$\boxed{\mathcal{L}\{y\} = \frac{4s^2 - 39s + 66}{(s-2)(s^2 - 7s + 12)}}$$

$$\mathcal{L}\{y\} = \frac{4s^2-39s+66}{(s-2)(s^2-7s+12)}$$

Partial Fractions Method

$$\frac{4s^2-39s+66}{(s-2)(s-3)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-3)} + \frac{D}{(s-4)}$$

$$4s^2-39s+66 = A(s-3)(s-4)+B(s-2)(s-4)+D(s-2)(s-3)$$

if $\boxed{s=2}$

$$4(2)^2-39(2)+66 = A(-1)(-2)+\cancel{B(0)(-2)}+\cancel{D(0)(-1)}$$

$$16-78+66 = 2A \qquad 4=2A \qquad \boxed{A=2}$$

if $\boxed{s=3}$

$$4(3)^2-39(3)+66 = \cancel{2(0)(-1)}+B(1)(-1)+\cancel{D(1)(0)}$$

$$36-117+66 = -B$$

$$-15=-B \qquad \boxed{B=15}$$

if. $\boxed{s=4}$

$$4(4)^2-39(4)+66 = D(2)(1)$$

$$64-156+66 = 2D$$

$$-26=2D \qquad \boxed{D=-13}$$

$$\mathcal{L}\{y\} = \frac{2}{(s-2)} + \frac{15}{s-3} - \frac{13}{s-4} \qquad y(0)=4$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{2}{s-2}+\frac{15}{s-3}-\frac{13}{s-4}\right\}$$

$$y(t) = 2\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} + 15\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} - 13\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$$\boxed{y(t) = 2e^{2t}+15e^{3t}-13e^{4t}}$$

Particular Solution of I.C.P.

$$y'(t) = 4e^{2t}+45e^{3t}-52e^{4t}$$

$$y'(0) = 4+45-52 \Rightarrow -3$$