Resolver el examen 2015-1-2
y enviarlo como "examen de casa"
para promediar con el realizado
el lunes 20. (fecha límite entrega
viernes 25 antes de las 23:59 hs.).

Existence and unicity of Laplace Transform. Theorem

Laplace Transform of $f(t)$ exist and is unique when

a) $f(t)$ is an exponential order function

$f(t)$ is

b) Sectional continous function

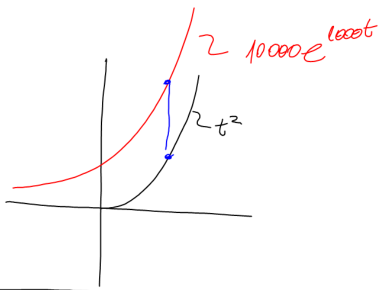
$f(t)$ is exponential order when

$$|f(t)| \leq Me^{At} \quad \text{where. } M, A \in \mathbb{R}$$

$0 < t < \infty$

$$\underbrace{\lim_{t\to\infty} f(t) \cdot \lim_{t\to\infty} e^{-st}}_{=0.}$$

$\Rightarrow e^{t^2} \longrightarrow e^{t^n} \quad n > 2$ don't exist Lap Transf. or there are not unique



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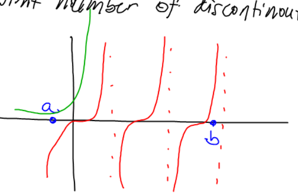
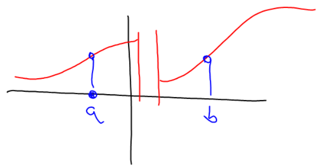
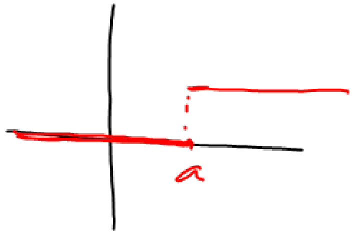
b) a $f(t)$ function is sectional continous when betwen $a \leq t < b$ there are a finit number of discontinouties.

$+ \begin{cases} \tan(t) \\ \cot(t) \\ \sec(t) \\ \csc(t) \end{cases}$





$$u(t-a) = \begin{cases} 0 \; ; & t<a \\ 1 \; ; & t>a \end{cases}$$



step function

Heaviside(t-a) (Maple)

$$\mathcal{L}\{u(t-a)\} = \int_0^\infty e^{-st}\cdot u(t-a)\,dt$$

$$= \int_0^a \cancelto{0}{e^{-st}\cdot(0)}\,dt + \int_a^\infty e^{-st}\cdot(1)\,dt$$

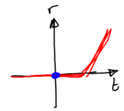
$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty \Rightarrow -\frac{1}{s}\left(\cancelto{0}{\lim_{t\to\infty} e^{-st}} - e^{-as}\right)$$

$$\boxed{\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

Rampa or slope functions

$$r(t-a)=\begin{cases} 0 \; ; & t<a \\ (t-a) \; ; & t>a \end{cases}$$



$$r(t-a)=(t-a)\cdot u(t-a)$$

$$\mathcal{L}\{r(t-a)\}=\frac{e^{-as}}{s^2}$$

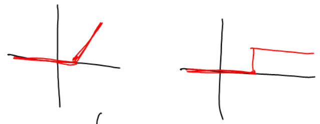
$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\}=s\,\mathcal{L}\{r(t-a)\}-r(t-a)\Big|_{t=0}$$

$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\}=s\left[\frac{e^{-ss}}{s^2}\right]-(0)$$

$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\}=\frac{e^{-ss}}{s}$$

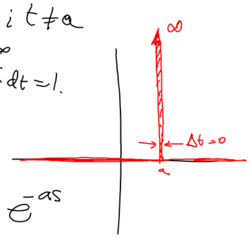
$$\mathcal{L}\left\{\frac{d}{dt}r(t-a)\right\}=\mathcal{L}\{u(t-a)\}$$

$$\frac{d}{dt}r(t-a)=u(t-a)$$



$$\delta(t-a)=\begin{cases} 0 \; ; & t\neq a \\ \int_{-\infty}^{\infty}\delta\,dt=1. \end{cases}$$

Dirac$(t-a)$



$$\mathcal{L}\{\mathrm{Dirac}(t-a)\}=e^{-as}$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\}=s\,\mathcal{L}\{u(t-a)\}-(0)$$

$$=\cancel{s}\left[\frac{e^{-as}}{\cancel{s}}\right]$$

$$=e^{-as}$$

$$\mathcal{L}\left\{\frac{d}{dt}u(t-a)\right\}=\mathcal{L}\{\delta(t-a)\}$$

$$\frac{d}{dt}u(t-a)=\delta(t-a).$$

Laplace Transform propierties

if: $\mathcal{L}\{f(t)\} = F(s)$ $\quad \mathcal{L}^{-1}\{F(s)\} = f(t)$

$\mathcal{L}\{g(t)\} = G(s)$ $\quad \mathcal{L}^{-1}\{G(s)\} = g(t)$

1) $\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s) \quad a, b \in \mathbb{R}$

2) $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

3) $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

4) $\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$

$$\mathcal{L}^{-1}\{F^{(n)}(s)\} = (-1)^n t^n f(t)$$

simple

5) $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{F(s)}{s}$

6) $\mathcal{L}^{-1}\left\{\int_s^\infty F(\sigma)\,d\sigma\right\} = \dfrac{f(t)}{t}$

7) $\mathcal{L}\{f(t-a)\cdot u(t-a)\} = e^{-as}F(s)$

8) $\mathcal{L}\{e^{bt}f(t)\} = F(s-b)$

9) $\mathcal{L}^{-1}\{F(s)\cdot G(s)\} = f(t) * g(t)$

convolution

$$f(t) * g(t) = \int_0^t f(\tau)\,g(t-\tau)\,d\tau.$$

$$\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

$$\mathcal{L}\{e^{t}\} = \frac{1}{s-1} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\downarrow$$

$$\mathcal{L}\{e^{3t}\} = \frac{1}{3}\left(\frac{1}{\left(\frac{s}{3}\right)-1}\right)$$

$$= \frac{1}{3}\left(\frac{1}{\frac{s-3}{3}}\right)$$

$$= \frac{1}{3}\left(\frac{3}{s-3}\right)$$

$$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{t\} = \frac{1}{s^2} \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{t^2\} = \frac{2!}{s^3} \Rightarrow \frac{2}{s^3}$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} \Rightarrow \frac{6}{s^4}$$

$$\mathcal{L}\{t^4\} = \frac{4!}{s^5} \Rightarrow \frac{24}{s^5}$$

$$\mathcal{L}\{t e^{2t}\} =$$

$$\Rightarrow \mathcal{L}\{e^{at} f(t)\} = F(s-a)$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{2t} \cdot 1\} = \frac{1}{s-2}$$

$$\mathcal{L}\{t\} = \frac{1}{s^2} \qquad \mathcal{L}\{t e^{2t}\} = \frac{1}{(s-2)^2}$$

$$\mathcal{L}\{t^2\} = \frac{2}{s^3} \qquad \mathcal{L}\{t^2 e^{2t}\} = \frac{2}{(s-2)^3}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2}$$

$$\mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2}$$

$$\mathcal{L}\{e^{at}\cos(bt)\} = \frac{(s-a)}{(s-a)^2+b^2}$$

$$\mathcal{L}\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2+b^2}$$