Por excepción

Viernes 14: Tercer examen parcial

que Cap. IV (Laplace) y V (Parciales).

en el J205A.

Hoy Serie 5 para entrega miércoles 19.

Exentar con promedio de 6.0.

$$\frac{\partial^2 z(x,y)}{\partial x^2} = 5\frac{\partial z}{\partial y}$$

Método de Separación Variables
(prueba y error).

Hipótesis inicial = $z(x,y) = F(x)\,G(y)$

$$z(x,y) = F(x) + G(y)$$

$$z(x,y) = F(x)^y$$

$$z(x,y) = G(y)^x$$

$$\frac{\partial^2 z}{\partial x^2} = 5\frac{\partial z}{\partial y} \qquad H_0 \quad z = F(x)\cdot G(y)$$

$$\frac{\partial z}{\partial x} = F'(x)\cdot G(y) \longrightarrow \frac{\partial^2 z}{\partial x^2} = F''(x)\cdot G(y)$$

$$\frac{\partial z}{\partial y} = F(x)\, G'(y)$$

$$F''(x)\cdot G(y) = 5F(x)\,G'(y)$$

$$\frac{F''(x)}{5F(x)} = \frac{G'(y)}{G(y)}$$

Se logró separar las variables

"Razones y Proporciones"

razón $= \alpha$

$$\frac{F''(x)}{5F(x)} = \alpha \qquad \frac{G'(y)}{G(y)} = \alpha \qquad \begin{cases} \alpha = 0 \\ \alpha < 0 \\ \alpha > 0 \end{cases}$$

$$\frac{F''(x)}{5F(x)} = \alpha \qquad \frac{G'(y)}{G(y)} = \alpha$$

$\alpha = 0$

$$\frac{F''(x)}{5F(x)} = 0 \qquad F(x) \neq 0 \qquad F''(x) = 0$$

$$F'(x) = k_1$$

$$\frac{G'(y)}{G(y)} = 0 \qquad G(y) \neq 0 \qquad F(x) = k_1 x + k_2$$

$$G'(y) = 0 \rightarrow G(y) = C_1$$

$$Z(x,y) = (k_1 x + k_2) C_1 \Rightarrow \boxed{Z(x,y) = C_{10} x + C_{20}}$$

$$\frac{\partial^2 z}{\partial x^2} = 5 \frac{\partial z}{\partial y} \qquad \frac{\partial z}{\partial x} = C_{10} \qquad \frac{\partial^2 z}{\partial x^2} = 0$$

$$\frac{\partial z}{\partial y} = 0$$

$$(0) = 5(0)$$

$$\underline{0 = 0.}$$

$$\frac{\partial^2 z}{\partial x^2} = 5\frac{\partial z}{\partial y} \qquad z = F\cdot G.$$

$$\frac{F''(x)}{5F(x)} = \alpha \qquad \frac{G'(y)}{G(y)} = \alpha$$

para $\alpha < 0$ $\qquad \alpha = -\beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''(x)}{5F(x)} = -\beta^2 \qquad F''(x) = -5\beta^2 F(x)$$

$$F''(x) + 5\beta^2 F(x) = 0 \quad \text{EDO (2) Lcc H.}$$

$$m^2 + 5\beta^2 = 0 \qquad m = \pm\sqrt{5}\beta i$$

$$F(x) = k_1 \cos(\sqrt{5}\beta x) + k_2 \operatorname{sen}(\sqrt{5}\beta x)$$

$$\frac{G'(y)}{G(y)} = -\beta^2 \qquad G'(y) = -\beta^2 G(y) \qquad m = -\beta^2$$

$$G'(y) + \beta^2 G(y) = 0 \quad \text{EDO(1) Lcc H}$$

$$G(y) = C_1 e^{-\beta^2 y}$$

$$Z(x,y) = \left(k_1 \cos(\sqrt{5}\beta x) + k_2 \operatorname{sen}(\sqrt{5}\beta x)\right) C_1 e^{-\beta^2 y}$$

$$\boxed{Z(x,y) = C_{10} e^{-\beta^2 y} \cos(\sqrt{5}\beta x) + C_{20} e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x)}$$

$$\frac{\partial z}{\partial x} = C_{10} e^{-\beta^2 y}\left(-\operatorname{sen}(\sqrt{5}\beta x)\sqrt{5}\beta\right) + C_{20} e^{-\beta^2 y}\left(\cos(\sqrt{5}\beta x)\sqrt{5}\beta\right)$$

$$\frac{\partial z}{\partial x} = -\sqrt{5}\beta C_{10} e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x) + \sqrt{5}\beta C_{20} e^{-\beta^2 y} \cos(\sqrt{5}\beta x).$$

$$\frac{\partial^2 z}{\partial x^2} = -5\beta^2 C_{10} e^{-\beta^2 y} \cos(\sqrt{5}\beta x) - 5\beta^2 C_{20} e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x).$$

$$\frac{\partial z}{\partial y} = -C_{10}\beta^2 e^{-\beta^2 y} \cos(\sqrt{5}\beta x) - C_{20}\beta^2 e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x)$$

$$\frac{\partial^2 z}{\partial x^2} = 5\frac{\partial z}{\partial y}$$

$$\left(-5\beta^2 C_{10} e^{-\beta^2 y} \cos(\sqrt{5}\beta x) - 5\beta^2 C_{20} e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x)\right) =$$

$$5\left(-\beta^2 C_{10} e^{-\beta^2 y} \cos(\sqrt{5}\beta x) - C_{20}\beta^2 e^{-\beta^2 y} \operatorname{sen}(\sqrt{5}\beta x)\right).$$

$$0 \equiv 0.$$

para $\alpha > 0$ $\quad \alpha = \beta^2 \quad \forall \beta \neq 0 \in \mathbb{R}$

$$\frac{F''(x)}{5F(x)} = \beta^2 \qquad F''(x) = 5\beta^2 F(x)$$

$F''(x) - 5\beta^2 F(x) = 0$ EDO(2)Lcc H.

$$m^2 - 5\beta^2 = 0 \qquad (m - \sqrt{5}\beta)(m + \sqrt{5}\beta) = 0$$

$$m_1 = \sqrt{5}\beta \qquad m_2 = -\sqrt{5}\beta.$$

$$F(x) = k_1 e^{\sqrt{5}\beta x} + k_2 e^{-\sqrt{5}\beta x}$$

$$\frac{G'(y)}{G(y)} = \beta^2 \qquad G(y) = C_1 e^{\beta^2 y}$$

$$Z(x,y) = \left(k_1 e^{\sqrt{5}\beta x} + k_2 e^{-\sqrt{5}\beta x}\right) C_1 e^{\beta^2 y}$$

$$\boxed{Z(x,y) = C_{10} e^{(\beta^2 y + \sqrt{5}\beta x)} + C_{20} e^{(\beta^2 y - \sqrt{5}\beta x)}.}$$

$$\frac{\partial^2 y}{\partial t^2} + 4\frac{\partial y}{x} = y$$

$$y(x,t) = F(x)\cdot G(t)$$

$$\frac{\partial y}{\partial t} = F\cdot G' \qquad \frac{\partial y}{\partial x} = F'G$$

$$\frac{\partial^2 y}{\partial t^2} = F\cdot G''$$

$$F\cdot G'' + 4F'G = FG \qquad FG'' - FG = 4F'G$$

$$F\cdot G'' = FG - 4F'G \qquad F(G'' - G) = 4F'G$$

$$= (F - 4F')G \qquad \frac{G'' - G}{G} = 4\frac{F'}{F}$$

$$\frac{G''}{G} = \frac{F - 4F'}{F}$$