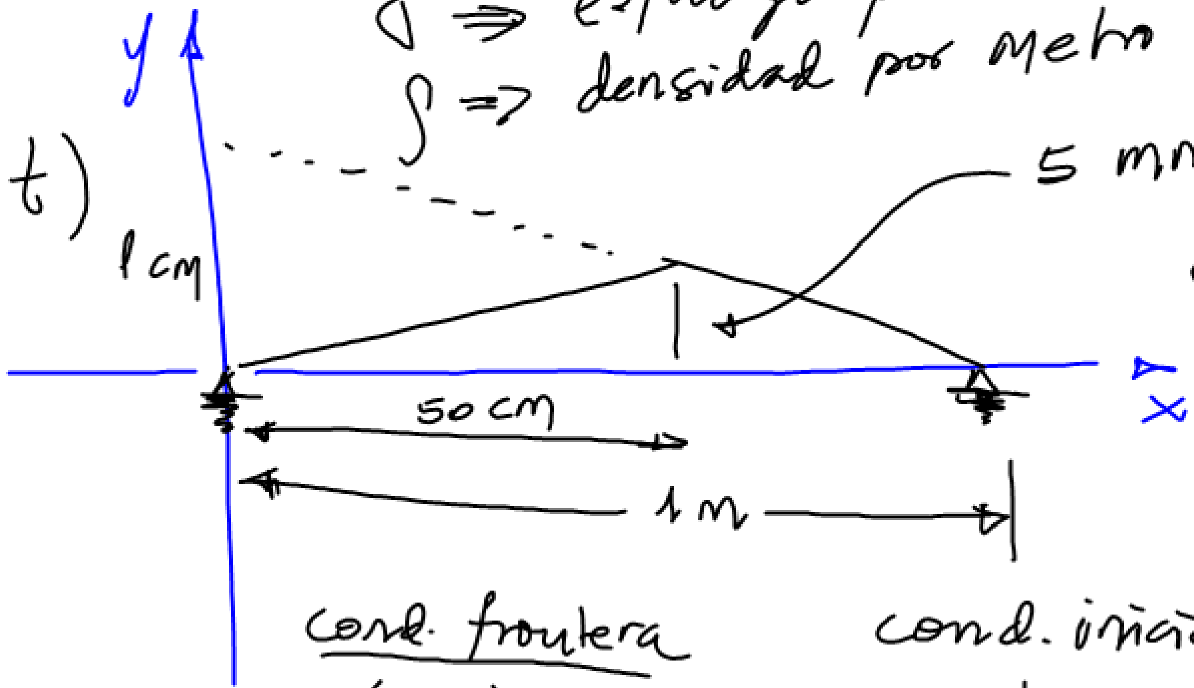
Problema de la Cuerda de Guitarra

$\sigma \Rightarrow$ esfuerzo por metro

$\rho \Rightarrow$ densidad por metro

$$\frac{\sigma}{\rho} = R^2$$

$y(x,t)$



$$\frac{\partial^2 y}{\partial t^2} = R^2 \frac{\partial^2 y}{\partial x^2}$$

cond. frontera

$y(0,t) = 0$

$y(1,t) = 0$

cond. iniciales tiempo

$$y(x,0) = \begin{cases} \frac{0.005}{0.5}x \ ; & 0 \le x < 0.5 \\ 0.01 - \frac{0.005}{0.5}x \ ; & 0.5 \le x \le 1 \end{cases}$$

$$v(0) = \left.\frac{\partial y}{\partial t}\right|_{t=0} = 0$$

$$\frac{\partial^2 y}{\partial t^2} = R^2 \frac{\partial^2 y}{\partial x^2}$$

$$y(x,t) = F(x) \cdot G(t)$$

$$\frac{\partial^2 y}{\partial t^2} = F G'' \qquad \frac{\partial^2 y}{\partial x^2} = F'' G$$

$$F G'' = R^2 F'' G$$

$$\frac{G''}{R^2 G} = \frac{F''}{F}$$

$$\frac{F''}{F} = \alpha$$

$$\alpha = 0$$

$$F'' = 0$$

$$F' = C_1$$

$$\boxed{F = C_1 x + C_2}$$

$$\left.\begin{array}{l} y(0,t)=0 \\ y(1,t)=0 \end{array}\right\} \quad \begin{array}{l} F(0)\,G(t)=0 \\ F(1)\,G(t)=0 \end{array} \qquad \begin{array}{l} G(t)\neq 0 \\ \forall t\in\mathbb{R}^+ \end{array}$$

$$y(x,t)=F(x)\,G(t)$$

$$\left.\begin{array}{l} F(0)=0 \\ F(1)=0 \end{array}\right\} \quad \underset{\alpha=0}{F(x)}=C_1x+C_2$$

$$C_1(0)+C_2=0 \longrightarrow \boxed{C_2=0}$$

$$C_1(1)+(0)=0 \longrightarrow \boxed{C_1=0}$$

$$\underset{\alpha=0}{F(x)}=0 \quad \forall x$$

$$\frac{G''}{R^2 G} = \frac{F''}{F} \qquad \alpha > 0 \qquad \frac{F''}{F} = \beta^2$$

$$F'' = \beta^2 F \qquad F'' - \beta^2 F = 0 \quad \text{EDO}(2)\,\text{LCCH}.$$

$$\boxed{F(x) = C_1 e^{\beta x} + C_2 e^{-\beta x}} \qquad m^2 - \beta^2 = 0 \qquad m_1 = \beta$$

$$(m-\beta)(m+\beta) = 0 \qquad m_2 = -\beta$$

$$\left.\begin{array}{l} F(0) = 0 \\ F(1) = 0 \end{array}\right\} \quad C_1 e^{\beta(0)} + C_2 e^{-\beta(0)} = 0$$

$$\boxed{F(x) = C_1 e^{\beta x} - C_1 e^{-\beta x}} \quad C_1 + C_2 = 0 \longrightarrow C_2 = -C_1$$

$$C_1 e^{\beta} - C_1 e^{-\beta} = 0 \qquad C_1 e^{\beta} = +C_1 e^{-\beta}$$

$$C_1 e^{\beta} = \frac{C_1}{e^{\beta}}$$

$$e^{2\beta} = \frac{C_1}{C_1}$$

$$e^{2\beta} = 1 \qquad \beta = 0$$

$$\frac{F''}{F}=\alpha \qquad \alpha<0 \qquad F''=-\beta^2 F$$

$F''+\beta^2 F=0$ EDO(2)LCCH.

$m^2+\beta^2=0 \qquad m=\pm\beta i$

$$\boxed{F(x)=C_1\cos(\beta x)+C_2\operatorname{sen}(\beta x)}$$

$F(0)=0$
$F(1)=0 \qquad C_1\cos(0)+C_2\operatorname{sen}(0)=0 \qquad C_1=0$

$F(x)=C_2\operatorname{sen}(\beta x)$

$C_2\operatorname{sen}(\beta\cdot(1))=0 \qquad \operatorname{sen}(\beta)=0 \qquad C_2\neq 0$

$\beta=n\pi \qquad n=1,\dots,\infty$

$$\boxed{F(x)=C_2\operatorname{sen}(n\pi x)}$$

$$\frac{G''}{R^2G}=-n^2\pi^2$$

$G''=-n^2\pi^2R^2G.$

$G''+n^2\pi^2R^2G=0$ EDO(2)LCCH.

$m^2+n^2\pi^2R^2=0 \qquad m=\pm n\pi R i$

$$\boxed{G(t)=k_1\cos(n\pi Rt)+k_2\operatorname{sen}(n\pi Rt)}$$

$y(x,t)=C_1\operatorname{sen}(n\pi x)\left(k_1\cos(n\pi Rt)+k_2\operatorname{sen}(n\pi Rt)\right)$

$y(x,t)=\operatorname{sen}(n\pi x)\left(C_{10}\cos(n\pi Rt)+C_{20}\operatorname{sen}(n\pi Rt)\right)$

$$\boxed{y(x,t)=\sum_{n=1}^{\infty}\operatorname{sen}(n\pi x)\left(b_n\cos(n\pi Rt)+a_n\sin(n\pi Rt)\right)}$$

$$y(x,0) = \begin{cases} \frac{0.005}{0.5}x & ; \ 0 \leq x \leq 0.5 \\ 0.01 - \frac{0.005}{0.5} & ; \ 0.5 < x \leq 1. \end{cases}$$

$$V(0) \Rightarrow \frac{\partial y}{\partial t}\Big|_{t=0} = 0.$$

$$y(x,t) = \sum_{n=1}^{\infty} \operatorname{sen}(n\pi x)\left(b_n \cos(n\pi R t) + a_n \operatorname{sen}(n\pi R t)\right)$$

$$y(x,0) = \sum_{n=1}^{\infty} \operatorname{sen}(n\pi x)\left(b_n + a_n(0)\right)$$

$$y(x,0) = \sum_{n=1}^{\infty} b_n \operatorname{sen}(n\pi x) = \begin{cases} \frac{0.005}{0.5}x & 0 \leq x < 0.5 \\ 0.01 - \frac{0.005}{0.5}x & 0.5 \leq x \leq 1. \end{cases}$$

$$L = 0.5.$$

$$\boxed{a_n = 0} \quad \frac{\partial y}{\partial t}\Big|_{t=0} = 0$$