



$$\boxed{\frac{dU}{dt} = -k U^2}$$

$$U_0(0) = 200 \frac{m}{s}$$

$$x_0(0) = 0 \quad m$$

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$$U(t_f) = 20 \frac{m}{s}$$

$$x(t_f) = 0.10 \quad m$$

$$\frac{dU}{dt} = k U^2$$

$$\frac{dU}{U^2} = k dt$$

Método  
Variables  
Separables.

$$\int \frac{dU}{U^2} = k \int dt$$

$$\int U^{-2} dU = kt + C_1$$

$$\frac{U^{-1}}{-1} = kt + C_1$$

$$U(0) = 200$$

$$-\frac{1}{U} = kt + C_1$$

$$U = -\frac{1}{(kt + C_1)}$$

Solución  
General

$$200 = -\frac{1}{k(0) + C_1}$$

$$200 = -\frac{1}{C_1}$$

$$C_1 = -\frac{1}{200}$$

$$U(t) = -\frac{1}{kt - \frac{1}{200}}$$

Solución  
Particular

$$V(t) = \frac{1}{\frac{1}{200} - kt}$$

$$\frac{dX}{dt} = \frac{1}{\frac{1}{200} - kt}$$

$$dx = \frac{dt}{-kt + \frac{1}{200}} \quad \text{Variables separables}$$

$$\int dx = \int \frac{dt}{-kt + \frac{1}{200}}$$

$$X(0) = 0 \quad \int dx = -\frac{1}{k} \int \frac{-k dt}{-kt + \frac{1}{200}}$$

$$X(t) = -\frac{1}{k} \ln\left(-kt + \frac{1}{200}\right) + C_2$$

$$0 = -\frac{1}{k} \ln\left(-k(0) + \frac{1}{200}\right) + C_2$$

$$0 = -\frac{1}{k} \ln\left(\frac{1}{200}\right) + C_2$$

$$0 = -\ln\left(\frac{1}{200}\right)^{\frac{1}{k}} + C_2$$

$$\underline{C_2 = \ln\left(\frac{1}{200}\right)^{\frac{1}{k}}}$$

$$X(t) = \ln\left(-kt + \frac{1}{200}\right)^{-\frac{1}{k}} + \ln\left(\frac{1}{200}\right)^{\frac{1}{k}}$$

$$X(t) = \ln\left(\frac{\frac{1}{200}}{-kt + \frac{1}{200}}\right)^{\frac{1}{k}}$$

$$e^X = \left(\frac{\frac{1}{200}}{-kt + \frac{1}{200}}\right)^{\frac{1}{k}}$$

$$e^{kX} = \frac{\frac{1}{200}}{-kt + \frac{1}{200}}$$

$$e^{kX} = \frac{1}{-200kt + 1}$$

$$\frac{1}{e^{kX}} = -200kt + 1$$

$$\frac{1}{e^{kX}} - 1 = -200kt$$

$$t = \frac{\frac{1}{e^{kX}} - 1}{-200k}$$

$$\boxed{t_f = \frac{\frac{1}{e^{\frac{k}{2}}} - 1}{-200k}}$$